Summer 2020

MATH 4581: Statistics and Stochastic Processes

Options and Design of Portfolios

Solutions

Figure 1: Who wins?

Problem 1. What follows are several portfolios denoted by $\mathcal{P}$. Graph the profit curve at $t = T$ for each of them. Determine the strengths and weaknesses of each portfolio. Namely, explain what a reasonable investor must be expecting to adopt each portfolio. In this listing, C_E denotes a call option with strike price E, $-C_E$ represents going short on a Call-the Call was sold, P_E is a put option, $-P_E$ is going short on a put, and the numbers E indicate the strike price.

(a) [5 **pts**] $\mathcal{P}_{Butterfly} = \$10 - C_{E_1} + 2C_{E_2} - C_{E_3}$, where $E_1 < E_2 < E_3$.

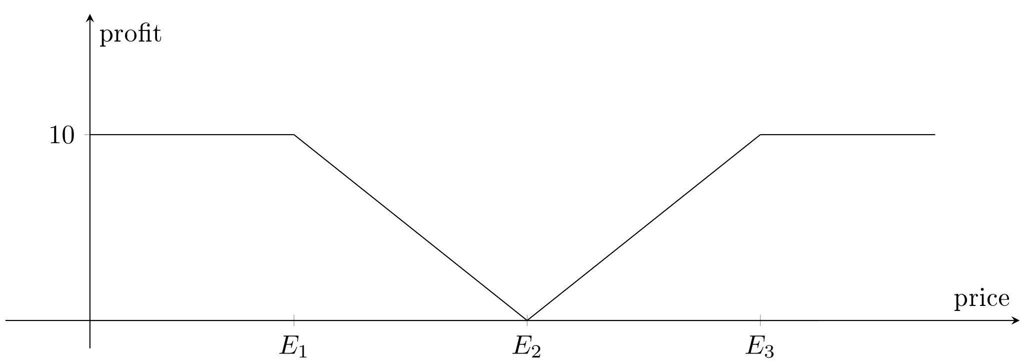

Figure 2: Portfolio (a)

(b) [5 **pts**] $\mathcal{P} = \$5 + C_{E_1} - C_{E_2} - C_{E_3} + C_{E_4}$, where $E_1 < E_2 < E_3 < E_4$.

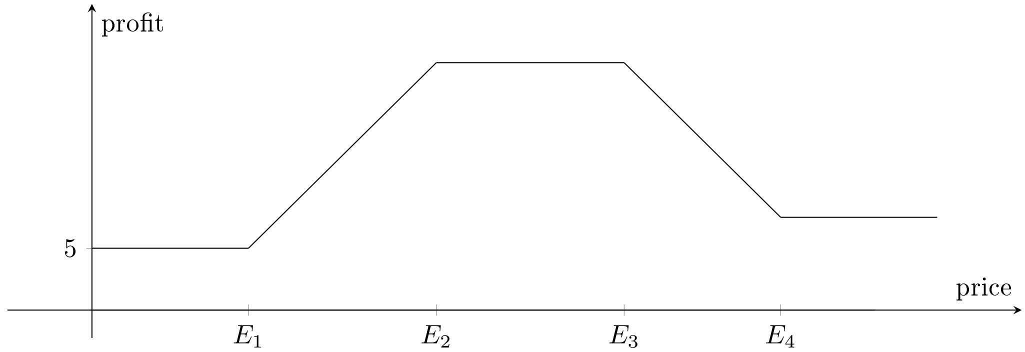

Figure 3: Portfolio (b)

(c)* [5 **pts**] $\mathcal{P}_{Box} = C_{E_1} - P_{E_1} - C_{E_2} + P_{E_2}$, where $E_1 < E_2$.

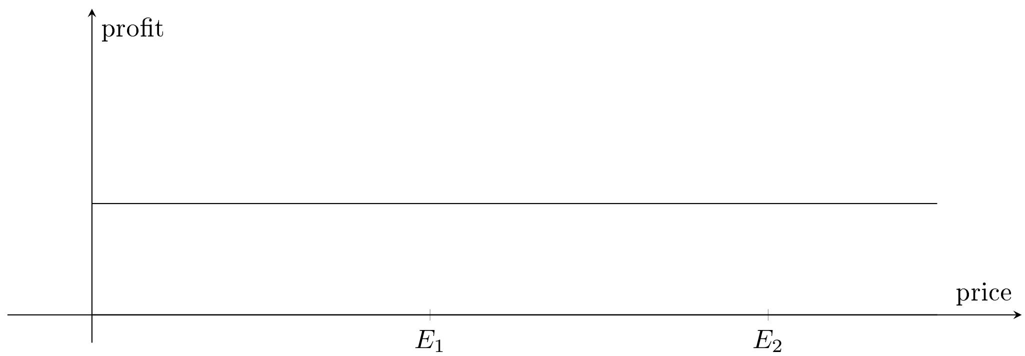

Figure 4: Portfolio (c)

Problem 2. For a given shape of the profit curve, design the portfolio and draw the graph of the profit as a function of price. The profit line is horizontal $\mathcal{P} = \$10$ until price \$60. At that point, it has slope 3 until price \$70. Then, the line has slope 2 until price \$100. Next, it has slope zero until price \$110. It then has a slope of -1 until \$120. After that, it has slope zero.

