Sine and cosine: definitions

Science of signs of sines and cosines

First trigonometric identity

Graphs of sine and cosine

Lecture 17

Cosine and sine

MATH 0200

Dr. Boris Tsvelikhovskiy

Outline

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First trigonometric identity

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2. Science of signs of sines and cosines
3. First trigonometric identity
4. Graphs of sine and cosine

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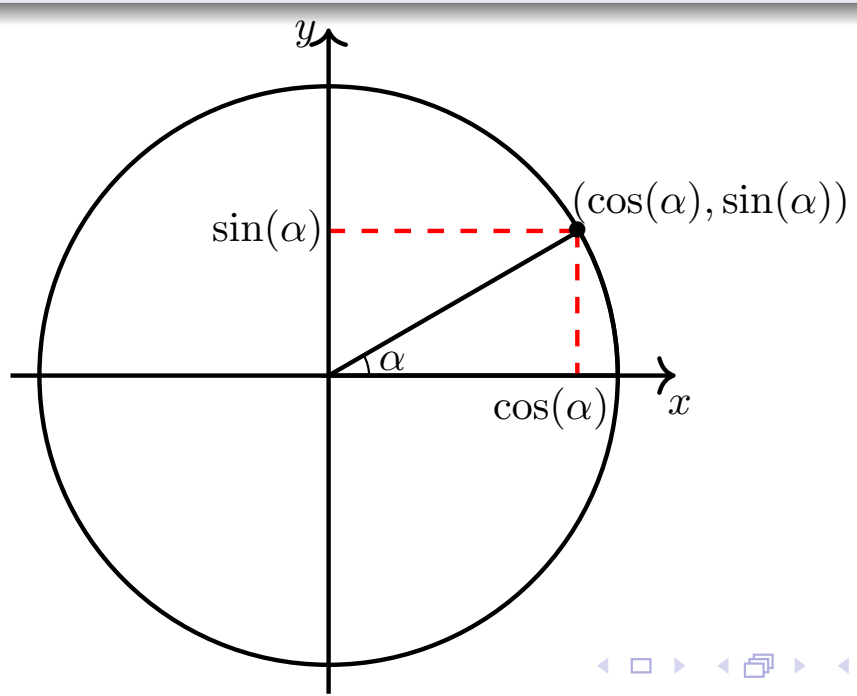

Special angles

α	$\sin(\alpha)$	$\cos(\alpha)$
$0°$	0	1
$\frac{\pi}{6} = 30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4} = 45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3} = 60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2} = 90°$	1	0
$\pi = 180°$	0	-1

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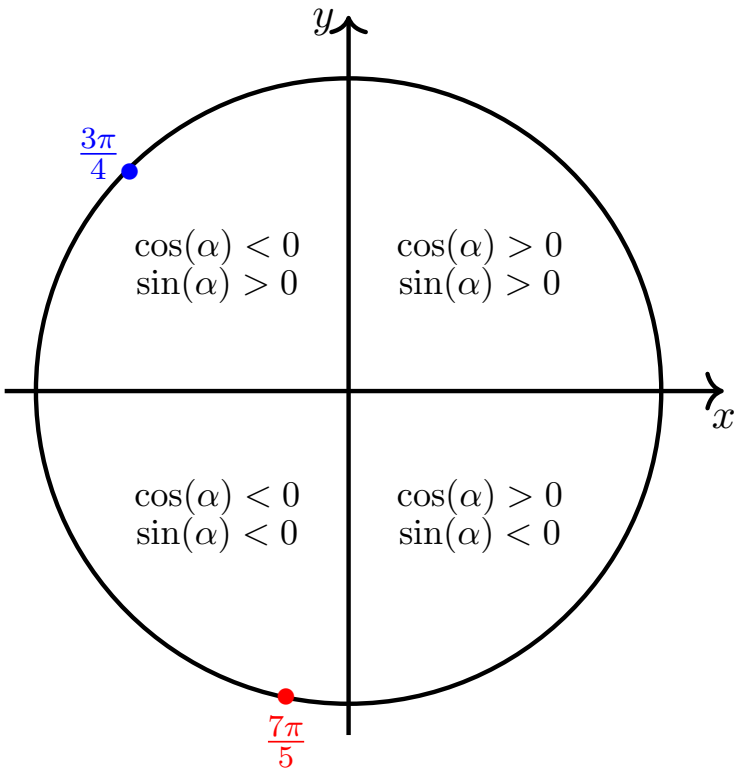

