Summer 2020

MATH 4581: Statistics and Stochastic Processes

Options and Design of Portfolios

Figure 1: Who wins?

Problem 1. What follows are several portfolios denoted by $\mathcal{P}$. Graph the profit curve at $t = T$ for each of them. In this listing, C_E denotes a call option with strike price E, $-C_E$ represents going short on a Call, P_E is a put option, $-P_E$ is going short on a put, and the numbers E_i indicate the strike prices.

(a) [5 **pts**] $\mathcal{P}_{Butterfly} = \$10 - C_{E_1} + 2C_{E_2} - C_{E_3}$, where $E_1 < E_2 < E_3$.

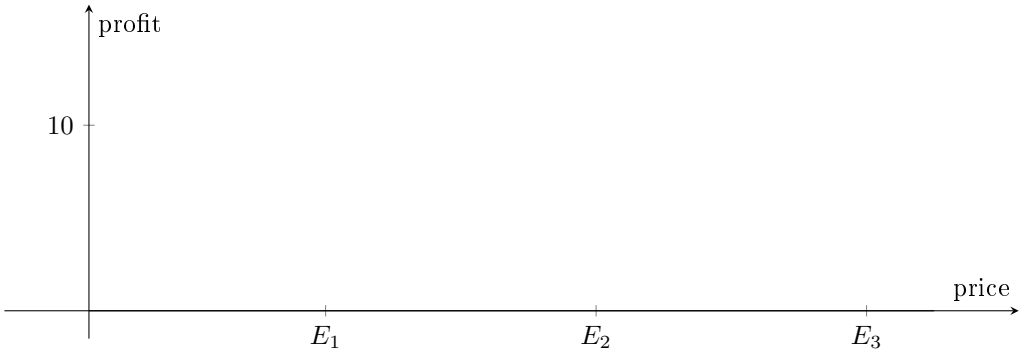

Figure 2: Portfolio (a)

(b) [5 **pts**] $\mathcal{P} = \$5 + C_{E_1} - C_{E_2} - C_{E_3} + C_{E_4}$, where $E_1 < E_2 < E_3 < E_4$.

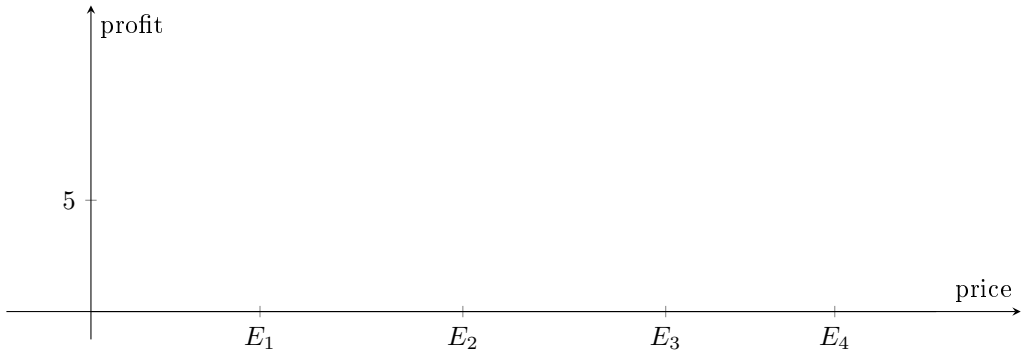

Figure 3: Portfolio (b)

(c)* [5 **pts**] $\mathcal{P}_{Box} = C_{E_1} - P_{E_1} - C_{E_2} + P_{E_2}$, where $E_1 < E_2$.

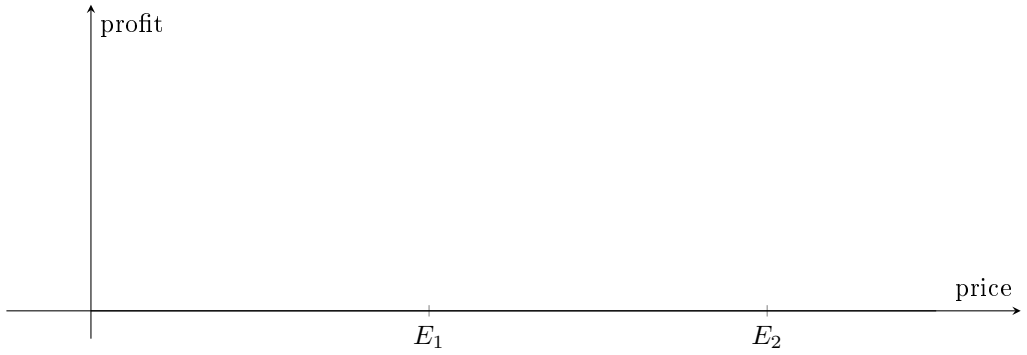

Figure 4: Portfolio (c)

Problem 2. For a given shape of the profit curve, design the portfolio and draw the graph of the profit as a function of price. The profit line is horizontal $\mathcal{P} = \$10$ until price \$60. At that point, it has slope 3 until price \$70. Then, the line has slope 2 until price \$100. Next, it has slope zero until price \$110. It then has a slope of -1 until \$120. After that, it has slope zero.

