Lecture 5

Operations on functions, composition of functions

MATH 0200

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Outline

Basic operations on functions

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The domain of the first three functions is the intersection of the domains of f and g, while for the quotient, there is an additional requirement that $g(x) \neq 0$.

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Example

Consider two functions $f(x) = \frac{1}{x-2}$ and $g(x) = x^2$. Then $(f \pm g)(x) = \frac{1}{x-2} \pm x^2$, $(fg)(x) = \frac{x^2}{x-2}$ and $(f/g)(x) = \frac{1}{x^2(x-2)}$. The domain of f is $x \neq 2$ and the domain of g is all real numbers. The intersection of the domains is the set of all real numbers except two, in the interval notation, $(-\infty, 2) \cup (2, \infty)$. This is the domain of the functions $f \pm g$ and fg. Finally, the domain of f/g is $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$.

Real-life example

Suppose that you need to exchange dollars to euros. However, you are in a bank that only provides transactions where one of the currencies is pounds. The exchange rates are given by the following formulas. For d dollars one gets $f(d) = 0.7d$ pounds and for p pounds $g(p) = 1.2p - 3$ euros.

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Question

How many euros does the bank give for \$100?

Composition of functions

Definition

Let A, B and C be three sets and $g : A \to B, f : B \to C$ two functions between them. The **composition** of f and g is a function $f \circ g : A \to C$ obtained by successive applications of g and f (here f is applied to the result of application of g).

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Example

Consider two functions $f(x) = \sqrt{x-5}$ and $g(x) = 7 - x^2$. Then $(f \circ g)(x) = \sqrt{(7-x^2)-5} = \sqrt{2-x^2}$ and $(g \circ f)(x) = 7 - (\sqrt{x-5})^2 = 7 - (x-5) = 12 - x$.

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Remark

Notice that $(g \circ f)(x) \neq (g \circ f)(x)$.

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- Let $h(x) = \frac{1}{|x|-7}$, then we can write $h(x) = (f \circ g)(x)$ with $f(x) = \frac{1}{x}$ and $g(x) = |x| - 7$.