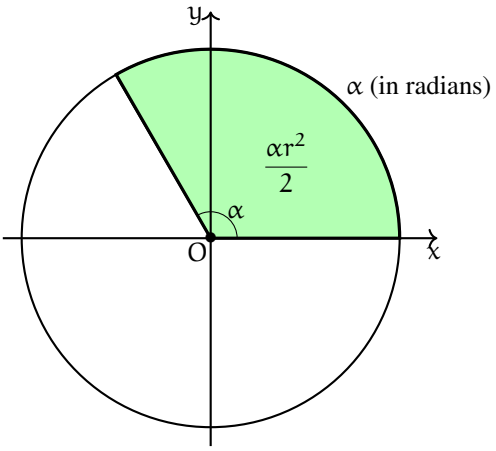
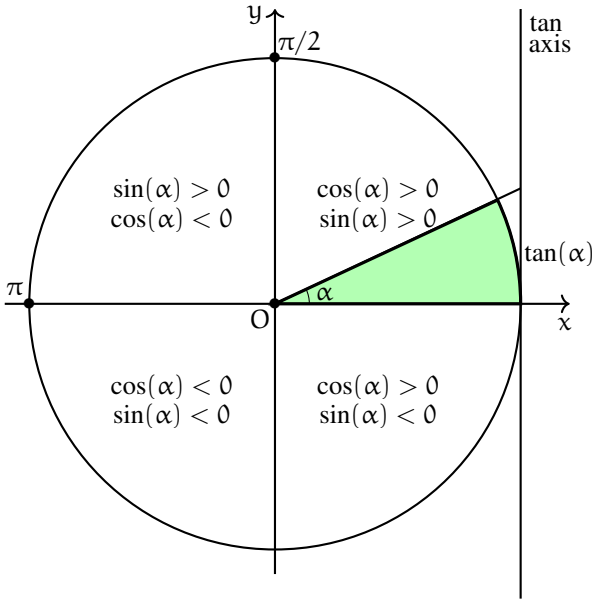
MATH 0200: Preparation for Scientific Calculus

Cheat Sheet

Arc Length and Circular Sector Area



Trigonometry



α	$\sin(\alpha)$	$\cos(\alpha)$	$\tan(\alpha)$
$0°$	0	1	0
$\pi/6 = 30°$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
$\pi/4 = 45°$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3 = 60°$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2 = 90°$	1	0	DNE

Basic values of trigonometric functions

- $\sin^2(a) + \cos^2(a) = 1$
- $\tan(a) = \dfrac{\sin(a)}{\cos(a)}, \quad \cot(a) = \dfrac{1}{\tan(a)}, \quad \sec(a) = \dfrac{1}{\cos(a)}, \quad \csc(a) = \dfrac{1}{\sin(a)}$
- $\sin(\pi/2 - a) = \cos(a), \quad \cos(\pi/2 - a) = \sin(a)$
- $\sin(-a) = -\sin(a), \quad \cos(-a) = \cos(a)$
- $\sin(a + b) = \sin(a)\cos(b) + \sin(b)\cos(a)$
- $\sin(a - b) = \sin(a)\cos(b) - \sin(b)\cos(a)$
- $\sin(2a) = 2\sin(a)\cos(a)$
- $\cos(a + b) = \cos(a)\cos(b) - \sin(b)\sin(a)$
- $\cos(a - b) = \cos(a)\cos(b) + \sin(b)\sin(a)$
- $\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$
- $\cos(a/2) = \sqrt{\dfrac{1 + \cos(a)}{2}}, \quad \sin(a/2) = \sqrt{\dfrac{1 - \cos(a)}{2}}$
- $\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$
- $1 + \tan^2(a) = \sec^2(a)$

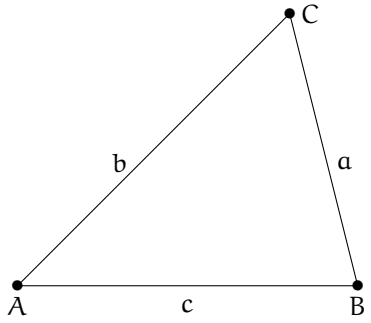
Inverse Trigonometric Functions

$f(x)$	Domain	Range
$\arcsin(x)$	$[-1, 1]$	$[-\pi/2, \pi, 2]$
$\arccos(x)$	$[-1, 1]$	$[0, \pi]$
$\arctan(x)$	$(-\infty, \infty)$	$(-\pi/2, \pi, 2)$

x	$\arcsin(x)$	$\arccos(x)$	$\arctan(x)$
0	0	$\pi/2$	0
1	$\pi/2$	0	$\pi/4$
-1	$-\pi/2$	π	$-\pi/4$

- $\arcsin(x) + \arccos(x) = \pi/2$
- $\cos(\arcsin(x)) = \sqrt{1 - x^2}, \quad \sin(\arccos(x)) = \sqrt{1 - x^2}$
- $\cos(\arctan(x)) = \dfrac{1}{\sqrt{1 + x^2}}, \quad \sin(\arctan(x)) = \dfrac{x}{\sqrt{1 + x^2}}$

Laws of Sines and Cosines



- $\dfrac{a}{\sin(A)} = \dfrac{b}{\sin(B)} = \dfrac{c}{\sin(C)}$
- $c^2 = a^2 + b^2 - 2ab\cos(C)$

Polar Coordinates $\leftrightarrow$ Cartesian Coordinates

- $(x, y) \to (\sqrt{x^2 + y^2}, \arctan(y/x))$ if $x > 0$
- $(x, y) \to (\sqrt{x^2 + y^2}, \pi + \arctan(y/x))$ if $x < 0$
- $(0, y) \to (y, \pi/2)$ if $y > 0$
- $(0, y) \to (y, 3\pi/2)$ if $y < 0$
- $(r, \varphi) \to (r\cos(\varphi), r\sin(\varphi))$

Dot Product

$\mathbf{v} = (a, b), \mathbf{w} = (c, d)$, then $\mathbf{v} \cdot \mathbf{w} = ac + bd = |\mathbf{v}||\mathbf{w}|\cos(\mathbf{v}, \mathbf{w})$

Complex numbers

- $i^2 = -1$
- $r = |a + bi| = \sqrt{a^2 + b^2}$
- Polar form: $a + bi = r(\cos(\varphi) + i\sin(\varphi))$
- $\overline{a + bi} = a - bi$

Properties of Logarithms

(1) $\ell\log_a(bc) = \ell\log_a(b) + \ell\log_a(c)$

(2) $\ell\log_a(b/c) = \ell\log_a(b) - \ell\log_a(c)$

(3) $\ell\log_a(b^c) = c\ell\log_a(b)$

(4) $\ell\log_a(b) = \dfrac{\ell\log_c(b)}{\ell\log_c(a)}$.