

Name: \_\_\_\_\_ Total \_\_\_\_\_/100

## MATH 4581: STATISTICS AND STOCHASTIC PROCESSES

## TEST 2

**Problem 1.** Let  $X_1(t)$ ,  $X_2(t)$  and  $X_3(t)$  be independent Poisson processes with means  $\lambda_1 = 1$ ,  $\lambda_2 = 2$  and  $\lambda_3 = 3$ , respectively. Let  $X(t)$  be the merged process  $X(t) = X_1(t) + X_2(t) + X_3(t)$ .

(a) [5 pts] Find the probability that  $X(1) = 2$  and  $X(2) = 4$ .

(b) [10 pts] Given that  $X(1) = 2$ , find the probability that  $X_1(1) = 1$ .

**Problem 2.** Consider an  $M/M/1$  system with  $\lambda = 6$  and  $\mu = 18$  (per minute).

(a) [5 pts] Find  $L$ , the average number of customers in the system and  $W$ , the average amount of time a customer spends in the system.

(b) [5 pts] Give the density function  $f_T(t)$ .

(c) [5 pts] Find the probability that the time a customer spends in the system exceeds 10 seconds.

**Problem 3.** Use Ito's formula to compute the differentials of the following functions ( $B(t)$  is a standard Brownian motion):

(a) [5 pts]  $X(t, B(t)) = t + B^5(t)$

(b) [5 pts]  $Y(t, B(t)) = \sin(t^2 + B^2(t))$

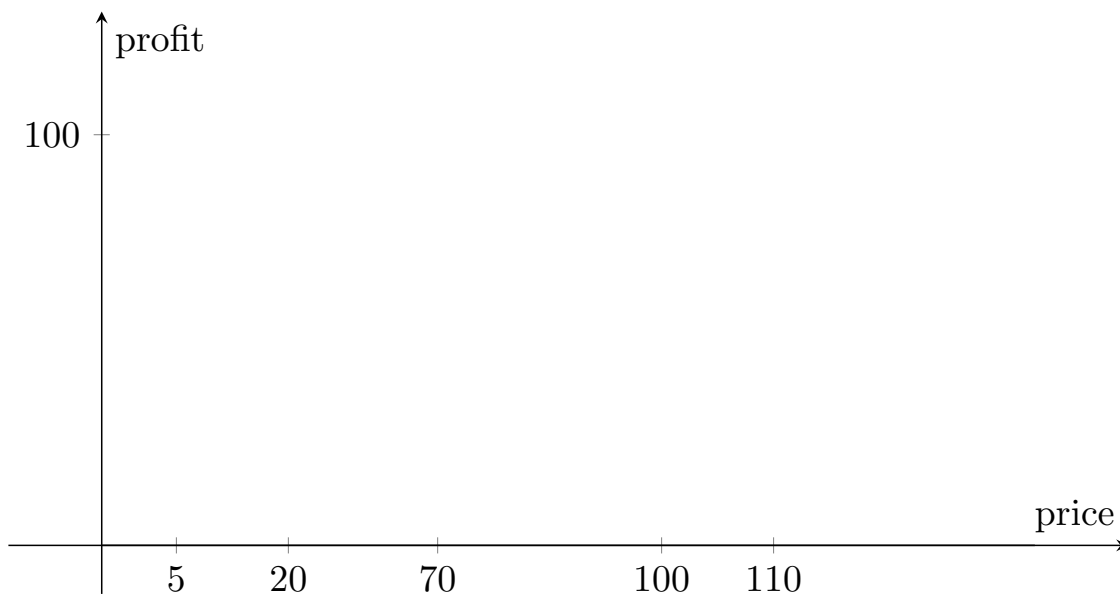
**Problem 4.** Let  $B(t)$  be a standard Brownian motion.

(a) [5 pts] Find  $P(B(9) < 2)$

(b) [10 pts] Find  $P(B(10) - B(6) > 4 \mid B(5) - B(2) = 2020)$ .

**Problem 5.** For a given shape of the profit curve, design the portfolio and draw the graph of the profit as a function of price. The profit line is horizontal  $\mathcal{P} = \$100$  until the price is \$5. Then the profit line has slope  $-2$  until price \$20. At that point, it has slope  $-1$  until price \$70. Then, the line is horizontal until price \$100. Next, it has slope 4 until price \$110. After that, it is horizontal.

(a) [5 pts] Draw the graph of the profit as a function of price.



(b) [5 pts] Design the portfolio with the above behavior using only call options.

(c) [5 pts] Design the portfolio with the above behavior using only put options.

**Problem 6** [15 pts] Peter read in the press this morning that, for an expiration date of a year from now (with 5% interest) that  $C_{60}(70, t) = 9$  and  $P_{60}(70, t) = 4$ . How can he use this information to make some money?

**Problem 7** Let  $P(t)$  be the price of a stock at time  $t$ . Assume that the current price of the stock is \$50 and it is modeled by a geometric Brownian motion with drift parameter  $\mu = -0.1$  and volatility  $\sigma = .49$ .

(a) [10 pts] Find the probability that the price of the stock in two years is between \$30 and \$60.

(b) [5 pts] If the yearly interest rate is  $r = .05$ , what should the selling price of a European 2 year Call option with strike price \$35 be, so there is no arbitrage opportunity?