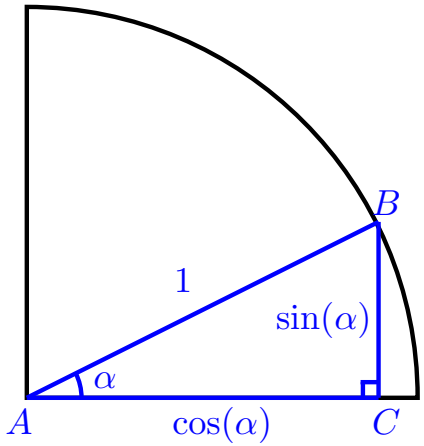
Lecture 19

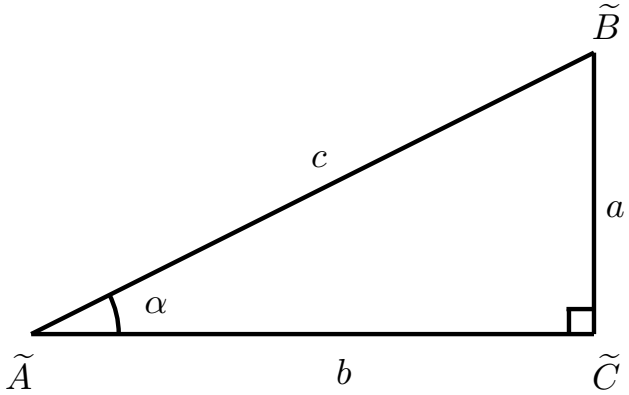
Trigonometry in right triangles

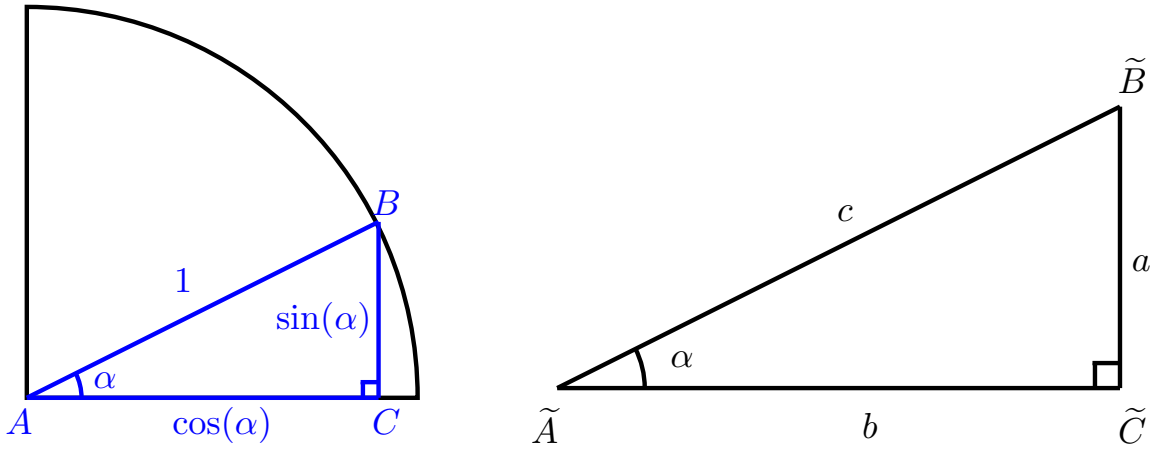
MATH 0200

Dr. Boris Tsvelikhovskiy

Outline

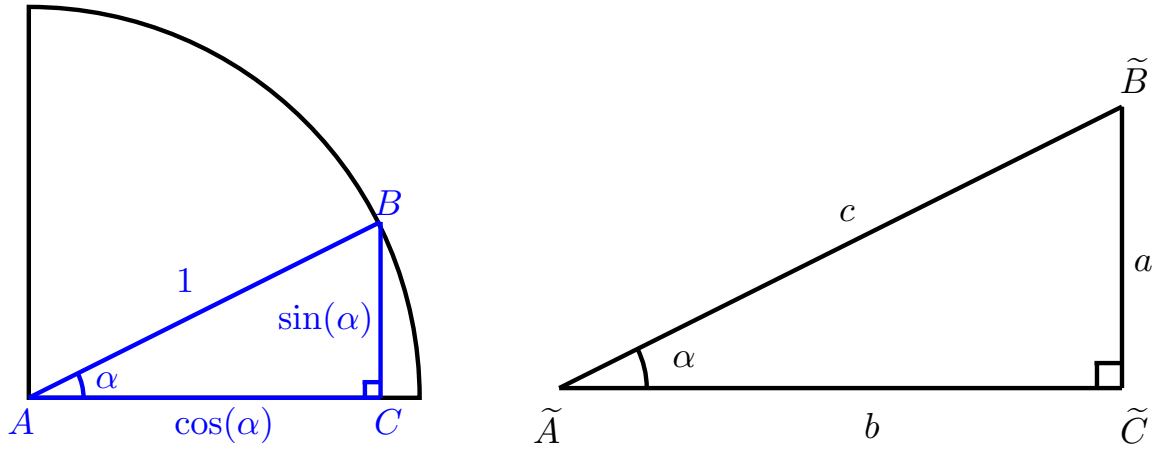

B
1
sin(α)
α
A
cos(α)
C


B̃
c
a
α
Ã
b
C̃



Remark

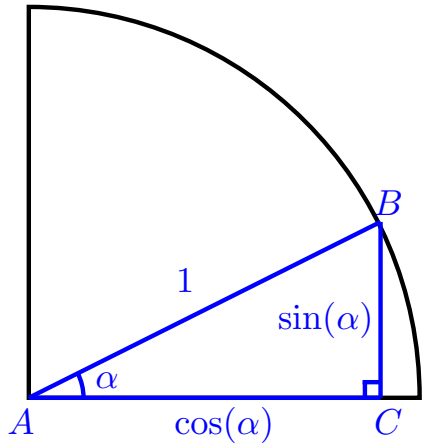
We observe that our first trigonometric identity $\sin^2(\alpha) + \cos^2(\alpha) = 