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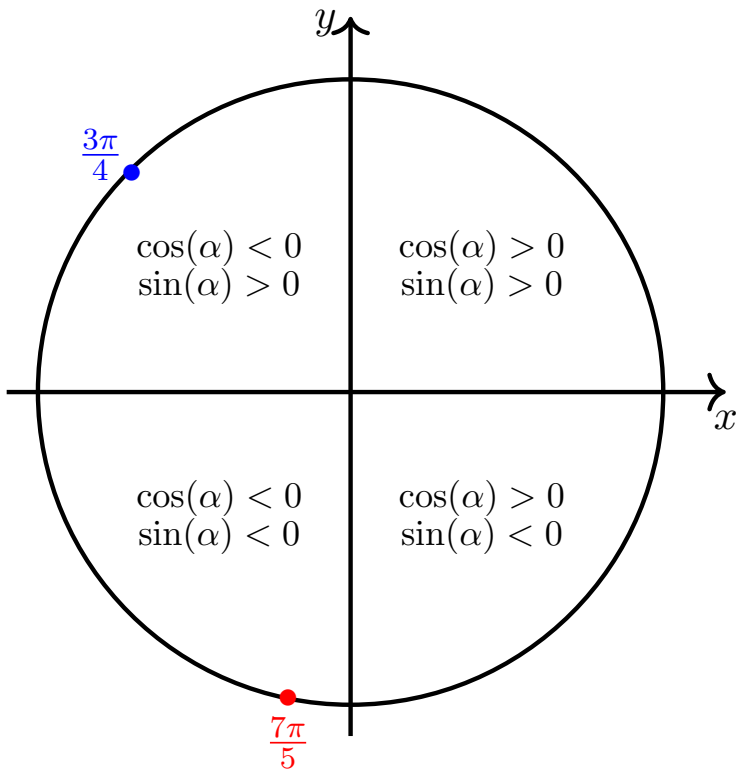

Example

1. $\cos\left(\frac{3\pi}{4}\right) < 0$ and $\sin\left(\frac{3\pi}{4}\right) > 0$;

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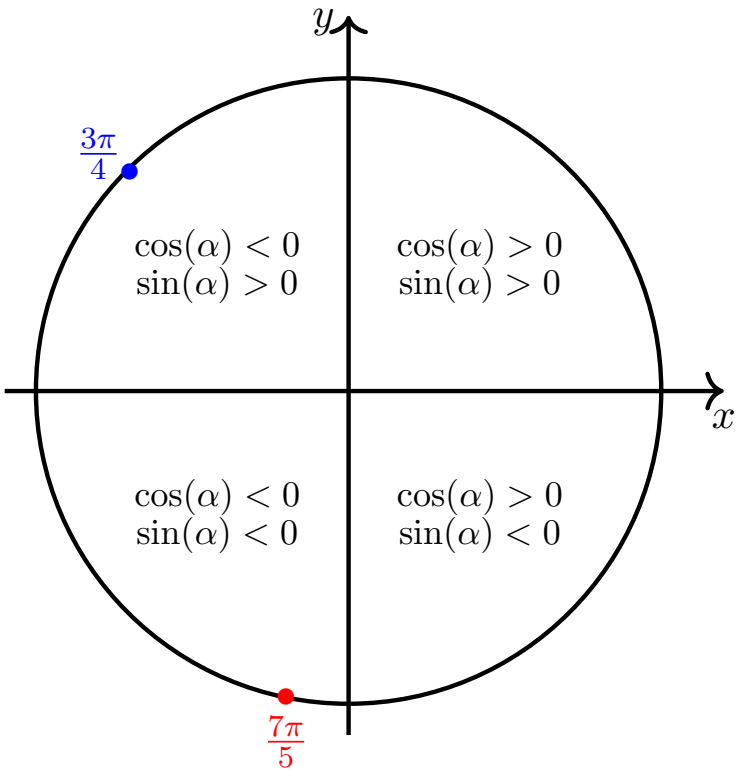

Example

1. $\cos\left(\frac{3\pi}{4}\right) < 0$ and $\sin\left(\frac{3\pi}{4}\right) > 0$;
2. $\cos\left(\frac{7\pi}{5}\right) < 0$ and $\sin\left(\frac{7\pi}{5}\right) < 0$;

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For every angle α:

$$\sin^2(\alpha) + \cos^2(\alpha) = 1.$$

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Given that $\sin(\alpha) = 0.8$ and α is between $\frac{\pi}{2}$ and π, find the value of $\cos(\alpha)$.

