Lecture 18

Other trigonometric functions

MATH 0200

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Outline

1 Tangent

2 Sign of tangent

3 Graph of tangent

4 More trigonometric functions

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Definition

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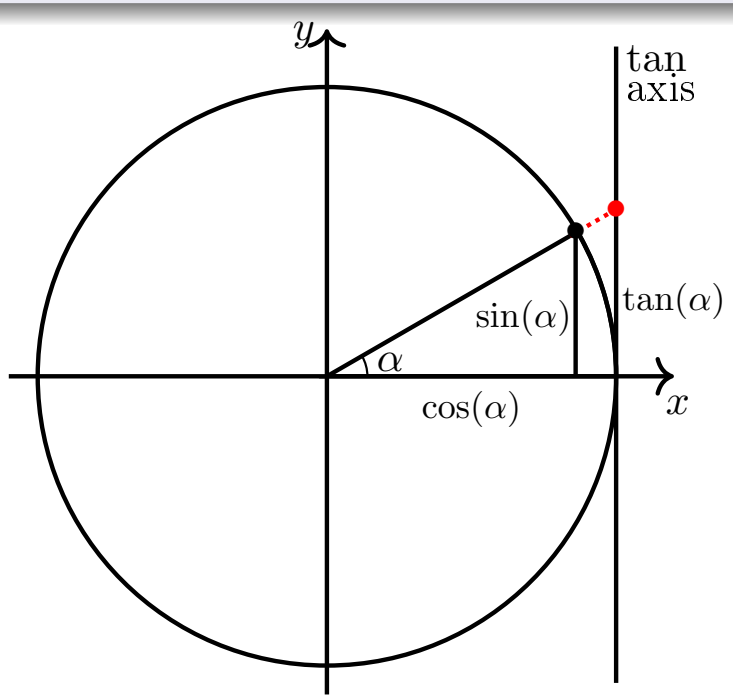

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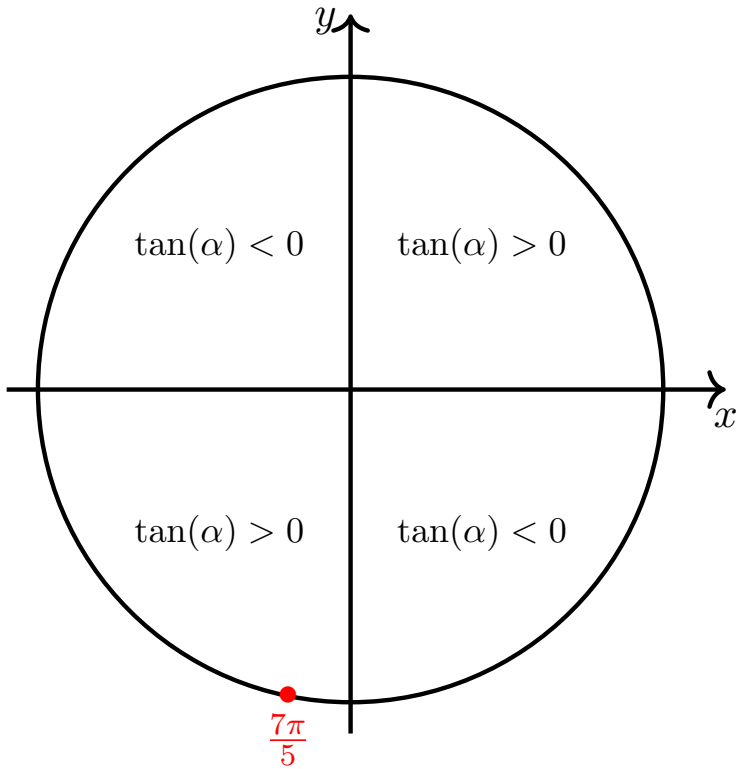
α	$\tan(\alpha)$
$0°$	0
$\frac{\pi}{6} = 30°$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4} = 45°$	1
$\frac{\pi}{3} = 60°$	$\sqrt{3}$
$\frac{\pi}{2} = 90°$	undefined
$\pi = 180°$	0

