Lecture 20

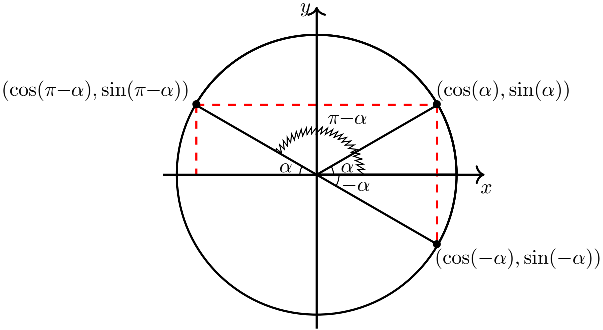
Trigonometric identities

MATH 0200

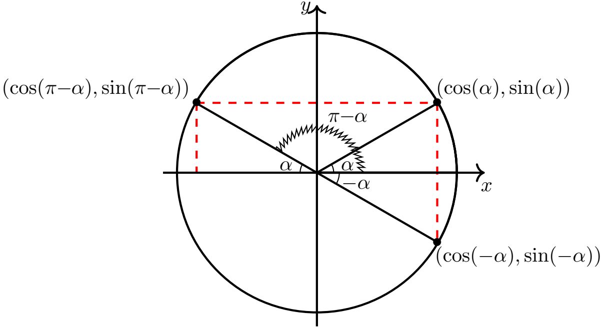
Dr. Boris Tsvelikhovskiy

Outline

Trigonometric identities for $\alpha, -\alpha$ and $\pi - \alpha$

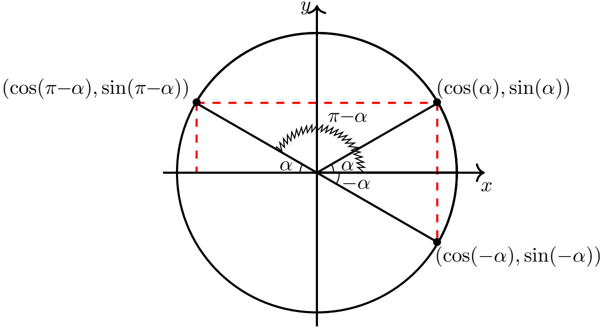


Trigonometric identities for α, $-\alpha$ and $\pi - \alpha$



We obtain the following identities.

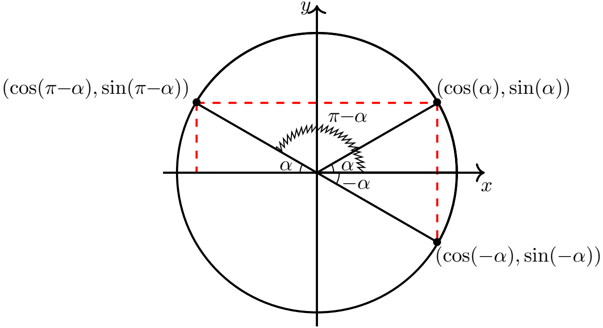
Trigonometric identities for α, $-\alpha$ and $\pi - \alpha$



We obtain the following identities.

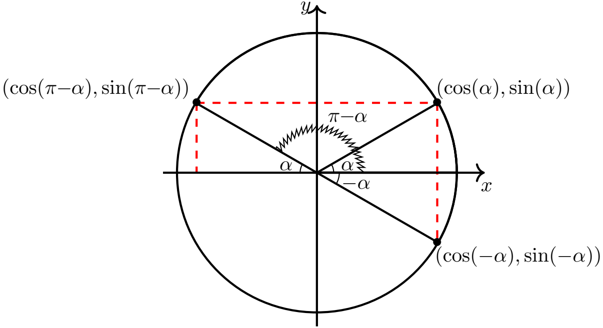
- $\cos(\alpha) = \cos(-\alpha)$ and $\cos(\pi - \alpha) = -\cos(\alpha)$;

Trigonometric identities for $\alpha, -\alpha$ and $\pi - \alpha$



We obtain the following identities.

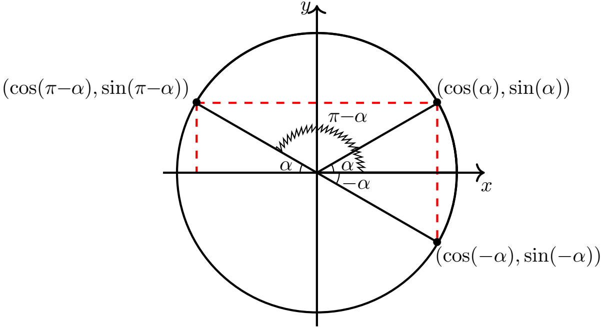
- $\cos(\alpha) = \cos(-\alpha)$ and $\cos(\pi - \alpha) = -\cos(\alpha)$;
- $\sin(\alpha) = \sin(\pi - \alpha)$ and $\sin(-\alpha) = -\sin(\alpha)$;



We obtain the following identities.