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$$\sin^2(\alpha) + \cos^2(\alpha) = 1.$$

Example

Given that $\sin(\alpha) = 0.8$ and α is between $\frac{\pi}{2}$ and π, find the value of $\cos(\alpha)$.

We find $\cos^2(\alpha) = 1 - \sin^2(\alpha) \Leftrightarrow \cos(\alpha) = \pm\sqrt{1 - \sin^2(\alpha)} = \pm\sqrt{1 - 0.8^2} = \pm\sqrt{0.36} = \pm 0.6$ and, as $\frac{\pi}{2} < \alpha < \pi$, we conclude that $\cos(\alpha)$ is negative, so $\cos(\alpha) = -0.6$.

Question

Given that $\cos(\alpha) = -\sqrt{0.19}$ and α is between π and $\frac{3\pi}{2}$, find the value of $\sin(\alpha)$.

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Answer: we compute $\sin^2(\alpha) = 1 - \cos^2(\alpha) \Leftrightarrow \sin(\alpha) = \pm\sqrt{1 - \cos^2(\alpha)} = \pm\sqrt{1 - (-\sqrt{0.19})^2} = \pm\sqrt{0.81} = \pm 0.9$ and, as $\pi < \alpha < \frac{3\pi}{2}$, we conclude that $\sin(\alpha)$ is negative, so $\sin(\alpha) = -0.9$.

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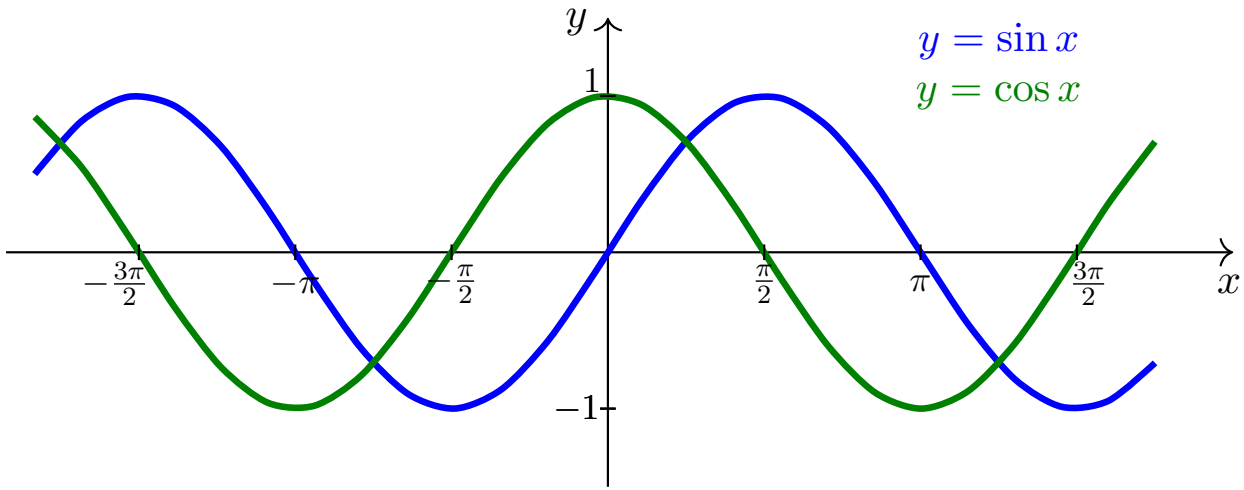

