Summer 2020

Name:_____ **Total**_____/10

MATH 4581: Statistics and Stochastic Processes

Bonus problems III

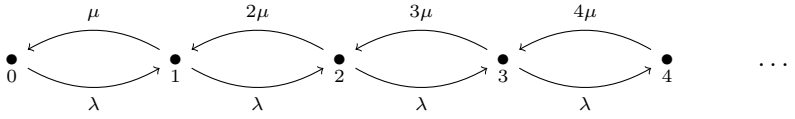
Problem 1 Three pottery shards from four widely scattered and now-extinct Native American tribes have been collected by a museum. Archaeologists were asked to estimate the age of the shards.

(a) [2 **pts**] Based on the results shown in the table below, use Bonferroni's method to test at the $\alpha = 5\%$ level of significance if the mean age of shards found in every tribe is the average mean age of shards found in the remaining three (i.e. use the contrasts $C_i = \mu_i - \frac{\sum_{j \neq i} \mu_j}{3}$).

Tribe 1	Tribe 2	Tribe 3	Tribe 4
1200	850	1800	950
800	900	1450	1200
950	1100	1150	1150

(b) [2 **pts**] If you were to test (using Bonferroni's method) at the α level of significance that the means in all possible pairs of tribes are the same, which critical value should you choose for each pair?

Problem 2 The goal is to derive the properties of an $M/M/\infty$ queue. In this system arrivals occur at rate λ according to a Poisson process and move the process from state i to $i+1$. Service times have an exponential distribution with parameter μ and there are always sufficient servers such that every arriving customer is served immediately, i.e. transitions from state i to $i-1$ are at rate $i\mu$.



(a) [2 **pts**] Write the balance equations and express p_n in terms of p_0. Do you recognize the distribution?

(b) [2 **pts**] Using the answer in (a) and the fact that $\sum_{n=0}^{\infty} p_n = 1$, find the steady state probabilities p_n.

(c) [2 **pts**] Derive the formula for the average number of customers in the system. [1]

[1] **Hint:** You may need the equality $\rho e^{\rho} = \sum_{n=1}^{\infty} \frac{\rho^n}{(n-1)!}$.