- $\cos(\alpha) = \cos(-\alpha)$ and $\cos(\pi - \alpha) = -\cos(\alpha)$;
- $\sin(\alpha) = \sin(\pi - \alpha)$ and $\sin(-\alpha) = -\sin(\alpha)$;
- $\tan(-\alpha) = \frac{\sin(-\alpha)}{\cos(-\alpha)} = \frac{-\sin(\alpha)}{\cos(\alpha)} = -\tan(\alpha)$;

Trigonometric identities for $\alpha, -\alpha$ and $\pi - \alpha$



We obtain the following identities.

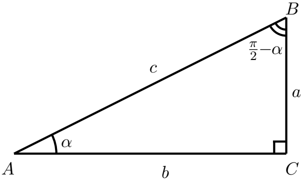
- $\cos(\alpha) = \cos(-\alpha)$ and $\cos(\pi - \alpha) = -\cos(\alpha)$;
- $\sin(\alpha) = \sin(\pi - \alpha)$ and $\sin(-\alpha) = -\sin(\alpha)$;
- $\tan(-\alpha) = \frac{\sin(-\alpha)}{\cos(-\alpha)} = \frac{-\sin(\alpha)}{\cos(\alpha)} = -\tan(\alpha)$;
- $\tan(\pi - \alpha) = \frac{\sin(\pi-\alpha)}{\cos(\pi-\alpha)} = \frac{\sin(\alpha)}{-\cos(\alpha)} = -\tan(\alpha)$.

Trigonometric identities for α and $\frac{\pi}{2} - \alpha$

Let's take a look at a right triangle (with angles $\alpha, \frac{\pi}{2} - \alpha$ and $\frac{\pi}{2}$).

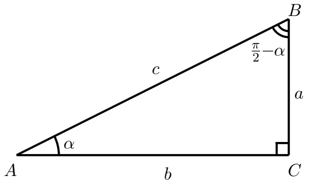
Trigonometric identities for α and $\frac{\pi}{2} - \alpha$

Let's take a look at a right triangle (with angles α, $\frac{\pi}{2} - \alpha$ and $\frac{\pi}{2}$).



Trigonometric identities for α and $\frac{\pi}{2} - \alpha$

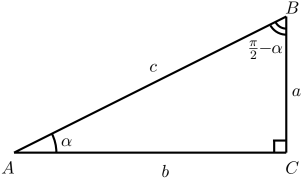
Let's take a look at a right triangle (with angles α, $\frac{\pi}{2} - \alpha$ and $\frac{\pi}{2}$).



We obtain the following identities.

Trigonometric identities for α and $\frac{\pi}{2} - \alpha$

Let's take a look at a right triangle (with angles α, $\frac{\pi}{2} - \alpha$ and $\frac{\pi}{2}$).

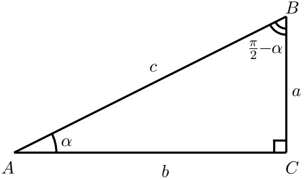


We obtain the following identities.

- $\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{a}{c} = \sin(\alpha)$ and $\sin\left(\frac{\pi}{2} - \alpha\right) = \frac{b}{c} = \cos(\alpha)$;

Trigonometric identities for α and $\frac{\pi}{2} - \alpha$

Let's take a look at a right triangle (with angles α, $\frac{\pi}{2} - \alpha$ and $\frac{\pi}{2}$).



We obtain the following identities.

- $\cos\left(\frac{\pi}{2} - \alpha\right) = \frac{a}{c} = \sin(\alpha)$ and $\sin\left(\frac{\pi}{2} - \alpha\right) = \frac{b}{c} = \cos(\alpha)$;
- $\tan\left(\frac{\pi}{2} - \alpha\right) = \frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\cos\left(\frac{\pi}{2}-\alpha\right)} = \frac{\cos(\alpha)}{\sin(\alpha)} = \cot(\alpha)$.

Question

Given that $\cos(u) = -0.6$ and $0 < \frac{\pi}{2} - u < \frac{\pi}{2}$, find $\cos\left(\frac{\pi}{2} - u\right)$.

Question

Given that $\cos(u) = -0.6$ and $0 < \frac{\pi}{2} - u < \frac{\pi}{2}$, find $\cos\left(\frac{\pi}{2} - u\right)$.