1$ is nothing else but Pythagorean theorem for triangle $\triangle ABC$.

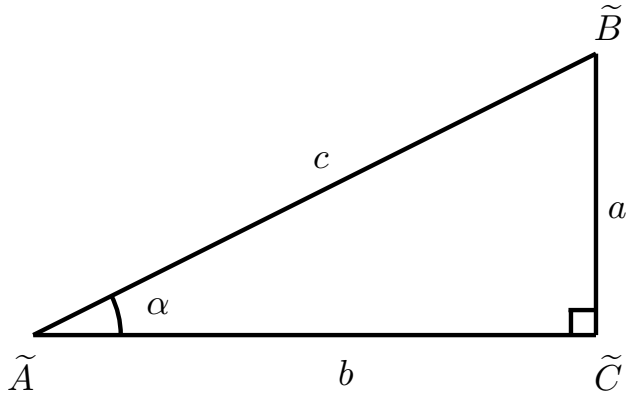


Remark

We observe that our first trigonometric identity $\sin^2(\alpha) + \cos^2(\alpha) = 1$ is nothing else but Pythagorean theorem for triangle $\triangle ABC$.

Notice that the two right triangles $\triangle ABC$ and $\triangle \widetilde{A}\widetilde{B}\widetilde{C}$ are similar.



