Name:_____ **Total**_____/50

MATH 4581: STATISTICS AND STOCHASTIC PROCESSES

QUIZ 1

Problem 1

(a) [5 **pts**] Find the moment-generating function of the random variable X having the distribution $P(X = k) = \frac{1}{8}\binom{3}{k}$ with $k \in \{0, 1, 2, 3\}$.

(b) [10 **pts**] Using the result in (a), find the expected value $\mathbb{E}(X)$, variance $Var(X)$ and standard deviation $\sigma(X)$.

Problem 2

(a) [5 **pts**] Check that the function $f_Y(y) = \frac{1}{3}$ for $0 \leq y \leq 2$ and $f_Y(y) = \frac{2y}{15}$ for $2 \leq y \leq 3$ is a probability distribution on the interval $[0, 3]$.

(b) [10 **pts**] Find the moment-generating function of the random variable Y having the distribution $f_Y(y)$ as above.

Problem 3[5 **pts**] Let X and Y be independent random variables. Express the moment-generating function of $W = 3X - 2Y + 2020$ in terms of $M_X(t)$ and $M_Y(t)$, the moment-generating functions of X and Y.[1]

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[1]**Hint:** use property (3) for the independent random variables $3X$ and $-2Y + 2020$ followed by the application of property (2) to the random variable $-2Y + 2020$.

Problem 4 In the dataset "Popular Kids", students in grades $5-7$ were asked whether good grades, athletic ability, or popularity was most important to them. A two-way table separating the students by grade and by choice of most important factor is shown below

Goals	Grade 5	Grade 6	Grade 7	Total
Grades	49	50	69	168
Popularity	24	36	38	98
Sports	19	22	28	69
Total	92	108	135	335

Table 1: Observed values

(a) [**7 pts**] Fill in the table of expected values.

Goals	Grade 5	Grade 6	Grade 7
Grades			
Popularity			
Sports			

Table 2: Expected values

(b) [**8 pts**] Use the χ^2 test and either the critical value or p-value to decide if there is a statistically significant difference at the level $\alpha = 5\%$ between the preferences of three groups.