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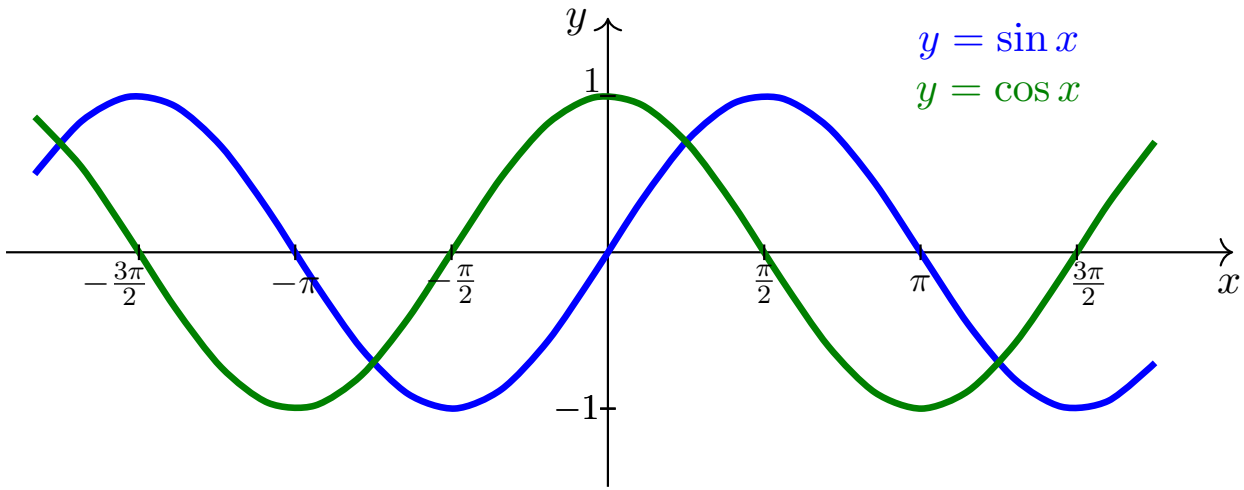

Domain: $\mathbb{R} = (-\infty, \infty)$.

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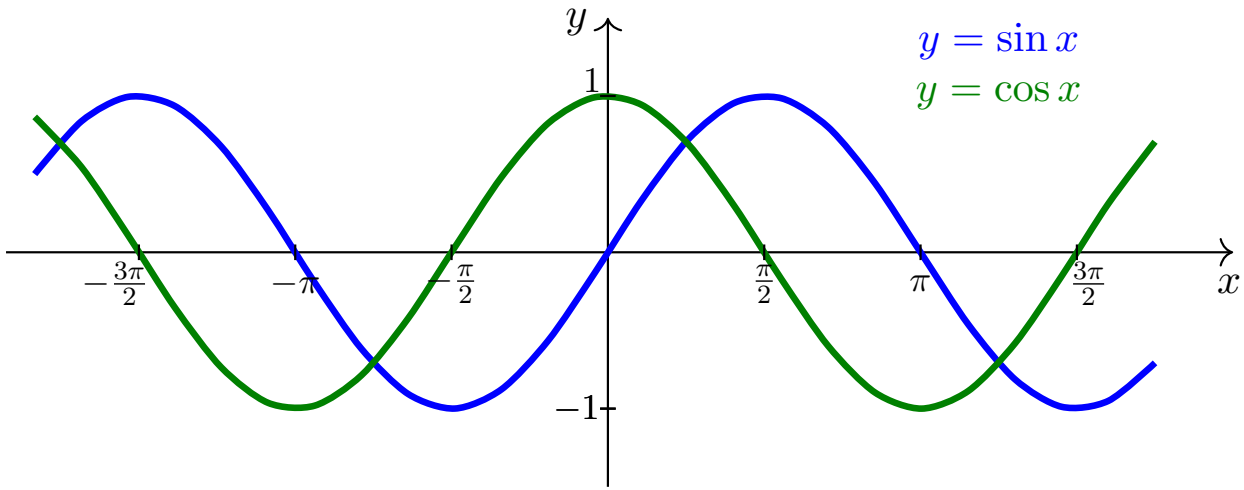

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Range: $[-1, 1]$.

