#### Lecture 16

MATH 0200

Unit circle

angles

Negative

Radian

Kadian

Special points on the unit circle

Length of a circular arc and area of

# Lecture 16 The unit circle

MATH 0200

Dr. Boris Tsvelikhovskiy

# Outline

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- 1 Unit circle
- 2 Positive angles
- 3 Negative angles
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- 5 Special points on the unit circle
- 6 Length of a circular arc and area of a sector

# Unit circle

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### Definition

The unit circle is the circle with radius 1 centered at the origin.

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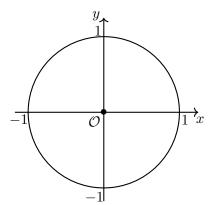
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Special points on the unit circle

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# Example

Find the points on the unit circle whose y-coordinate is equal to 0.5.

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# Example

Find the points on the unit circle whose y-coordinate is equal to 0.5. Let (x, 0.5) be a point on the unit circle. Then  $x^2 + 0.5^2 = 1 \Leftrightarrow x^2 = 1 - 0.25 = 0.75$  and  $x = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$ .

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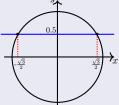
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# Positive angles

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#### Definition

For a number  $\alpha > 0$ , the radius of the unit circle corresponding to  $\alpha$  degrees is the radius that has angle  $\alpha$  degrees with the positive horizontal axis, when measured counterclockwise from the positive horizontal axis:

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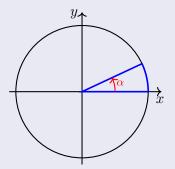
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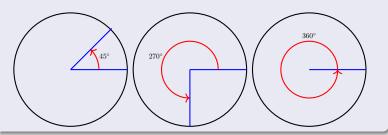
# Example

Sketch the radius of the unit circle corresponding to each of the following angles:  $45^{\circ}, 270^{\circ}$  and  $360^{\circ}$ .

Length of a circular arc and area of a sector

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# Negative angles

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#### Definition

For a number  $\alpha < 0$ , the radius of the unit circle corresponding to  $\alpha$  degrees is the radius that has angle  $|\alpha|$  degrees with the positive horizontal axis, when measured clockwise from the positive horizontal axis:

# Negative angles

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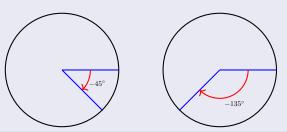
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#### Remark

The radius corresponding to an angle  $\alpha$  corresponds to angles  $\alpha + 360k$  for any integer k as well.

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#### Remark

The radius corresponding to an angle  $\alpha$  corresponds to angles  $\alpha + 360k$  for any integer k as well.

#### Definition

**Radians** are a unit of measurement for angles such that  $2\pi$  radians correspond to  $360^{\circ}$ .

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**Radians** are a unit of measurement for angles such that  $2\pi$  radians correspond to  $360^{\circ}$ .

Using the equality  $2\pi$  rad =  $360^{\circ}$ , we get conversion formulas:

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• 1 rad = 
$$\left(\frac{180}{\pi}\right)^{\circ}$$
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- 1 rad =  $\left(\frac{180}{\pi}\right)^{\circ}$  and
- $1^{\circ} = \frac{\pi}{180}$  rad.

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# Example

• Convert 120 degrees to radians.

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### Example

• Convert 120 degrees to radians.

We use the second formula to get  $120^{\circ} = \frac{\pi}{180} \cdot 120 = \frac{120\pi}{180} = \frac{2\pi}{3}$  rad.

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2 Convert 
$$\frac{5\pi}{6}$$
 radians to degrees.

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$$120^{\circ} = \frac{\pi}{180} \cdot 120 = \frac{120\pi}{180} = \frac{2\pi}{3} \text{ rad.}$$

2 Convert  $\frac{5\pi}{6}$  radians to degrees.

This time we use the first formula and get  $\frac{5\pi}{6}$  rad  $=\frac{180}{\pi} \cdot \frac{5\pi}{6} = \frac{180 \cdot 5\pi}{6\pi} = 30 \cdot 5 = 150^{\circ}$ .

**3** Convert 
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This time we use the first formula and get

$$\frac{2\pi}{15}$$
 rad =  $\frac{180}{\pi} \cdot \frac{2\pi}{15} = \frac{180 \cdot 2\pi}{15\pi} = 12 \cdot 2 = 24^{\circ}$ .

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# Question

What is the radian measure of the angle  $\alpha = \frac{18}{\pi}$  degrees?

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angle	endpoint of radius
$0 = 0^{\circ}$	(1,0)
$\frac{\pi}{6} = 30^{\circ}$	$\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$
$\frac{\pi}{4} = 45^{\circ}$	$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
$\frac{\pi}{3} = 60^{\circ}$	$\left(rac{1}{2},rac{\sqrt{3}}{2} ight)$
$\frac{\pi}{2} = 90^{\circ}$	(0, 1)
$\pi=180^{\circ}$	(-1, 0)

# Length of a circular arc and area of a sector

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Length of a circular arc and area of a sector Let  $0 \le \alpha \le 2\pi$ , then the length a circular arc on the circle of radius r corresponding to  $\alpha$  radians is equal to  $\alpha r$  and the area of sector with angle  $\alpha$  radians is equal to  $\frac{\alpha r^2}{2}$ .

# Length of a circular arc and area of a sector

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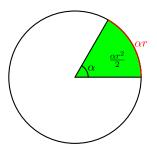
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# Examples

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# Example

Suppose the distance from the center of a wall clock to the endpoint of the hour hand is 4 inches and the length of the minute hand is 7 inches.

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# Example

Suppose the distance from the center of a wall clock to the endpoint of the hour hand is 4 inches and the length of the minute hand is 7 inches.

• How far does the endpoint of the hour hand of the clock travel in five hours?

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# Example

Suppose the distance from the center of a wall clock to the endpoint of the hour hand is 4 inches and the length of the minute hand is 7 inches.

• How far does the endpoint of the hour hand of the clock travel in five hours?

$$\ell = \frac{5}{12} \cdot 2\pi \cdot 4 = \frac{10\pi}{3} \text{ inches.}$$

Length of a circular arc and area of a sector

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• How far does the endpoint of the minute hand of the clock travel in 3 hours 15 minutes?

$$\ell = 3 \cdot 2\pi \cdot 7 + \frac{15}{60} \cdot 2\pi \cdot 7 = 45.5\pi$$
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For a 14-inch pizza 60, find the area of a slice with angle  $\frac{4}{7}$  radians.

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 =  $\frac{1}{2} \cdot \frac{4}{7} \cdot 7^2 = 14 \text{ in}^2$ .

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### Question

Suppose an ant walks clockwise on the unit circle from the point (0,1) to the endpoint of the radius corresponding to  $\left(2+\frac{\pi}{2}\right)$  radians. How far has the ant walked?

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