

Name: _____ Total _____/50

MATH 4581: STATISTICS AND STOCHASTIC PROCESSES

QUIZ 2

Problem 1 Consider the $M/M/1$ system with average arrival rate 5 people per hour and mean service time 6 minutes.

(a) [5 pts] Compute the steady state probabilities.

(b) [5 pts] Find the average number of customers in the system and the average amount of time a customer spends in the system.

Problem 2 On Monday morning in the Bank of Wonderland customers arrive at the average rate of 8 per 6 minutes. Three tellers are operating with mean service time of 90 seconds. In addition, the bank president Alice does not allow more than 4 customers inside the bank simultaneously.

(a) [5 pts] Determine the type of the queue described above, i.e. fill in the missing numbers indicated by the question marks: $M/M/?/?$.

(b) [10 pts] Find the steady state probabilities p_i . Show your work.

(c) [5 pts] Find L , the average number of customers in the system. Show your work.

(d) [5 pts] Find W , the average amount of time a customer has to wait in the system and W_Q , the average amount of time a customer has to wait in the queue.¹

Problem 3 Northeastern students send or receive an average of 48 text messages per day.

(a) [2 pts] How many text messages does a Northeastern student receive or send on average per hour?

(b) [5 pts] What is the probability that a Northeastern student receives or sends four messages per hour?

(c) [8 pts] What is the probability that a Northeastern student receives or sends at least three messages per hour?

¹**Hint:** find W using Little's formula, then $W_Q = W - \frac{1}{\mu_a}$, where $\mu_a = \sum_{i=1}^4 p_i \mu_i$ is the average departure rate and $\frac{1}{\mu_a}$ is the mean service time.