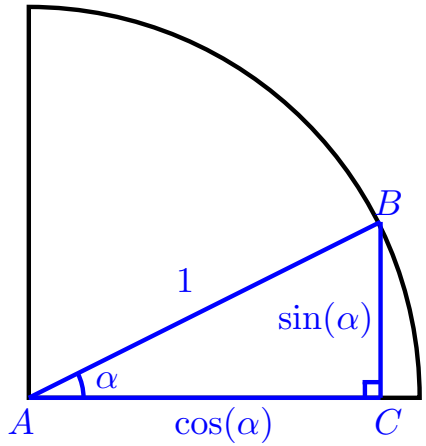


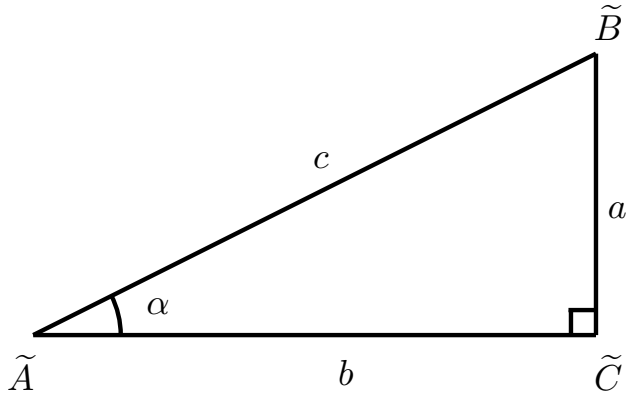
Remark

We observe that our first trigonometric identity $\sin^2(\alpha) + \cos^2(\alpha) = 1$ is nothing else but Pythagorean theorem for triangle $\triangle ABC$.

Notice that the two right triangles $\triangle ABC$ and $\triangle \widetilde{A}\widetilde{B}\widetilde{C}$ are similar. We conclude that

- $\sin(\alpha) = \frac{a}{c} = \frac{\text{opposite leg}}{\text{hypothenuse}}$;



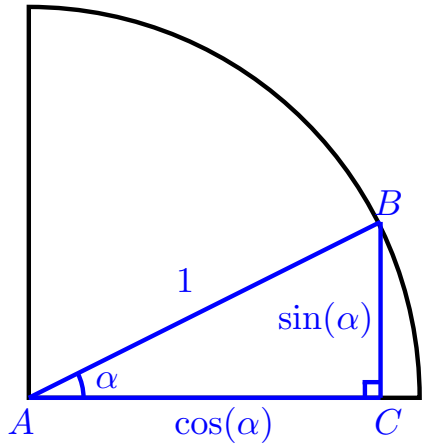


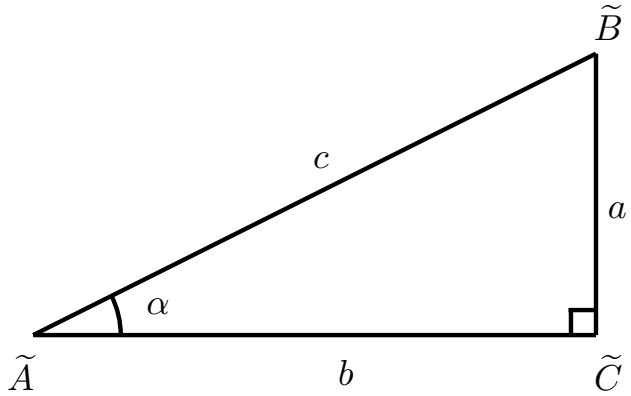
Remark

We observe that our first trigonometric identity $\sin^2(\alpha) + \cos^2(\alpha) = 1$ is nothing else but Pythagorean theorem for triangle $\triangle ABC$.

Notice that the two right triangles $\triangle ABC$ and $\triangle \widetilde{A}\widetilde{B}\widetilde{C}$ are similar. We conclude that

- $\sin(\alpha) = \frac{a}{c} = \frac{\text{opposite leg}}{\text{hypothenuse}}$;
- $\cos(\alpha) = \frac{b}{c} = \frac{\text{adjacent leg}}{\text{hypothenuse}}$;





Remark

We observe that our first trigonometric identity $\sin^2(\alpha) + \cos^2(\alpha) = 1$ is nothing else but Pythagorean theorem for triangle $\triangle ABC$.

Notice that the two right triangles $\triangle ABC$ and $\triangle \widetilde{A}\widetilde{B}\widetilde{C}$ are similar. We conclude that