Answer: first we use the identity $\sin\left(\frac{\pi}{2} - u\right) = \cos(u) = -0.6$ and then find $\cos\left(\frac{\pi}{2} - u\right) = \pm\sqrt{1 - \sin^2\left(\frac{\pi}{2} - u\right)} = \pm\sqrt{1 - (-0.6)^2} = \pm\sqrt{1 - 0.36} = \pm\sqrt{0.64} = \pm 0.8$. As $0 < \frac{\pi}{2} - u < \frac{\pi}{2}$, we choose the positive value $\cos\left(\frac{\pi}{2} - u\right) = 0.8$.

Trigonometric identities involving 2π-periodicity

Remark

Notice that the radius corresponding to an angle α is the same as the radius corresponding to angle $\alpha + 2\pi n$ for any integer n.

Trigonometric identities involving 2π-periodicity

Remark

Notice that the radius corresponding to an angle α is the same as the radius corresponding to angle $\alpha + 2\pi n$ for any integer n.

Definition

A function $f(x)$ is called **periodic** if it repeats its values at regular intervals: $f(x) = f(x + P)$ for a constant P and all values of x in the domain. The smallest positive constant P for which this is the case is called the **period** of the function.

Trigonometric identities involving 2π-periodicity

Remark

Notice that the radius corresponding to an angle α is the same as the radius corresponding to angle $\alpha + 2\pi n$ for any integer n.

Definition

A function $f(x)$ is called **periodic** if it repeats its values at regular intervals: $f(x) = f(x + P)$ for a constant P and all values of x in the domain. The smallest positive constant P for which this is the case is called the **period** of the function.

The trigonometric functions $\sin(x), \cos(x), \tan(x), \cot(x)$, $\sec(x)$ and $\csc(x)$ are periodic with period 2π:

Trigonometric identities involving 2π-periodicity

Remark

Notice that the radius corresponding to an angle α is the same as the radius corresponding to angle $\alpha + 2\pi n$ for any integer n.

Definition

A function $f(x)$ is called **periodic** if it repeats its values at regular intervals: $f(x) = f(x + P)$ for a constant P and all values of x in the domain. The smallest positive constant P for which this is the case is called the **period** of the function.

The trigonometric functions $\sin(x), \cos(x), \tan(x), \cot(x)$, $\sec(x)$ and $\csc(x)$ are periodic with period 2π:

- $\sin(x) = \sin(x + 2\pi)$;

Trigonometric identities involving 2π-periodicity

Remark

Notice that the radius corresponding to an angle α is the same as the radius corresponding to angle $\alpha + 2\pi n$ for any integer n.

Definition

A function $f(x)$ is called **periodic** if it repeats its values at regular intervals: $f(x) = f(x + P)$ for a constant P and all values of x in the domain. The smallest positive constant P for which this is the case is called the **period** of the function.

The trigonometric functions $\sin(x), \cos(x), \tan(x), \cot(x)$, $\sec(x)$ and $\csc(x)$ are periodic with period 2π:

- $\sin(x) = \sin(x + 2\pi)$;
- $\cos(x) = \cos(x + 2\pi)$;

Trigonometric identities involving 2π-periodicity

Remark

Notice that the radius corresponding to an angle α is the same as the radius corresponding to angle $\alpha + 2\pi n$ for any integer n.

Definition

A function $f(x)$ is called **periodic** if it repeats its values at regular intervals: $f(x) = f(x + P)$ for a constant P and all values of x in the domain. The smallest positive constant P for which this is the case is called the **period** of the function.

The trigonometric functions $\sin(x), \cos(x), \tan(x), \cot(x)$, $\sec(x)$ and $\csc(x)$ are periodic with period 2π:

- $\sin(x) = \sin(x + 2\pi)$;
- $\cos(x) = \cos(x + 2\pi)$;
- $\tan(x) = \tan(x + 2\pi) \ldots$

Example

Find the smallest number α larger than 7π such that $\tan(\alpha) = -1$.

Example

Find the smallest number α larger than 7π such that $\tan(\alpha) = -1$.

We know that $\tan\left(\frac{3\pi}{4} + n\pi\right) = -1$ and, therefore, need to find the smallest integer n with $\frac{3\pi}{4} + n\pi > 7\pi$. As $\frac{3\pi}{4} + n\pi > 7\pi \Leftrightarrow \frac{3}{4} + n > 7 \Leftrightarrow n > 6.25$, the smallest integer value of n satisfying the inequality is 7. The answer is $\alpha = \frac{3\pi}{4} + 7\pi = \frac{31\pi}{4}$.