Spring 2023

MATH 0200: Preparation for Scientific Calculus

Review Midterm Exam 2

1. Find the length of the circular arc of the unit circle connecting the point $P_1 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and the point whose radius corresponds to 1 radian.

2. For a 14-inch pizza  , find the area of a slice with angle $4/7$ radians.

3. Suppose y is a number such that $\tan(y) = -\frac{5}{7}$. Find $\tan(-y)$.

4. Suppose that $\sin(\alpha) = \frac{3}{5}$ and α is in the second quadrant. Use trigonometric identities to find the exact values of the following quantities.

   (*a*) $\cos(\alpha)$

   (*b*) $\sin(2\alpha)$

   (*c*) $\cos(2\alpha)$

(d) $\tan(2\alpha)$

5. Let α be an angle in the first quadrant, and suppose $\sin(\alpha) = a$. Evaluate the following expressions in terms of a. For example, $\sin(\alpha + \pi) = -a$. Your answers need to be expressions that involve a.

   (a) $\sin\left(\alpha + \dfrac{3\pi}{2}\right)$

   (b) $\cos\left(\dfrac{\pi}{2} - \alpha\right)$

   (c) $\sin(\pi - \alpha)$

   (d) $\sin\left(\dfrac{\pi}{2} - \alpha\right)$

   (e) $\cos(2\pi - \alpha)$

6. A 12-foot ladder leans against a vertical wall forming an angle of 60° with the ground. How high above the ground does the ladder touch the wall?

7. Use trigonometric identities to find the exact value of each expression.

   (*a*) $\cos(48^\circ)\cos(12^\circ) - \sin(48^\circ)\sin(12^\circ)$

   (*b*) $\dfrac{\tan(78^\circ) + \tan(112^\circ)}{1 - \tan(78^\circ)\tan(112^\circ)}$

8. Show (without using a calculator) that

$$\sin\left(\frac{\pi}{7}\right)\cos\left(\frac{4\pi}{21}\right) + \cos\left(\frac{\pi}{7}\right)\sin\left(\frac{4\pi}{21}\right) = \frac{\sqrt{3}}{2}.$$

9. (*a*) Use the half angle formula to find the value of $\sin\left(\dfrac{\pi}{8}\right)$.

   (*b*) Find the area of a regular 16-sided polygon whose vertices are 16 equally spaced points on a circle with radius 3.

10. Find the perimeter of a regular 12-sided polygon whose vertices are 12 equally spaced points on a circle with radius 3.[1]

11. Find the smallest number t such that $\cos(3^t) = 0$.

[1] **Hint:** use the law of cosines