Sign of tangent



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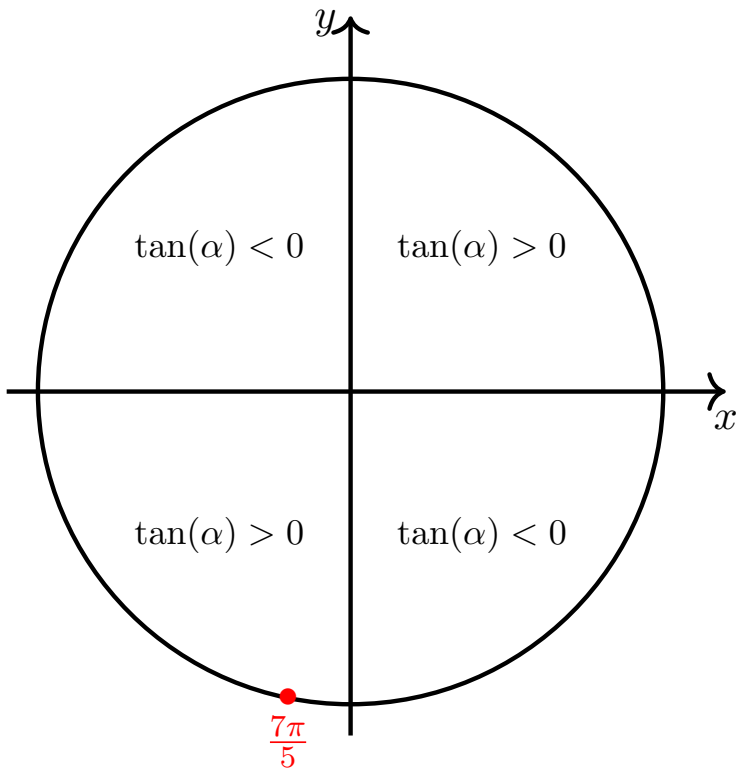

Example

For instance, $\tan\left(\frac{7\pi}{5}\right) > 0$.

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Using the second equation, we find $\sin(\alpha) = -2\cos(\alpha)$ and plug it into the first equation to get

$\cos^2(\alpha) + (-2\cos(\alpha))^2 = 5\cos^2(\alpha) = 1 \Leftrightarrow \cos^2(\alpha) = 0.2 \Leftrightarrow$ $\cos(\alpha) = \pm\sqrt{0.2}$.

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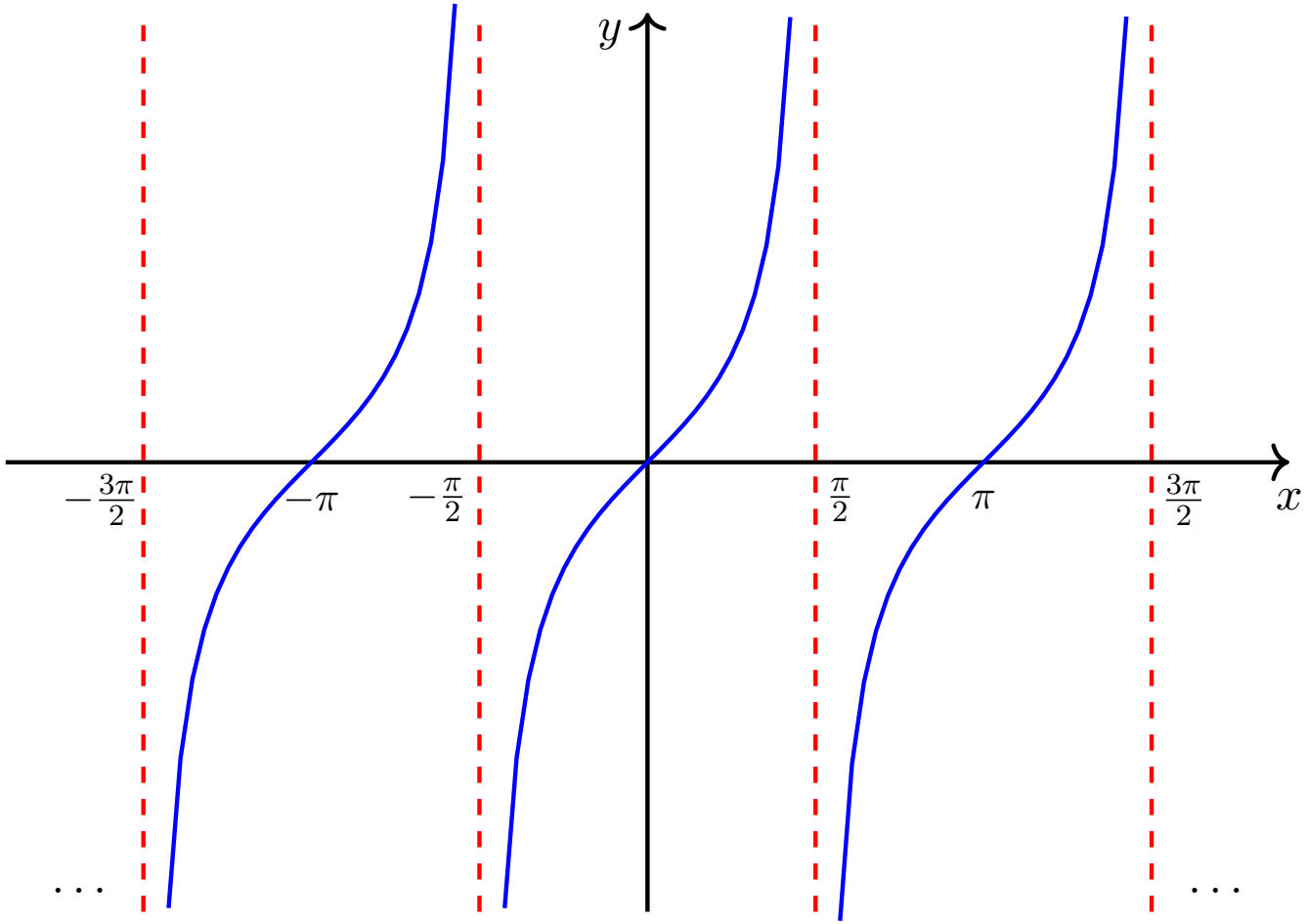
$\cos^2(\alpha) + (-2\cos(\alpha))^2 = 5\cos^2(\alpha) = 1 \Leftrightarrow \cos^2(\alpha) = 0.2 \Leftrightarrow$ $\cos(\alpha) = \pm\sqrt{0.2}$. As $\frac{3\pi}{2} < \alpha < 2\pi$, we have $\cos(\alpha) > 0$, so $\cos(\alpha) = \sqrt{0.2}$ and $\sin(\alpha) < 0$ with

