Lecture 4

Transformations of functions and graphs thereof

MATH 0200

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Outline

1 Shifting

2 Flipping

3 Stretching

4 Even and odd functions

Shifting

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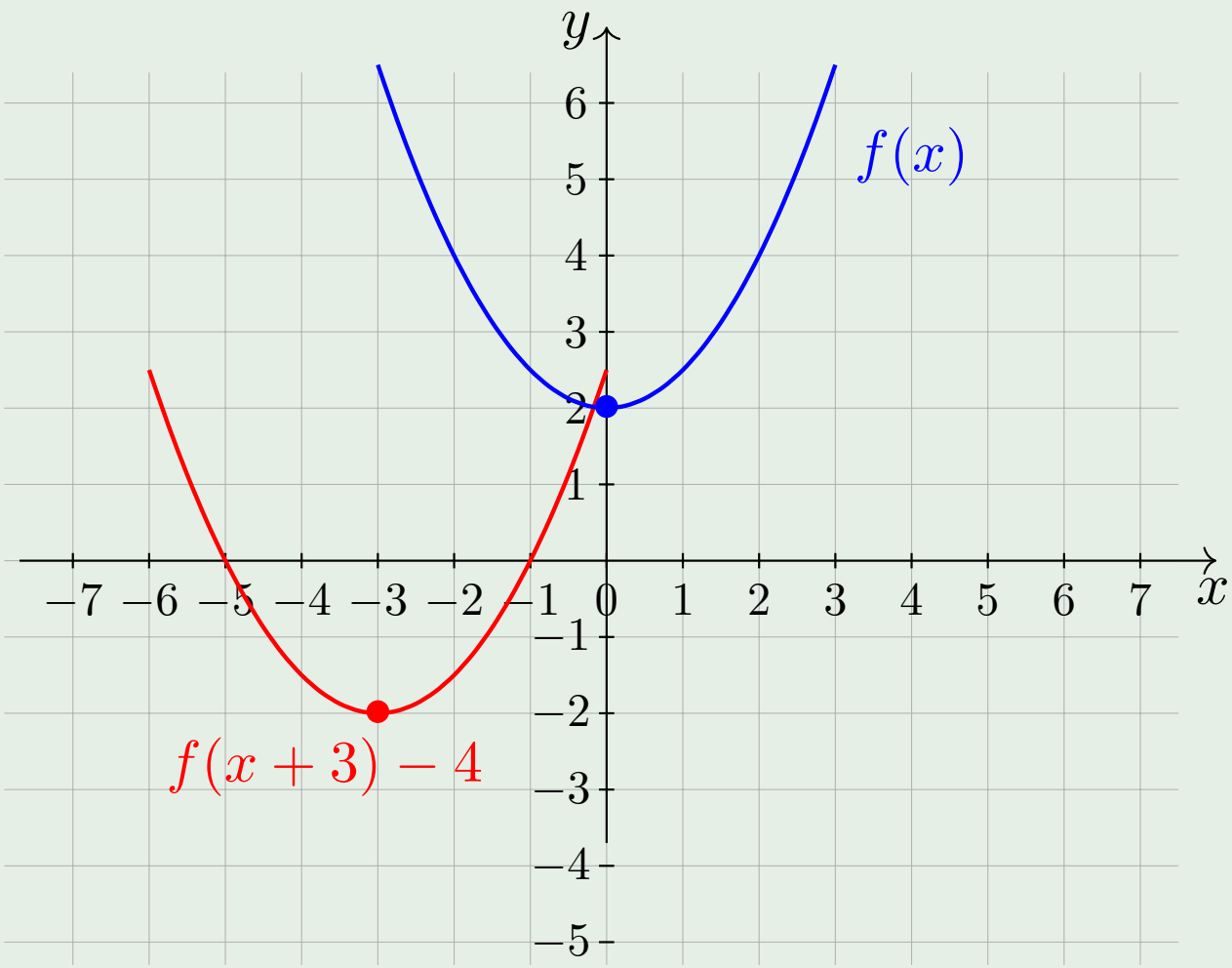
- the graph of $f(x)-c$ is obtained by shifting the graph of $f(x)$ by c units down.

