Lecture 23

Using trigonometry to compute area

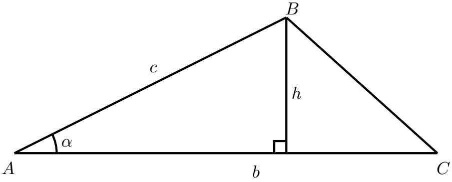
MATH 0200

Dr. Boris Tsvelikhovskiy

Outline

1. Area of a triangle
2. Area of a parallelogram
3. Area of a regular polygon

Area of a triangle



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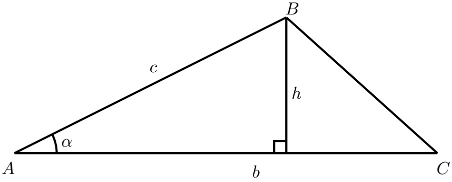

Recall that a triangle with *base* b and *height* h (the term 'base' refers to any side, and 'height' to the length of a perpendicular from the vertex opposite the base onto the line containing the base) has area $\mathcal{S} = \frac{bh}{2}$. Using that $\sin(\alpha) = \frac{h}{c} \Leftrightarrow h = c\sin(\alpha)$, we can write $\mathcal{S} = \frac{bc\sin(\alpha)}{2}$.

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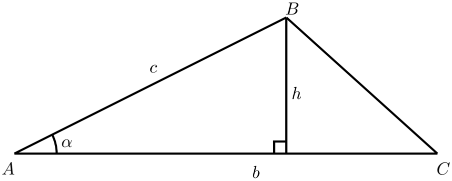

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Formula

A triangle with sides of length b and c and with angle α between those two sides has area $\mathcal{S} = \frac{bc\sin(\alpha)}{2}$.

Example

Find radian measure of angle α between two sides of a triangle with lengths $b = 5$ and $c = 4$, if the area of the triangle equals 3.

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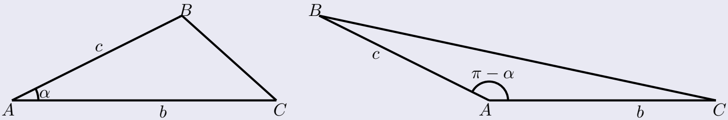

Question

Find the area of a triangle that has sides of length $3\sqrt{2}$ and 4, with a $45°$ angle between those sides.

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Answer: $\mathcal{S} = \frac{bc\sin(\alpha)}{2} = \frac{3\sqrt{2}\cdot 4\cdot \sin(45^\circ)}{2} = \frac{3\sqrt{2}\cdot 4\cdot \frac{1}{\sqrt{2}}}{2} = \frac{3\cdot 4}{2} = 6.$

Area of a parallelogram

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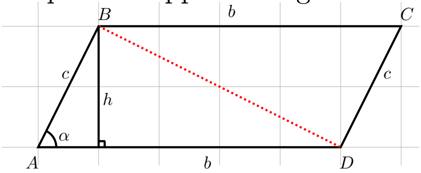

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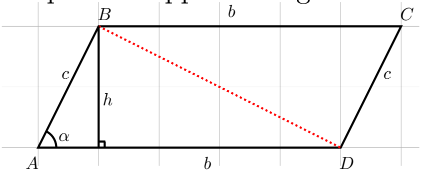

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Formula

A parallelogram with *adjacent* sides (sides with a common vertex) of lengths b and c and angle α between those two sides has area $\mathcal{S} = bc\sin(\alpha)$.

Example

Find the length of one of the sides of parallelogram if the side adjacent to it has length 10, the angle between two adjacent sides is $\alpha = \frac{\pi}{3}$, and the area of the parallelogram equals 7.

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We use the equation from previous slide (with one of the sides of length $b = 10$):

$\mathcal{S} = bc\sin(\alpha) \Leftrightarrow 7 = 10c\sin\left(\frac{\pi}{3}\right) \Leftrightarrow 0.7 = \frac{\sqrt{3}}{2}\cdot c \Leftrightarrow c = 1.4\cdot\frac{1}{\sqrt{3}}.$

Area of a regular polygon

One way to find the area of a polygon is to decompose ('cut') the polygon into triangles and then compute the sum of the areas of those triangles. This procedure works particularly well for a **regular polygon**, which is a polygon with all sides of the same length and all angles of the same measure.

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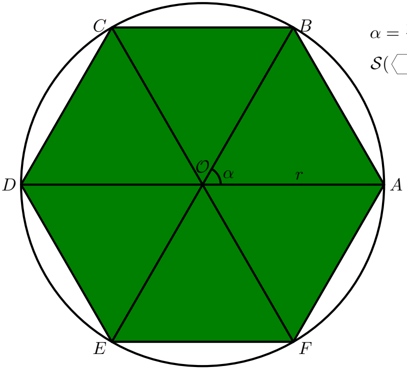

$\alpha = \frac{2\pi}{6} = \frac{\pi}{3}$

$\mathcal{S}(⬡) = 6\mathcal{S}(\triangle \mathcal{O}AB) = \frac{6r^2\sin(\alpha)}{2} = \frac{3\sqrt{3}r^2}{2}$

Formula

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Example

Find the area of a regular dodecagon (twelve-sided polygon) whose vertices are on the unit circle.

A dodecagon has $n = 12$ sides and the fact that its vertices lie on the unit circle implies $r = 1$. We get

$$\mathcal{S} = \frac{12 \cdot 1^2 \cdot \sin\left(\frac{2\pi}{12}\right)}{2} = 6 \sin\left(\frac{2\pi}{12}\right) = 6 \cdot \frac{1}{2} = 3.$$