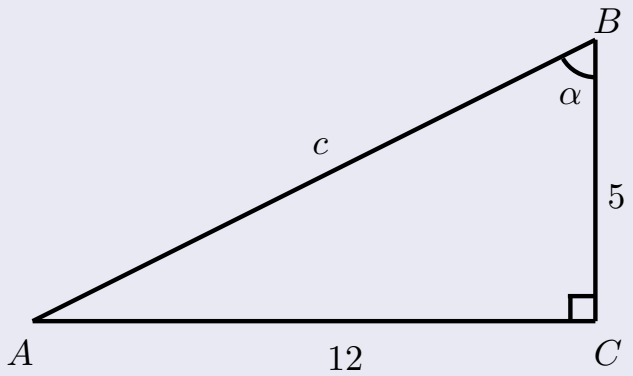
- $\sin(\alpha) = \frac{a}{c} = \frac{\text{opposite leg}}{\text{hypothenuse}}$;
- $\cos(\alpha) = \frac{b}{c} = \frac{\text{adjacent leg}}{\text{hypothenuse}}$;
- $\tan(\alpha) = \frac{a}{b} = \frac{\text{opposite leg}}{\text{adjacent leg}}$;

Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.

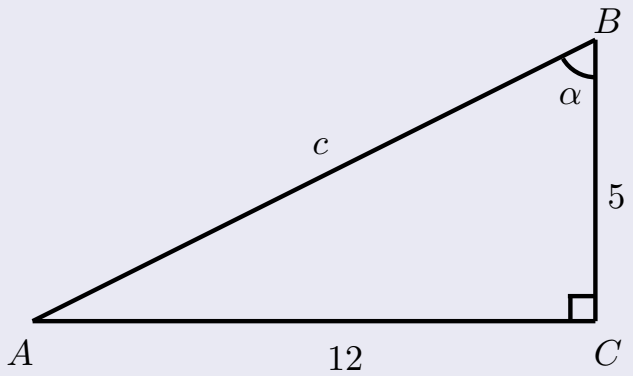
Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.



Example

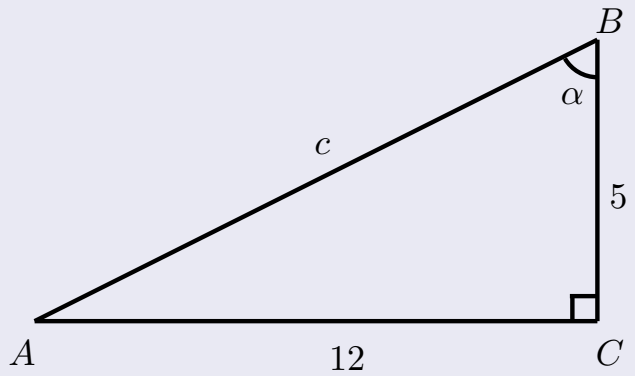
Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.



(a) Find the length of the hypotenuse AC.

Example

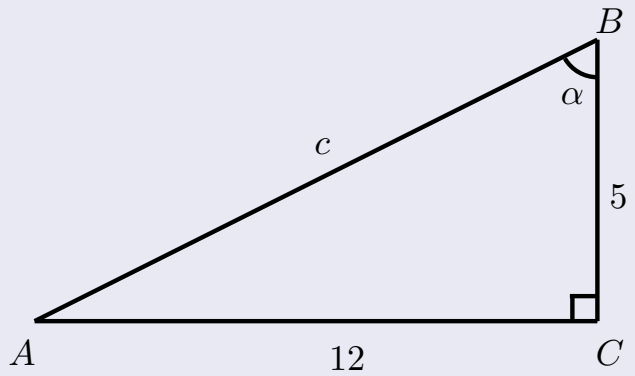
Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.



(a) Find the length of the hypotenuse AC.
We use Pythagorean theorem to get
$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.

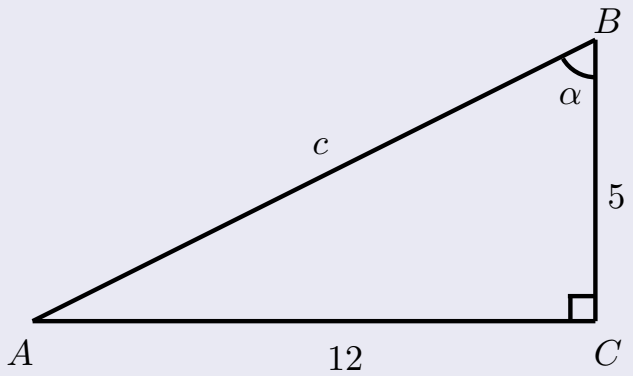


(a) Find the length of the hypotenuse AC.
We use Pythagorean theorem to get
$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

(b) Evaluate $\sin(\alpha)$.

Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.

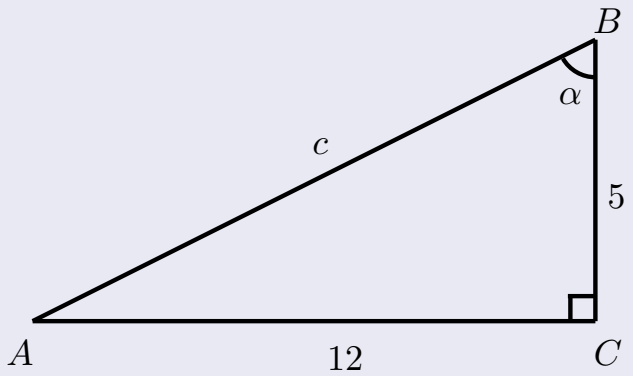


(a) Find the length of the hypotenuse AC.
We use Pythagorean theorem to get
$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

(b) Evaluate $\sin(\alpha)$. We get $\sin(\alpha) = \frac{12}{13}$.

Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.



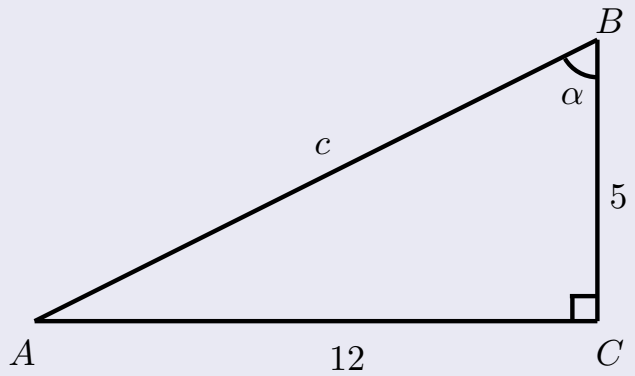
(a) Find the length of the hypotenuse AC.
We use Pythagorean theorem to get
$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

(b) Evaluate $\sin(\alpha)$. We get $\sin(\alpha) = \frac{12}{13}$.

(c) Evaluate $\cos(\alpha)$.

Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.



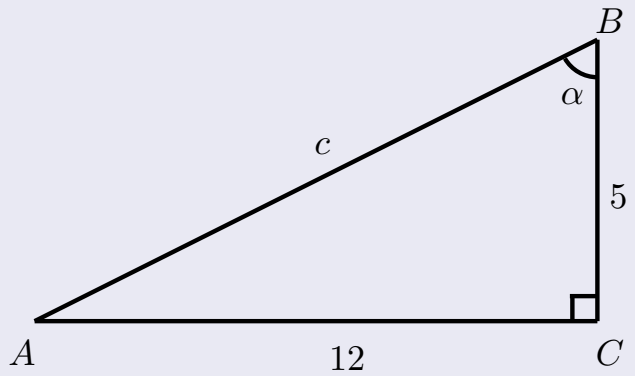
(a) Find the length of the hypotenuse AC.
We use Pythagorean theorem to get
$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

(b) Evaluate $\sin(\alpha)$. We get $\sin(\alpha) = \frac{12}{13}$.

(c) Evaluate $\cos(\alpha)$. We get $\cos(\alpha) = \frac{5}{13}$.

Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.



(a) Find the length of the hypotenuse AC.
We use Pythagorean theorem to get
$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

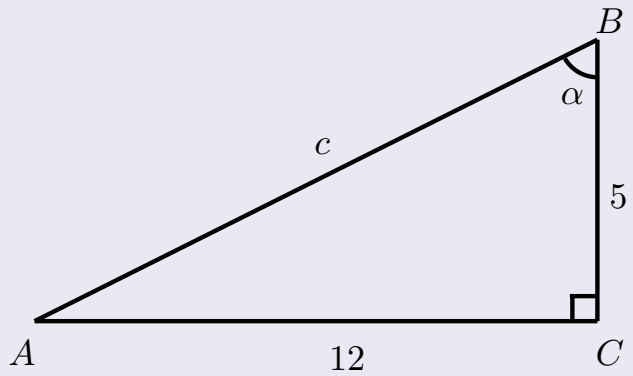
(b) Evaluate $\sin(\alpha)$. We get $\sin(\alpha) = \frac{12}{13}$.

