Lecture 26
Transformation of trigonometric functions

MATH 0200

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Outline

1 Amplitude

2 Period

Amplitude

Definition

The **amplitude** of a function is one-half the difference between the maximum and minimum values of the function.

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Example

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Let's find the amplitude of the function $f(x) = 5\cos(x)$. Recall that the range of cosine is the closed interval $[-1, 1]$, hence, the range of $f(x)$ is $[-5, 5]$ and the amplitude equals $(5 - (-5))/2 = 5$.

Remark

A shift of a function $f(x)$ will likely change the maximal and minimal values, but will **NOT** change the difference between them. Therefore shifts preserve amplitude.

Example

Let's find the amplitude of the function
$f(x) = 5\cos(x - 2) + 7$.

Remark

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Example

Let's find the amplitude of the function
$f(x) = 5\cos(x - 2) + 7$.
The range of cosine $\cos(x - 2)$ is still the closed interval
$[-1, 1]$, hence, the range of $f(x)$ is $[2, 12]$ (here
$2 = 5 \cdot (-1) + 7$ and $12 = 5 \cdot 1 + 7$), so the amplitude equals
$(12 - 2)/2 = 5$ (as before).

Period

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Remark

Some functions do not repeat their behavior at regular intervals and thus do not have a period. For instance, any linear function $f(x) = mx + b$ (with $m \neq 0$) does not have a period. A function is called **periodic** if it has a period.

Example

1. The period of the function $f(x)$ whose graph is depicted below is equal to $\frac{\pi}{2}$.
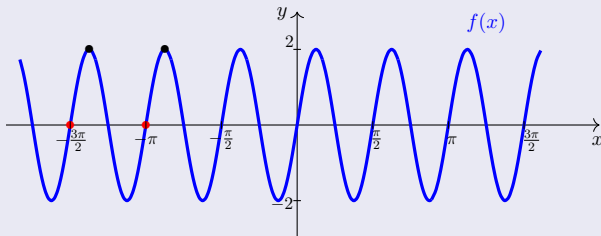
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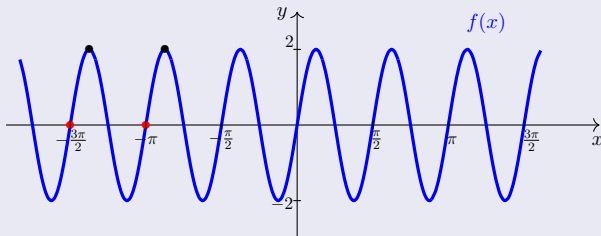
Example

① The period of the function $f(x)$ whose graph is depicted below is equal to $\frac{\pi}{2}$.

② The amplitude of $f(x)$ is equal to $\frac{2-(-2)}{2} = 2$.

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Recall that the period of $\cos(x)$ is $p = 2\pi$ and the graph of $g(x)$ is obtained from the graph of $\cos(x)$ by stretching it 10 times horizontally and 3 times vertically. While vertical stretch has no effect on the period, the horizontal one increases it 10 times, so we get $p = 20\pi$.

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Remark

- Shifts and vertical stretch have no effect on the period.
- The period of $\sin(mx)$ and $\cos(mx)$ is $p = \frac{2\pi}{m}$.

Question

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Question

Find the amplitude and period of $f(x) = 3\sin(7x) - 2$.

Answer: the range of $f(x)$ is the closed interval $[-5, 1]$, so the amplitude equals $\frac{1-(-5)}{2} = 3$. The period is $p = \frac{2\pi}{7}$.