(a) [3 **pts**] Draw the graph of the profit as a function of price.

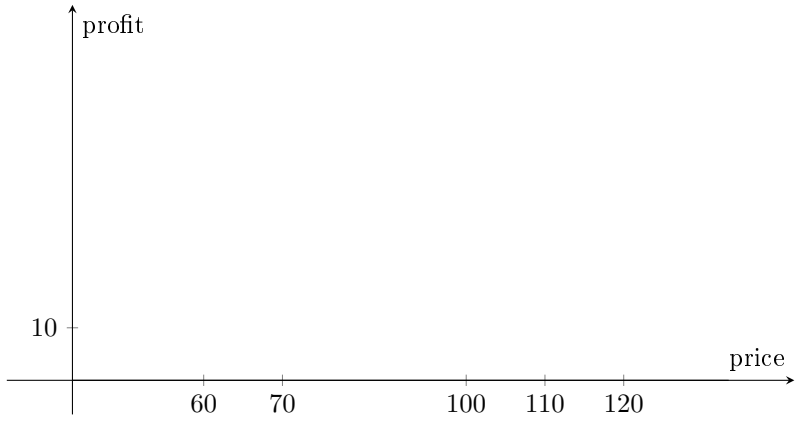

Figure 5: Profit as a function of price

(b) [4 **pts**] Design the portfolio with the above behavior using only call options.

(c) [4 **pts**] Design the portfolio with the above behavior using only put options.

(d) [4 **pts**] Design a portfolio with the above behavior using both call and put options.

Problem 3 [5 **pts**]. Design the portfolios according to the table and graphs below.

Behavior type	Calls only	Puts only
Bear		
Bull		

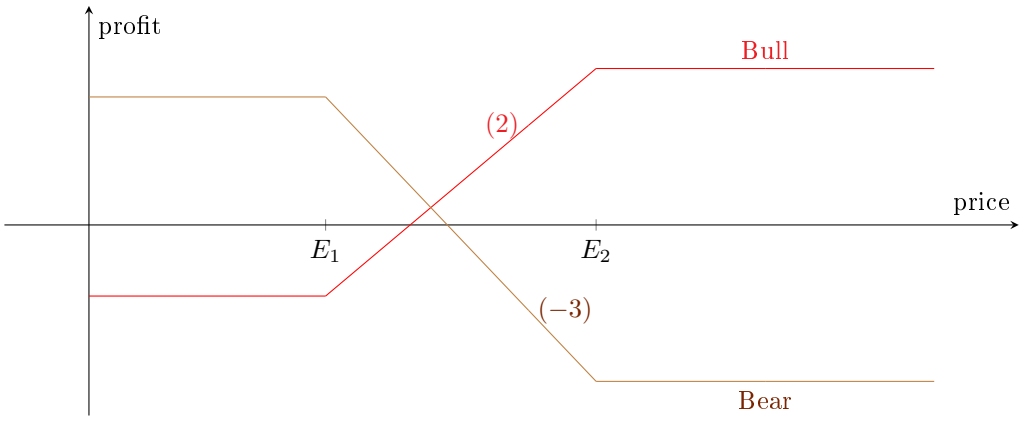

Figure 6: Bear vs Bull

Problem 4. Edward and Jeffrey are friends. Every morning they read a newspaper. Edward's choice is The Washington Post, while Jeffrey prefers The New York Times.

(a) [7 **pts**] The Washington Post published this morning that for an expiration date of a year from now (with 5% interest) the price of a call option $C_{60}(70,t)$ is \$9 and the price of a put option $P_{60}(70,t)$ is \$4. How can Edward use this information to make some money?[1]

(b) [8 **pts**] The New York Times assure that for an expiration date of a year from now (with 5% interest) $C_{70}(68,t)$ is \$9 and $P_{70}(68,t)$ is \$4. How can Jeffrey use this information to his advantage?

The following problem does not contribute to the grade.

Problem 5. Let $B(t)$ be a standard Brownian motion.

(a) Find $P(B(4) > 1)$

(b) Find $P(B(4) > 1 \cap B(7) - B(4) < 2)$

(c) Find $P(B(4) > 3 | B(2) = 1)$.

[1] **Hint:** use the put–call parity equation $P_E(S,t) + S = C_E(S,t) + Ee^{-r(T-t)}$