(a) [3 **pts**] Draw the graph of the profit as a function of price.

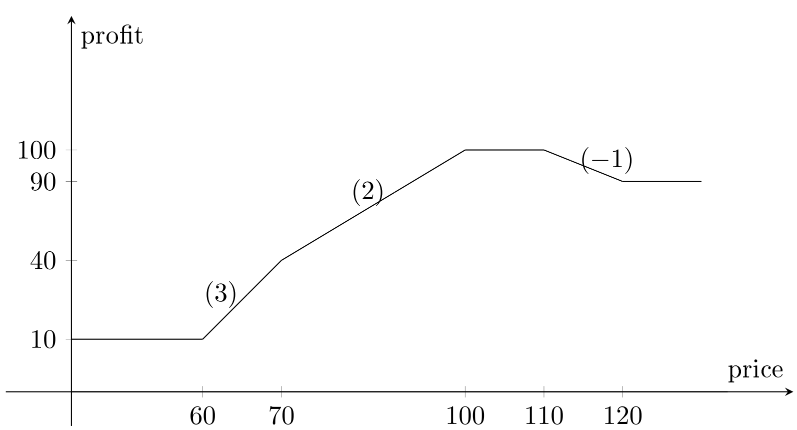

Figure 5: Profit as a function of price

(b) [4 **pts**] Design the portfolio with the above behavior using only call options.

$$\mathcal{P} = \$10 + 3C_{60} - C_{70} - 2C_{100} - C_{110} + C_{120}.$$

(c) [4 **pts**] Design the portfolio with the above behavior using only put options.

$$\mathcal{P} = \$10 + 3P_{60} - P_{70} - 2P_{100} - P_{110} + P_{120}.$$

(d) [4 **pts**] Design a portfolio with the above behavior using both call and put options.

$$\mathcal{P} = \$10 + 3C_{60} - C_{70} - 2C_{100} - P_{110} + P_{120}.$$

Problem 3 [5 **pts**]. Design the portfolios according to the table and graphs below.

Behavior type	Calls only	Puts only
Bear	$-3C_{E_1} + 3C_{E_2}$	$-3P_{E_1} + 3P_{E_2}$
Bull	$2C_{E_1} - 2C_{E_2}$	$2P_{E_1} - 2P_{E_2}$

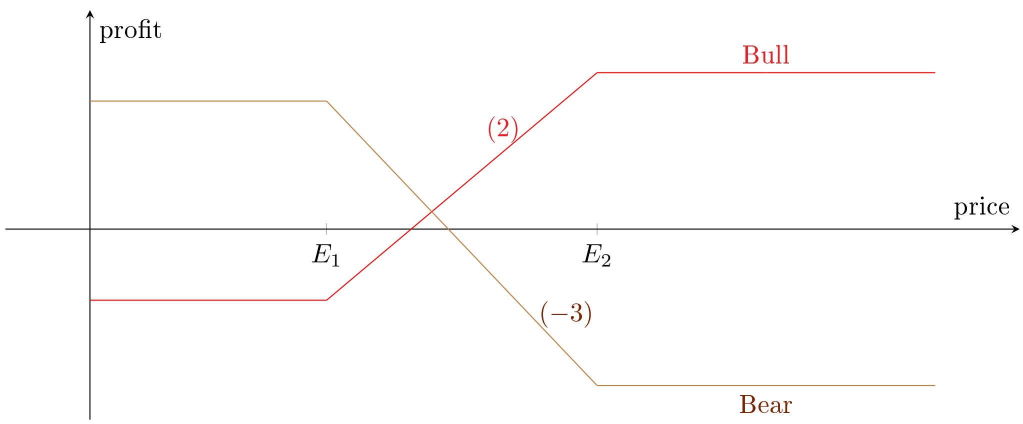

Figure 6: Bear vs Bull

Problem 4. Edward and Jeffrey are friends. Every morning they read a newspaper. Edward's choice is The Washington Post, while Jeffrey prefers The New York Times.

(a) [7 **pts**] The Washington Post published this morning that for an expiration date of a year from now (with 5% interest) the price of a call option $C_{60}(70,t)$ is \$9 and the price of a put option $P_{60}(70,t)$ is \$4. How can Edward use this information to make some money?[1]

Notice that one has $P_{60}(70,t)+S=4+70=74>66.07=9+57.07=C_{60}(70,t)+60e^{-0.05}$. Let us show that there is an arbitrage opportunity. This can be done in two steps.

1. Go short on put options and sell stocks.
2. Buy call options and put money in the bank, each time keep the \$5 difference.

At the expiration time $t=T$ the parity equation becomes $P_E(S,T)+S=C_E(S,t)+E$ and holds by the definition of the call and put contracts.

(b) [8 **pts**] The New York Times assure that for an expiration date of a year from now (with 5% interest) $C_{70}(68,t)$ is \$9 and $P_{70}(68,t)$ is \$4. How can Jeffrey use this information to his advantage?

This time use the inequality $P_{70}(68,t)+S=4+68=72<75.59=9+66.59=C_{70}(68,t)+70e^{-0.05}$.

1. Go short on call options and withdraw the money from the bank.
2. Buy put options and stocks, each time keep the \$3.59 change.

[1] **Hint:** use the put–call parity equation $P_E(S,t)+S=C_E(S,t)+Ee^{-r(T-t)}$