# Lecture 2

## Functions

MATH 0200

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# Outline

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- The **domain** of a function $f$ is the collection of elements in the source set that get assigned some value in the target by $f$ (possible inputs).
- The **range** of a function $f$ is the collection of elements in the target set that get assigned to some element(s) in the source set by $f$ (attainable outcomes).

## Example

Consider two sets $S = \{\text{students in a calculus class}\}$ and $T = \{0, 1, 2, 3, 4, 5\}$ with the function $f : S \to T$ given by

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Let's make our example more explicit. Suppose there are 20 students, five really enjoy calculus and gave it a rating of 5, seven students hate math, so their response was 0, and the remaining part of the class decided not to participate. Then the domain of $f$ consists of $12 = 5 + 7$ students who took part in the survey and the range of $f$ is $\{0, 5\}$.

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- $g(t) = 5 - t - t^3$, giving $g(1) = 5 - 1 - 1^3 = 3$. For both functions $f$ and $g$ the domain and range are all real numbers.

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- $h(s) = \frac{5}{5-2s}$. This time the domain is $s \neq 2.5$ (as the denominator cannot be zero) and the range consists of all real numbers except 0.

## Question

Consider the function $f(x) = \frac{21}{x^2+3} + x$. What is the value of $f(2)$?

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|---|---|
| $-2$ | 3 |
| 0 | 1 |
| 3 | 4 |
| 5 | 3 |
| 15 | 3 |
| 19 | $-7$ |

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The domain of $f$ is $\{-2, 0, 3, 5, 15, 19\}$ and the range of $f$ is $\{-7, 1, 3, 4\}$.