Summer 2020

Name:____ **Total**______/10

MATH 4581: Statistics and Stochastic Processes

Bonus problems II

Problem 1[**3 pts**] Recall that the Gamma function is given by $\Gamma(z) = \int\limits_0^\infty x^{z-1}e^{-x}dx$. Show that for any positive integer n one has $\Gamma(n) = (n-1)!$ [1].

Problem 2[**3 pts**] The probability density function for the F-distribution with m and n degrees of freedom is

$$f_{F_{m,n}}(x) = \frac{\Gamma\left(\frac{m+n}{2}\right) m^{\frac{m}{2}} n^{\frac{n}{2}} x^{\frac{m}{2}-1}}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)(n+mx)^{\frac{m+n}{2}}}.$$

Find the point at which $f_{F_{m,n}}(x)$ attains its maximal value. [2].

Problem 3[**4 pts**] Let $A_1, A_2, \dots A_n$ be a collection of events in a probability space (Ω, p) and let $\bar{A}_i$ stand for the complement of A_i. Prove the Bonferroni inequality[3]

$$P(\bigcap_{i=1}^{n} A_i) \geq 1 - \sum_{i=1}^{n} P(\bar{A}_i).$$

[1]**Hint:** use induction on n and integration by parts

[2]**Hint:** The number $\frac{\Gamma\left(\frac{m+n}{2}\right) m^{\frac{m}{2}} n^{\frac{n}{2}}}{\Gamma(\frac{m}{2})(\frac{n}{2})}$ is a constant, denote it by C and find the critical point of $f_{F_{m,n}}(x) = C\frac{x^{\frac{m}{2}-1}}{(n+mx)^{\frac{m+n}{2}}}$ by equating the derivative to zero

[3]**Hint:** see the proof for $n=2$ on page 2 of the notes