Lecture 22	
MATH 0200	
Trigonometri functions composed with their inverses Arccosine plus arcsine	Lecture 22 Inverse trigonometric identities
	MATH 0200

Dr. Boris Tsvelikhovskiy

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# Outline

Lecture 22

MATH 0200

Trigonometric functions composed with their inverses

Arccosine plus arcsine

# 1 Trigonometric functions composed with their inverses

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**2** Arccosine plus arcsine

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Trigonomet: functions composed with their inverses

Arccosine plus arcsine

# Recall that if f and $f^{-1}$ are inverse functions, then

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Trigonometr functions composed with their inverses

Arccosine plus arcsine Recall that if f and  $f^{-1}$  are inverse functions, then •  $(f \circ f^{-1})(x) = x$  for any x in the domain of  $f^{-1}$ ;

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Trigonometri functions composed with their inverses

Arccosine plus arcsine Recall that if f and f<sup>-1</sup> are inverse functions, then
(f ∘ f<sup>-1</sup>)(x) = x for any x in the domain of f<sup>-1</sup>;
(f<sup>-1</sup> ∘ f)(x) = x for any x in the domain of f.

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#### Lecture 22

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Trigonometri functions composed with their inverses

Arccosine plus arcsine Recall that if f and f<sup>-1</sup> are inverse functions, then
(f ∘ f<sup>-1</sup>)(x) = x for any x in the domain of f<sup>-1</sup>;
(f<sup>-1</sup> ∘ f)(x) = x for any x in the domain of f.

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## Example

Evaluate  $\arccos(\cos(400^\circ))$ .

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Trigonometri functions composed with their inverses

Arccosine plus arcsine Recall that if f and f<sup>-1</sup> are inverse functions, then
(f \circ f^{-1})(x) = x for any x in the domain of f<sup>-1</sup>;
(f<sup>-1</sup> \circ f)(x) = x for any x in the domain of f.

## Example

Evaluate  $\arccos(\cos(400^\circ))$ . First we need to find  $0 \le \alpha \le \pi$  (or  $180^\circ$ ) with  $\cos(\alpha) = \cos(400^\circ)$ . As  $\cos(400^\circ) = \cos((400 - 360)^\circ) =$   $= \cos(40^\circ)$ , we get  $\alpha = 40^\circ = \frac{80\pi}{360} = \frac{2\pi}{9}$  and  $\arccos(\cos(400^\circ)) = \arccos(\cos\left(\frac{2\pi}{9}\right)) = \frac{2\pi}{9}$ .

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#### Trigonometr functions composed with their inverses

Arccosine plus arcsine

# Example

Find the inverse of  $f(x) = 5 - 6\sin(3x)$  on interval  $\left[0, \frac{\pi}{6}\right]$ .

#### Trigonomet: functions composed with their inverses

Arccosine plus arcsine

# Example

Find the inverse of  $f(x) = 5 - 6\sin(3x)$  on interval  $\left[0, \frac{\pi}{6}\right]$ . • Write  $y = 5 - 6\sin(3x)$ .

Trigonomet functions composed with their inverses

Arccosine plus arcsine

# Example

Find the inverse of  $f(x) = 5 - 6\sin(3x)$  on interval  $\left[0, \frac{\pi}{6}\right]$ . • Write  $y = 5 - 6\sin(3x)$ . • Transform  $y = 5 - 6\sin(3x) \Leftrightarrow y - 5 = -6\sin(3x) \Leftrightarrow$ 

$$\frac{y-5}{-6} = \sin(3x) \Leftrightarrow 3x = \arcsin\left(\frac{y-5}{-6}\right) \Leftrightarrow x = \frac{1}{3}\arcsin\left(\frac{y-5}{-6}\right) = \frac{1}{3}\arcsin\left(\frac{5-y}{6}\right).$$

Trigonometri functions composed with their inverses

Arccosine plus arcsine

# Example

Find the inverse of  $f(x) = 5 - 6\sin(3x)$  on interval  $\begin{bmatrix} 0, \frac{\pi}{6} \end{bmatrix}$ . • Write  $y = 5 - 6\sin(3x)$ . • Transform  $y = 5 - 6\sin(3x) \Leftrightarrow y - 5 = -6\sin(3x) \Leftrightarrow \frac{y-5}{-6} = \sin(3x) \Leftrightarrow 3x = \arcsin\left(\frac{y-5}{-6}\right) \Leftrightarrow x = \frac{1}{3}\arcsin\left(\frac{y-5}{-6}\right) = \frac{1}{3}\arcsin\left(\frac{5-y}{6}\right)$ .

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**3** 
$$f^{-1}(x) = \frac{1}{3} \arcsin\left(\frac{5-x}{6}\right)$$

Trigonometri functions composed with their inverses

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# Example

Find the inverse of  $f(x) = 5 - 6\sin(3x)$  on interval  $\left[0, \frac{\pi}{6}\right]$ . • Write  $y = 5 - 6\sin(3x)$ . • Transform  $y = 5 - 6\sin(3x) \Leftrightarrow y - 5 = -6\sin(3x) \Leftrightarrow \frac{y-5}{-6} = \sin(3x) \Leftrightarrow 3x = \arcsin\left(\frac{y-5}{-6}\right) \Leftrightarrow x = \frac{1}{3}\arcsin\left(\frac{y-5}{-6}\right) = \frac{1}{3}\arcsin\left(\frac{5-y}{6}\right)$ . •  $f^{-1}(x) = \frac{1}{3}\arcsin\left(\frac{5-x}{6}\right)$ . • Check:  $f \circ f^{-1}(x) = 5 - 6\sin(3 \cdot \frac{1}{3}\arcsin\left(\frac{5-x}{6}\right)) = 5 - 6\sin(\arcsin\left(\frac{5-x}{6}\right)) = 5 - 6\sin(\arcsin\left(\frac{5-x}{6}\right)) = 5 - 6\cdot(5-x) = x \checkmark$ 

