Lecture 12

Exponential functions and logarithms

MATH 0200

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Outline

Exponential functions

Definition

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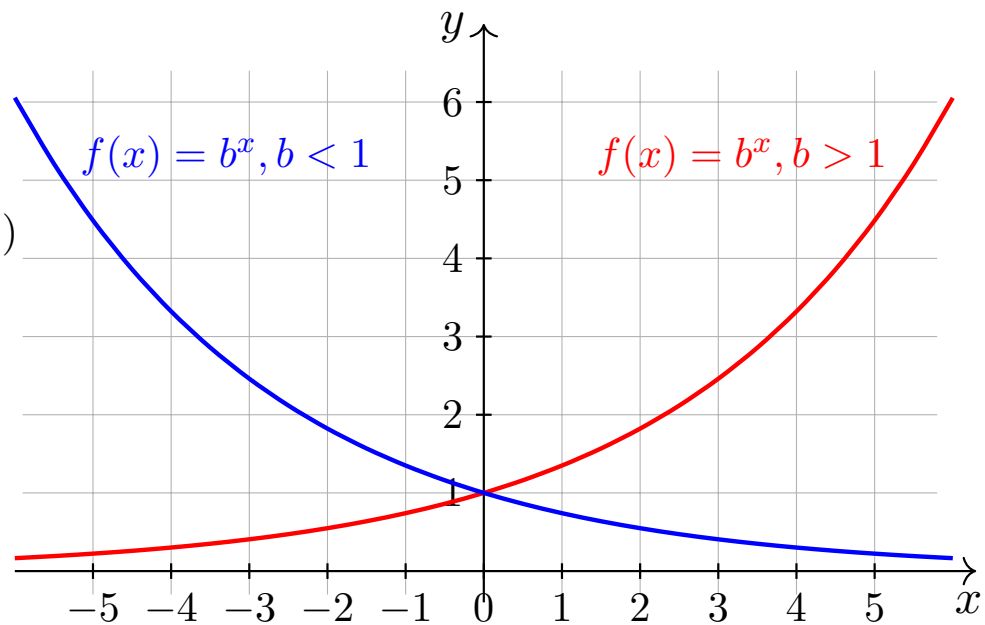
Range: $(0, \infty)$



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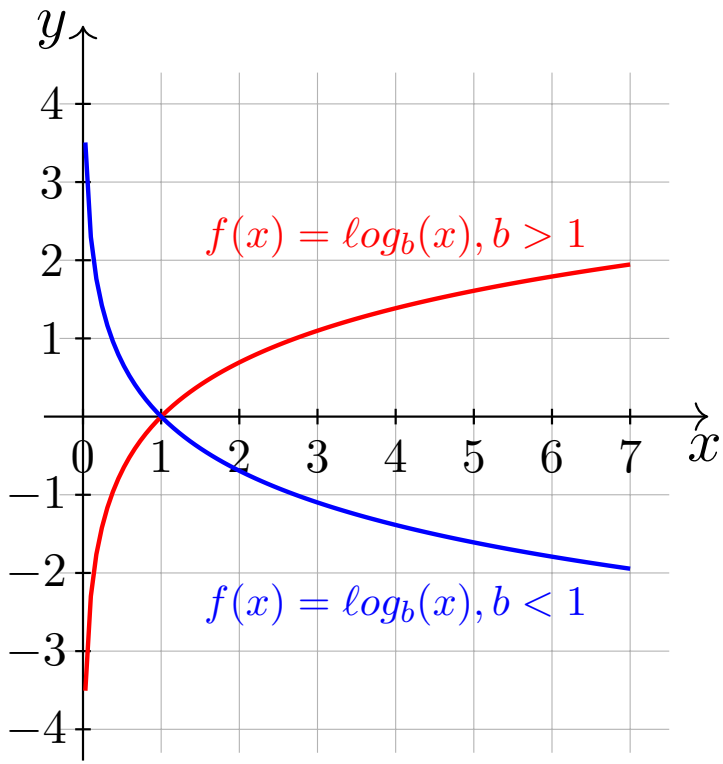

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- Find a number t with $7^{5-t} = 3$.
  $7^{5-t} = 3 \Leftrightarrow 5 - t = \ell og_7(3) \Leftrightarrow t = 5 - \ell og_7(3)$.

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Answer: $5^{t+4} = \dfrac{1}{25} \Leftrightarrow t+4 = \ell og_5\left(\dfrac{1}{25}\right) = \ell og_5(5^{-2}) \Leftrightarrow$
$t+4 = -2 \Leftrightarrow t = -2-4 = -6.$

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2. $\ell og_b(b) = 1$ for any $b > 0, b \neq 1$;
3. The logarithmic and exponential functions are inverse:

$$\ell og_b(b^x) = b^{\ell og_b(x)} = x.$$