(c) Evaluate $\cos(\alpha)$. We get $\cos(\alpha) = \frac{5}{13}$.

(d) Evaluate $\cot(\alpha)$.

Example

Consider the right triangle $\triangle ABC$ below with legs $AC = 12$ and $BC = 5$.



(a) Find the length of the hypotenuse AC.
We use Pythagorean theorem to get
$c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$.

(b) Evaluate $\sin(\alpha)$. We get $\sin(\alpha) = \frac{12}{13}$.

(c) Evaluate $\cos(\alpha)$. We get $\cos(\alpha) = \frac{5}{13}$.

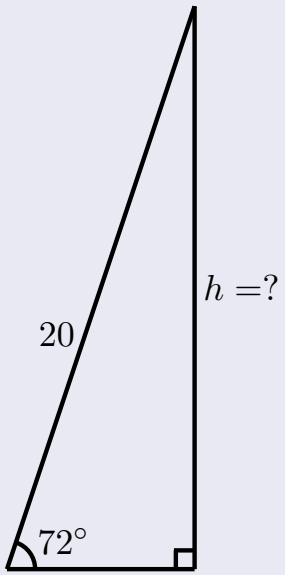
(d) Evaluate $\cot(\alpha)$. We get $\cot(\alpha) = \frac{\cos(\alpha)}{\sin(\alpha)} = \frac{5}{13} \cdot \frac{13}{12} = \frac{5}{12}$.

Example

Suppose a 20-foot ladder is leaning against a wall, making a 72° angle with the ground. How high up the wall is the end of the ladder?

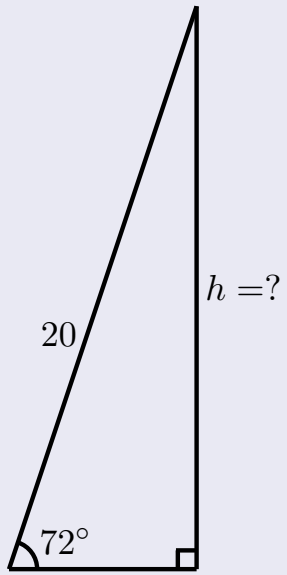
Example

Suppose a 20-foot ladder is leaning against a wall, making a 72° angle with the ground. How high up the wall is the end of the ladder?



Example

Suppose a 20-foot ladder is leaning against a wall, making a 72° angle with the ground. How high up the wall is the end of the ladder?

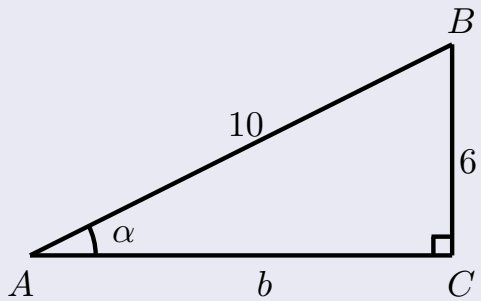


Solution: $\sin(72^\circ) = \frac{h}{20} \Leftrightarrow h = 20\sin(72^\circ) \approx 19.021$ ft.

Consider the right triangle depicted below.

Consider the right triangle depicted below.

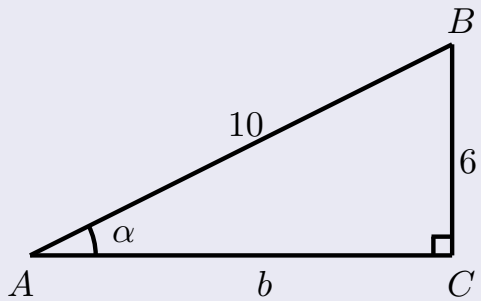
Question



- Evaluate $\sin(\alpha)$.

Consider the right triangle depicted below.

Question



- Evaluate $\sin(\alpha)$.
- Evaluate $\tan(\alpha)$.