Trigonometri functions composed with their inverses

Arccosine plus arcsine

# Example

Find the inverse of  $f(x) = 5 - 6\sin(3x)$  on interval  $[0, \frac{\pi}{6}]$ . • Write  $y = 5 - 6\sin(3x)$ . 2 Transform  $y = 5 - 6\sin(3x) \Leftrightarrow y - 5 = -6\sin(3x) \Leftrightarrow y - 5 = -6\sin(3x)$  $\frac{y-5}{-6} = \sin(3x) \Leftrightarrow 3x = \arcsin\left(\frac{y-5}{-6}\right) \Leftrightarrow x =$  $\frac{1}{3}\arcsin\left(\frac{y-5}{-6}\right) = \frac{1}{3}\arcsin\left(\frac{5-y}{6}\right).$ **3**  $f^{-1}(x) = \frac{1}{3} \arcsin\left(\frac{5-x}{6}\right)$ . **Check:**  $f \circ f^{-1}(x) = 5 - 6\sin(3 \cdot \frac{1}{2}\arcsin(\frac{5-x}{6})) =$  $5 - 6\sin(\arcsin\left(\frac{5-x}{6}\right)) = 5 - 6 \cdot \left(\frac{5-x}{6}\right) = 5 - (5-x) = x \checkmark$ Range of  $f^{-1}$  = domain of  $f = \begin{bmatrix} 0, \frac{\pi}{6} \end{bmatrix}$ .

Trigonometri functions composed with their inverses

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# Example

Find the inverse of  $f(x) = 5 - 6\sin(3x)$  on interval  $\left[0, \frac{\pi}{6}\right]$ . • Write  $y = 5 - 6\sin(3x)$ . 2 Transform  $y = 5 - 6\sin(3x) \Leftrightarrow y - 5 = -6\sin(3x) \Leftrightarrow$  $\frac{y-5}{-6} = \sin(3x) \Leftrightarrow 3x = \arcsin\left(\frac{y-5}{-6}\right) \Leftrightarrow x =$  $\frac{1}{3}\arcsin\left(\frac{y-5}{-6}\right) = \frac{1}{3}\arcsin\left(\frac{5-y}{6}\right).$ **3**  $f^{-1}(x) = \frac{1}{3} \arcsin\left(\frac{5-x}{6}\right)$ . **Check:**  $f \circ f^{-1}(x) = 5 - 6\sin(3 \cdot \frac{1}{2}\arcsin(\frac{5-x}{6})) =$  $5 - 6\sin(\arcsin\left(\frac{5-x}{6}\right)) = 5 - 6 \cdot \left(\frac{5-x}{6}\right) = 5 - (5-x) = x \checkmark$ Range of  $f^{-1}$  = domain of  $f = \begin{bmatrix} 0, \frac{\pi}{6} \end{bmatrix}$ . As  $\sin(3x)$  attains all values between  $0 = \sin(0)$  and  $1 = \sin\left(\frac{3\pi}{6}\right)$ , we get domain of  $f^{-1} =$  range of f is [-1, 5].

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#### Trigonometric functions composed with their inverses

#### Arccosine plus arcsine

Let t be a number between 0 and 1. Consider a right triangle with the ratio of lengths of its legs equal to t.

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Trigonometric functions composed with their inverses

Arccosine plus arcsine Let t be a number between 0 and 1. Consider a right triangle with the ratio of lengths of its legs equal to t.



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Trigonometric functions composed with their inverses

Arccosine plus arcsine Let t be a number between 0 and 1. Consider a right triangle with the ratio of lengths of its legs equal to t.



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We get  $\arcsin(t) + \arccos(t) = \arcsin\left(\frac{a}{b}\right) + \arccos\left(\frac{a}{b}\right) = \alpha + \left(\frac{\pi}{2} - \alpha\right) = \frac{\pi}{2}.$ 

#### Lecture 22

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Trigonometric functions composed with their inverses

Arccosine plus arcsine Let t be a number between 0 and 1. Consider a right triangle with the ratio of lengths of its legs equal to t.



We get  $\arcsin(t) + \arccos(t) = \arcsin\left(\frac{a}{b}\right) + \arccos\left(\frac{a}{b}\right) = \alpha + \left(\frac{\pi}{2} - \alpha\right) = \frac{\pi}{2}.$ The equation  $\arcsin(t) + \arccos(t) = \frac{\pi}{2}$  holds true for any  $-1 \le t \le 1.$ 

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Trigonometric functions composed with their inverses

#### Arccosine plus arcsine

# Question

Evaluate  $\cos(\arcsin\left(\frac{\pi}{2}-1\right))$ .

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# Question

Evaluate  $\cos(\arcsin\left(\frac{\pi}{2}-1\right))$ .

**Answer:** using that  $\cos(\arcsin(\frac{\pi}{2}-1)) + \sin(\arcsin(\frac{\pi}{2}-1)) = \frac{\pi}{2}$ , we get  $\cos(\arcsin(\frac{\pi}{2}-1)) = \frac{\pi}{2} - (\frac{\pi}{2}-1) = \frac{\pi}{2} - \frac{\pi}{2} + 1 = 1$ .