

Partial Differential Equations 2 – Fall 2019  
Exercise Sheet 6 — Due Date: Oct 29

Work in groups, write in L<sup>A</sup>T<sub>E</sub>X!

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**Problem 12** Prove Grönwall's inequality in the following version:

Let  $\eta$  be *nonnegative*, absolutely continuous function on  $[0, T]$  such that for almost every  $t$ ,

$$\eta'(t) \leq \phi(t)\eta(t) + \psi(t),$$

for  $\phi(t)$  and  $\psi(t)$  nonnegative, integrable functions on  $[0, T]$ . Then

$$\eta(t) \leq \exp\left(\int_0^t \phi(s) ds\right)\left(\eta(0) + \int_0^t \psi(s) ds\right).$$

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