MATH 0200: Preparation for Scientific Calculus

Review for the Final Exam

1. Solve $e^{2x} + e^x = 6$.

2. Solve $\dfrac{\ell n(11x)}{\ell n(4x)} = 2$.

3. Solve $\ell n(\ell nx) = 5$.

4. How many digits does $5^{999} \cdot 17^{222}$ have?

5. Suppose a savings account pays 5% interest per year, compounded four times per year. If the savings account starts with \$600, how many years would it take for the savings account to exceed \$1400?

6. A baseball card bought for \$50 increases by 10% in value each year. How long does it take for the card to quadruple in price?

7. (*a*) A baby alligator 🐊 is born 9 inches long and grows by 5% each month. How old will the alligator be when it doubles its initial length?

(*b*) Our alligator has eaten a slice 🍕 of a 20-inch pizza 🍕 with angle 2 radians. The weight of the alligator grows by approximately 250 grams per square inch of pizza eaten. How much weight did the alligator gain 🐊🍕...???

8. (*a*) Show that $\cos(15°) = \dfrac{\sqrt{2 + \sqrt{3}}}{2}$

(*b*) Show that $\sin(15°) = \dfrac{\sqrt{2 - \sqrt{3}}}{2}$

9. Suppose that $\sin(\alpha) = -\dfrac{2}{7}$ and α is in the second quadrant. Use trigonometric identities to find the exact values of the following quantities.
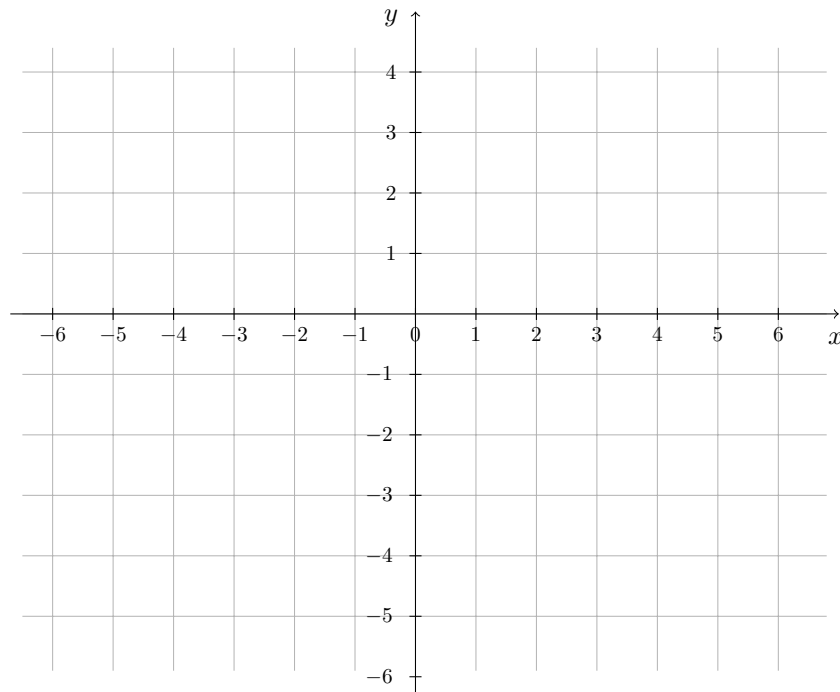
(a) $\cos(\alpha)$