Example



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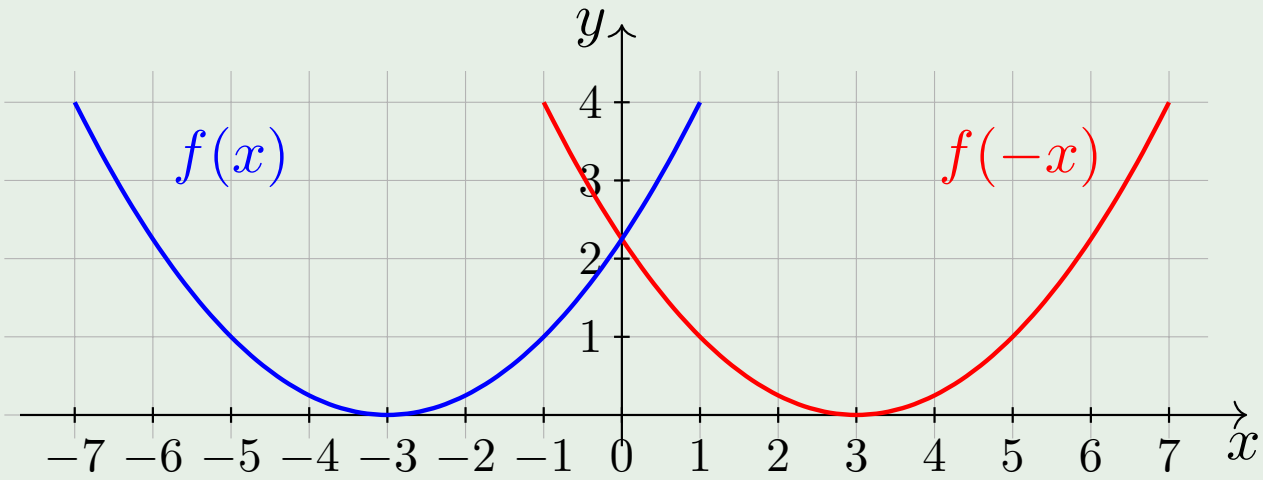

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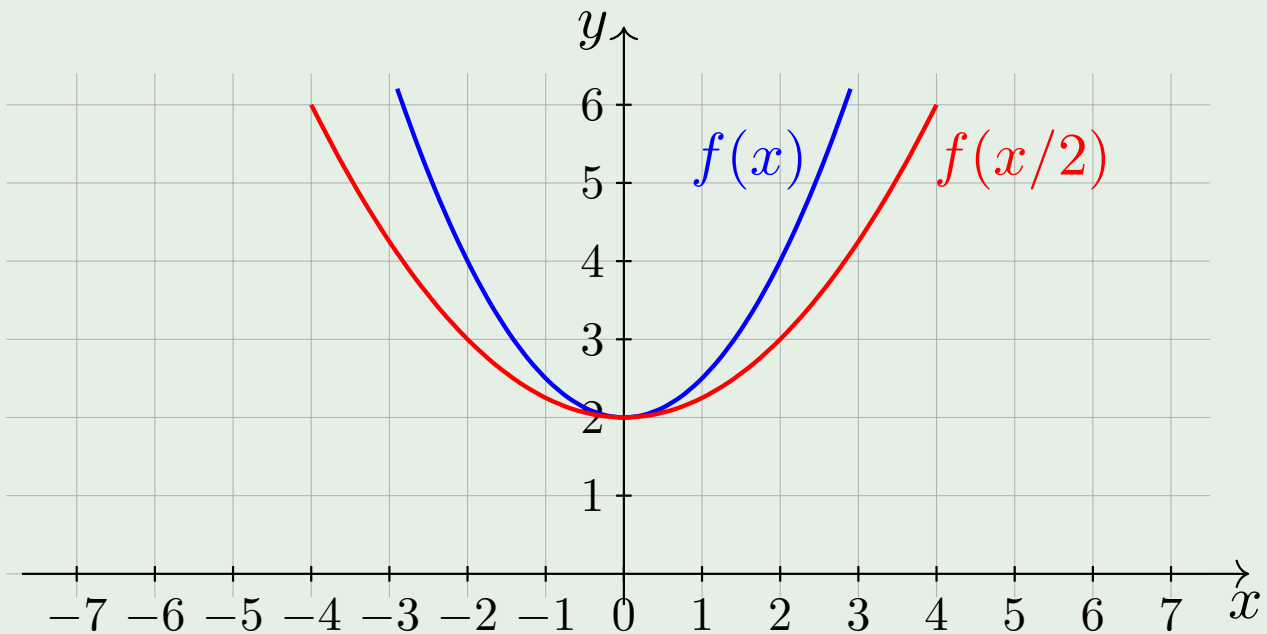

Question

Let $f(x)$ be a function with domain $[-3, 6]$ and range $[0, 5]$. What are the domain and range of the function $f(3x) + 2$?