$$\sin(\alpha) = -\sqrt{1 - (\sqrt{0.2})^2} = -\sqrt{1 - 0.2} = -\sqrt{0.8}.$$

Graph of tangent



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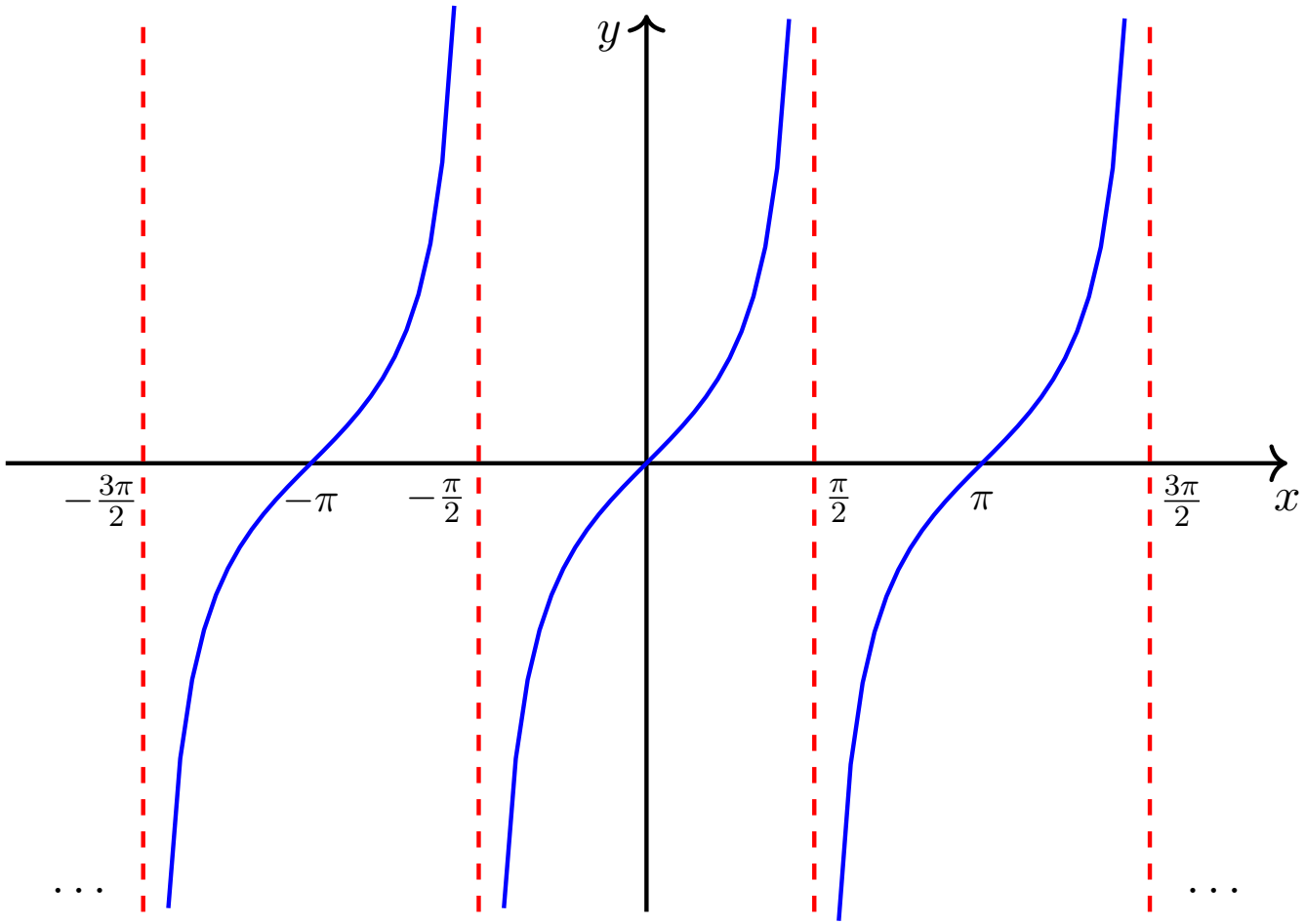