(b) $\sin(2\alpha)$

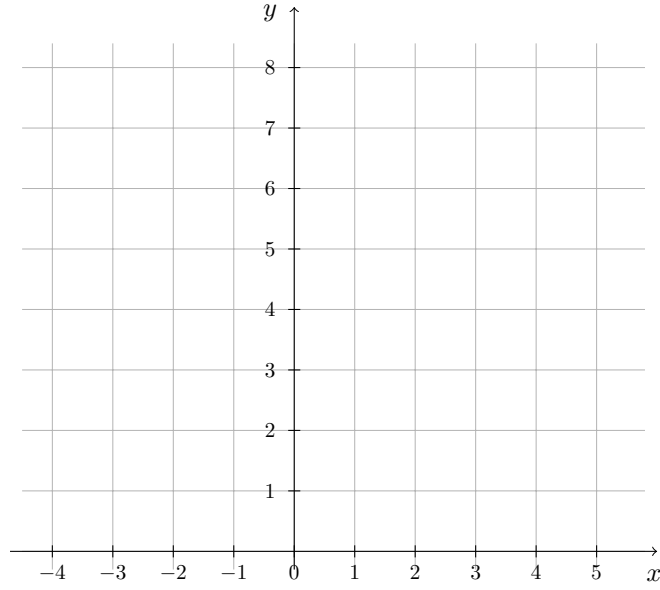(c) $\cos(2\alpha)$

10. Find the smallest number x such that $\cos(e^x + 1) = 0$.

11. Find the amplitude and period of the given function $f(x)$ on the given interval $[a, b]$. Sketch the graph and mark any line segments corresponding to amplitude and period.

(a) $f(x) = 5\sin\left(\dfrac{\pi x}{3}\right) - 1$ on the interval $[-6, 6]$.

(b) $f(x) = -2\cos(3\pi x) + 6$ on the interval $[-2, 2]$.

y

8

7

6

5

4

3

2

1

−4   −3   −2   −1   0   1   2   3   4   5 x

12. Find the value of t for which the vectors $\mathbf{u}$ and $\mathbf{v}$ are perpendicular.

(a) $\mathbf{u} = (2\ell n(t), -3)$ and $\mathbf{v} = (1, 6)$.

(b) $\mathbf{u} = (56, 2)$ and $\mathbf{v} = (-1, 7^t)$.

(c) $\mathbf{u} = \left(-\dfrac{\pi}{3}, 2\right)$ and $\mathbf{v} = (1, \arccos(t))$.

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13. Rewrite the following equations in polar coordinates.

    (a) $x^2 + y^2 = 49$.

    (b) $(x - 5)^2 + y^2 = 9$.

    (c) $x^2 + (y + 3)^2 = 25$.

14. Rewrite the following equations in Cartesian coordinates.

    (a) $r = 3\cos(\theta)$.

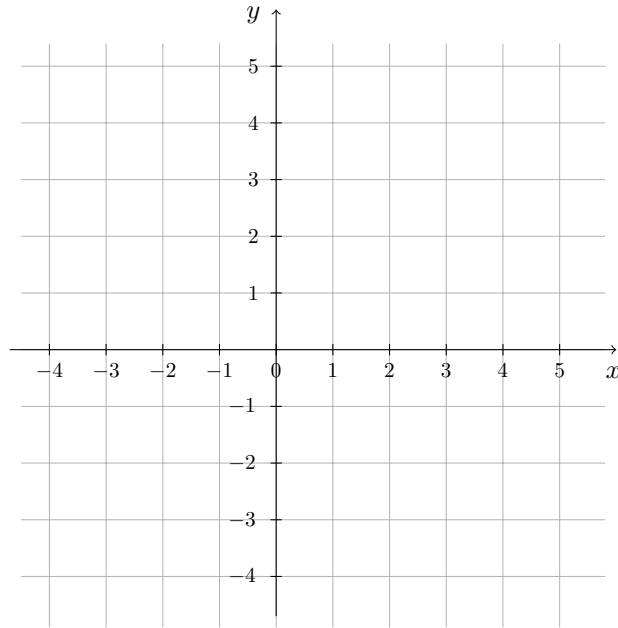   (b) $r = 2\sin(\theta)$.

    (c) $r = 5$.

15. Sketch the radius vectors corresponding to the following complex numbers and their conjugates.

    (a) $z = 1 + i$.

    (b) $w = -3i - 4$.

    (c) $\ell = 4i$.

16. Write the following complex numbers in the form $a + bi$.

    (a) $1 + i - (\overline{i - 5})$.

    (b) $(5i - 2)(3 - i)$.

    (c) $(3 - i)^2$.

    (d) $\dfrac{5 + 2i}{2 - i}$.

17. Write the following complex numbers in the form $z = r(\cos(\theta) + i\sin(\theta))$, where $r = |z|$ and θ is the angle that z forms with the x-axis.

(a) $z = \dfrac{1+i}{\sqrt{2}}$.

(b) $w = -5 - 5\sqrt{3}i$.