

Lecture 7

MATH 0200

Slope and  
*y*-intercept

Equation of  
a line

# Lecture 7

## Lines and linear functions

MATH 0200

Dr. Boris Tselikhovskiy

# Outline

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Slope and  
 $y$ -intercept

Equation of  
a line

① Slope and  $y$ -intercept

② Equation of a line

# Slope

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In this lecture we will go over linear functions and their graphs (lines on the plane).

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## Definition

Let  $P = (x_1, y_1)$  and  $Q = (x_2, y_2)$  be two points on a line  $\ell$ .  
The number  $m = \frac{y_2 - y_1}{x_2 - x_1}$  is called the **slope** of  $\ell$ .

# Slope

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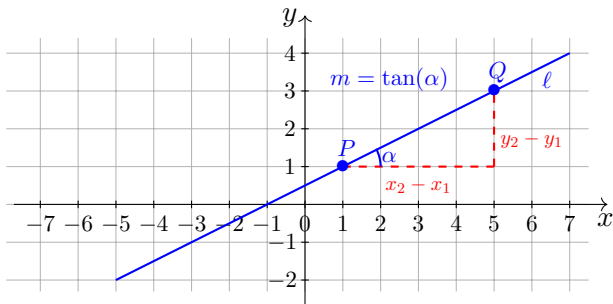
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# Data defining a line

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## Remark

The slope depends only on the line  $\ell$ , but not the two points on that line chosen to compute it.

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## Remark

The slope depends only on the line  $\ell$ , but not the two points on that line chosen to compute it.

Any line (except vertical lines) on  $xy$ -plane can be given by equation in the form  $y = mx + b$ , where  $m$  is the slope and  $b$  is the  **$y$ -intercept** ( $y$ -coordinate of the point where the line intersects the  $y$ -axis).

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- 1 its slope and a point;
- 2 two not coinciding points.

# Line with given slope and a point on it

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## Example

Let's find an equation of the line with slope 3 and passing through the point  $(1, 4)$ .

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## Example

Let's find an equation of the line with slope 3 and passing through the point  $(1, 4)$ . As the slope is  $m = 3$ , equation of the line must be  $y = 3x + b$ . It remains to find the value of  $y$ -intercept ( $b$ ). Here we use the fact that point  $(1, 4)$  is on the line:

$$4 = 3 \cdot 1 + b \Leftrightarrow b = 4 - 3 = 1,$$

and the equation becomes  $y = 3x + 1$ .

# Line containing two given points

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## Example

Find an equation of the line containing points  $(-1, 3)$  and  $(2, 2)$ .

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$$m = \frac{2 - 3}{2 - (-1)} = -\frac{1}{3}.$$

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Non coinciding lines on a plane are called **parallel** if their slopes are equal.

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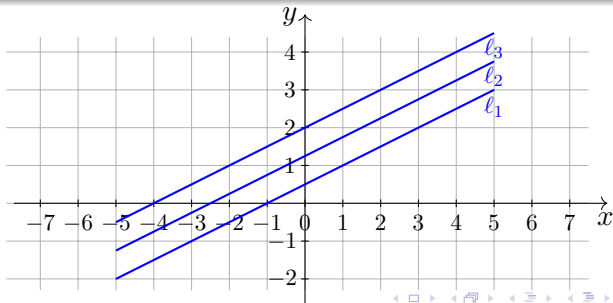
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Two lines on a plane are called **perpendicular** if they intersect at a right angle (90 degrees or  $\frac{\pi}{2}$  radians).

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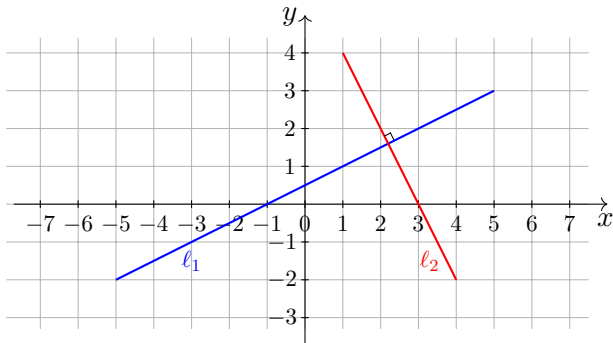
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Find an equation of the line passing through the point  $P = (1, 2)$  and perpendicular to the line given by equation  $y = 3x - 7$ .

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Find an equation of the line passing through the point  $P = (1, 2)$  and perpendicular to the line given by equation  $y = 3x - 7$ . First, we find the slope:  $m = -\frac{1}{3}$  and, using that the line contains  $P$ :

$$-\frac{1}{3} \cdot 1 + b = 2 \Leftrightarrow b = 2 + \frac{1}{3} = \frac{7}{3}.$$

The equation is  $y = -\frac{1}{3}x + \frac{7}{3}$ .

## Question

Consider the line  $\ell$  given by equation  $y = 7 - 5x$ .

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- 2 What is the slope of lines perpendicular to  $\ell$ ?