Even functions

Definition

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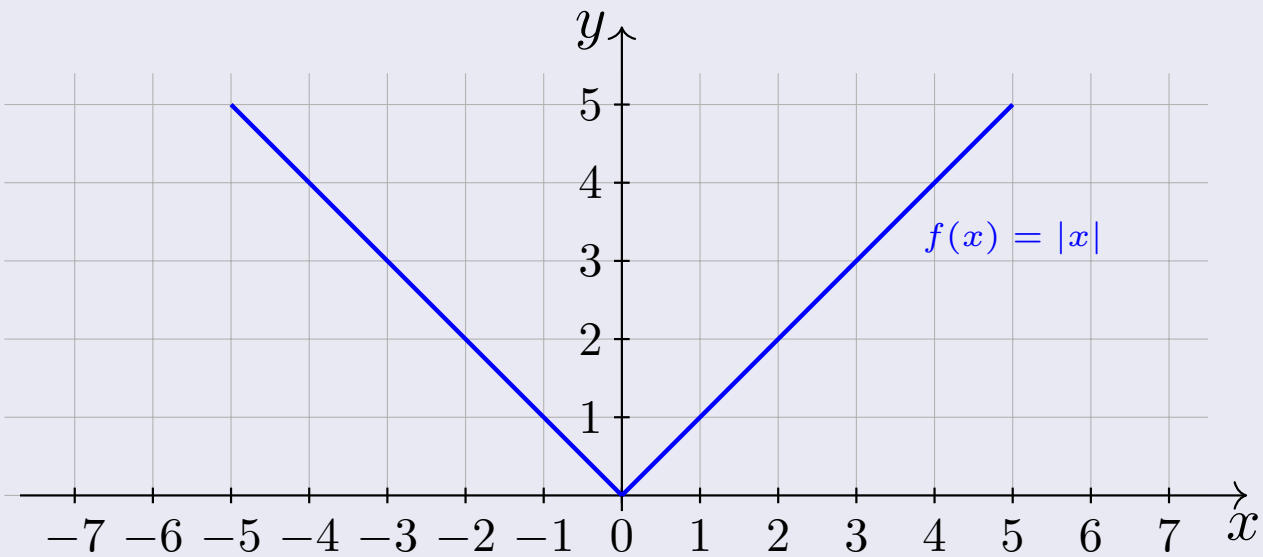

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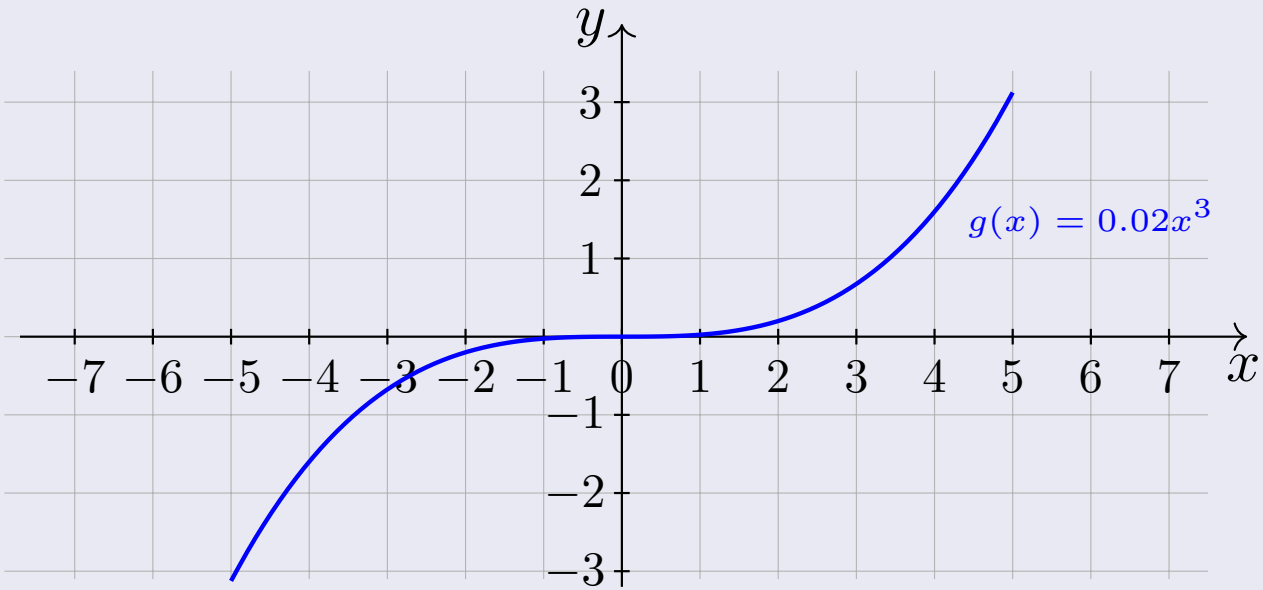

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- The graph of an even function is symmetric with respect to the $y-$axis (remains unchanged after reflection about the y-axis).
- The graph of an odd function has rotational symmetry with respect to the origin (remains unchanged after rotation of 180 degrees about the origin).