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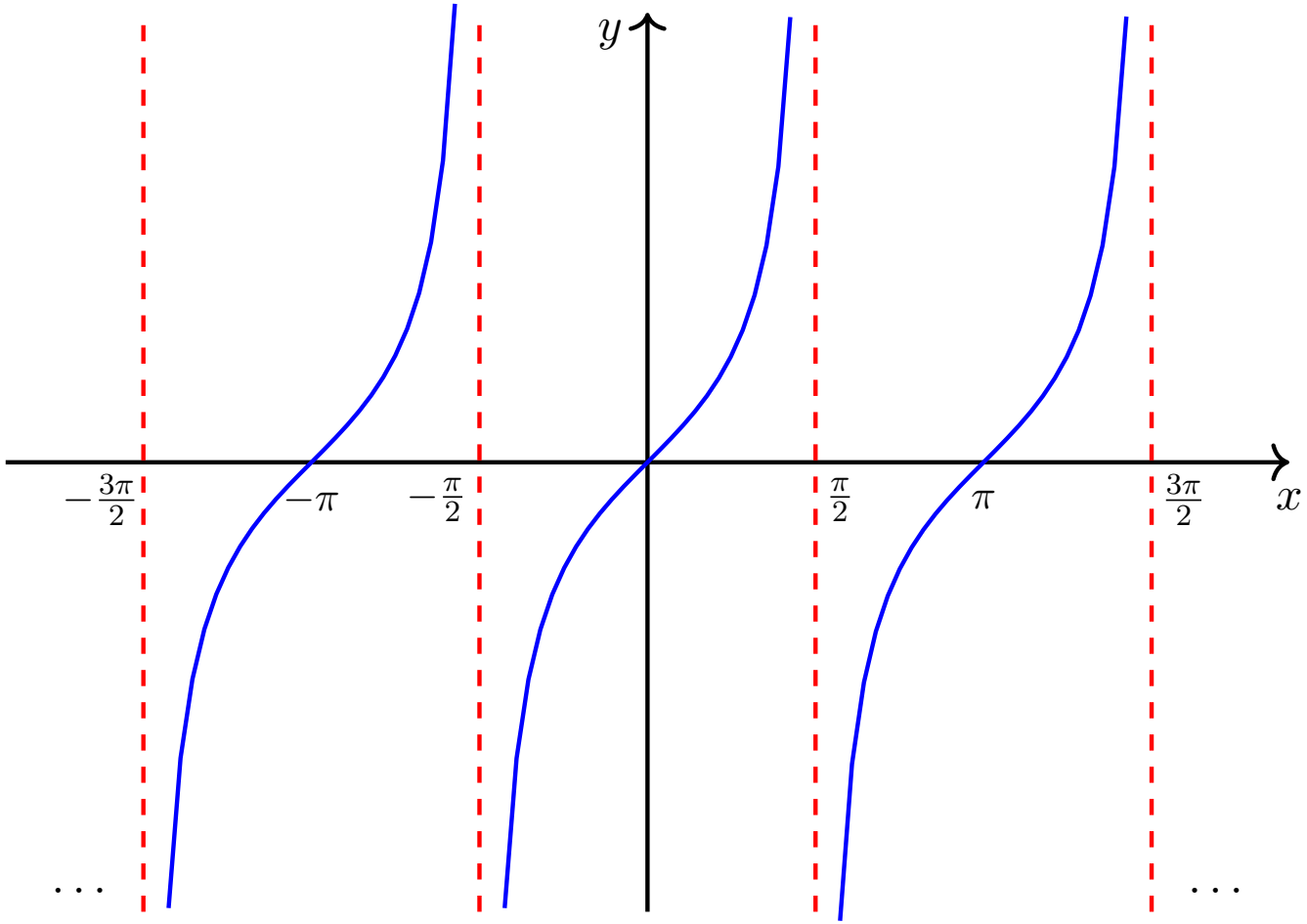

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Range: $\mathbb{R} = (-\infty, \infty)$.

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Evaluate $\csc\left(\frac{\pi}{6}\right)$.
We compute $\csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{6}\right)} = \frac{1}{0.5} = 2$.