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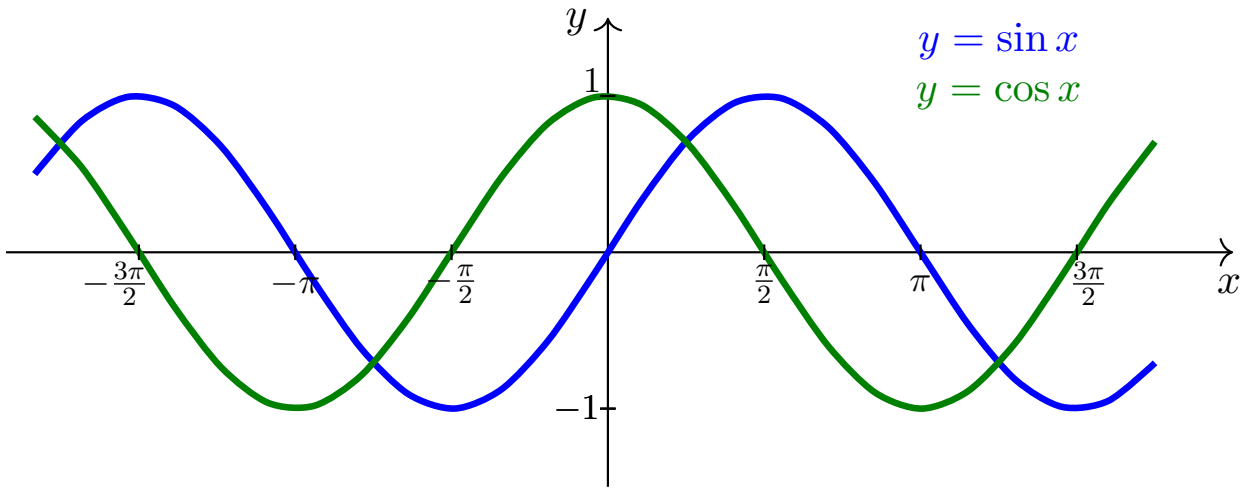

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Remark

- $\cos(-x) = \cos(x)$, so $\cos(x)$ is an even function;

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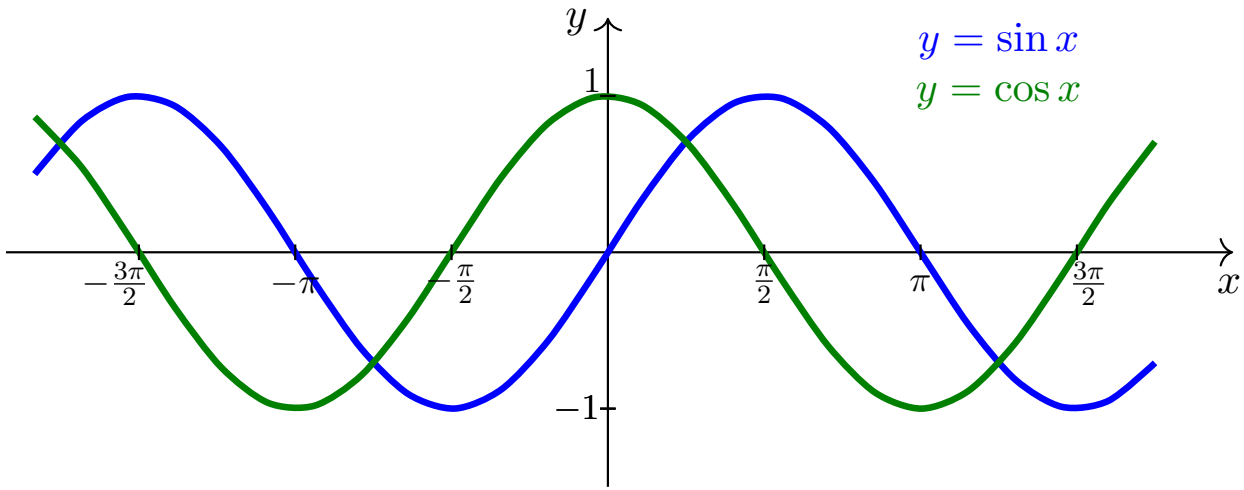

Domain: $\mathbb{R} = (-\infty, \infty)$.
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Remark

- $\cos(-x) = \cos(x)$, so $\cos(x)$ is an even function;
- $\sin(-x) = -\sin(x)$, so $\sin(x)$ is an odd function.