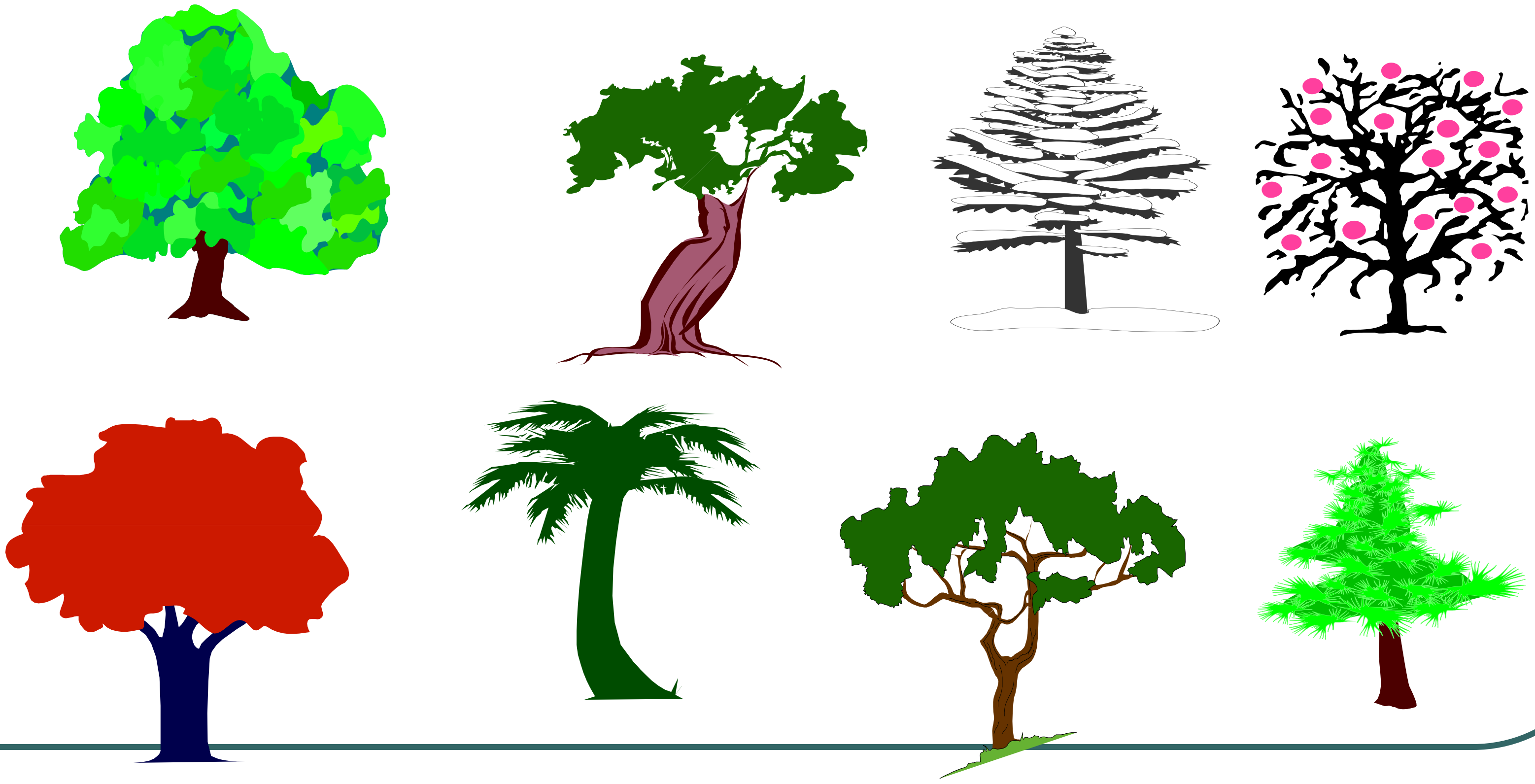
Section 11.1

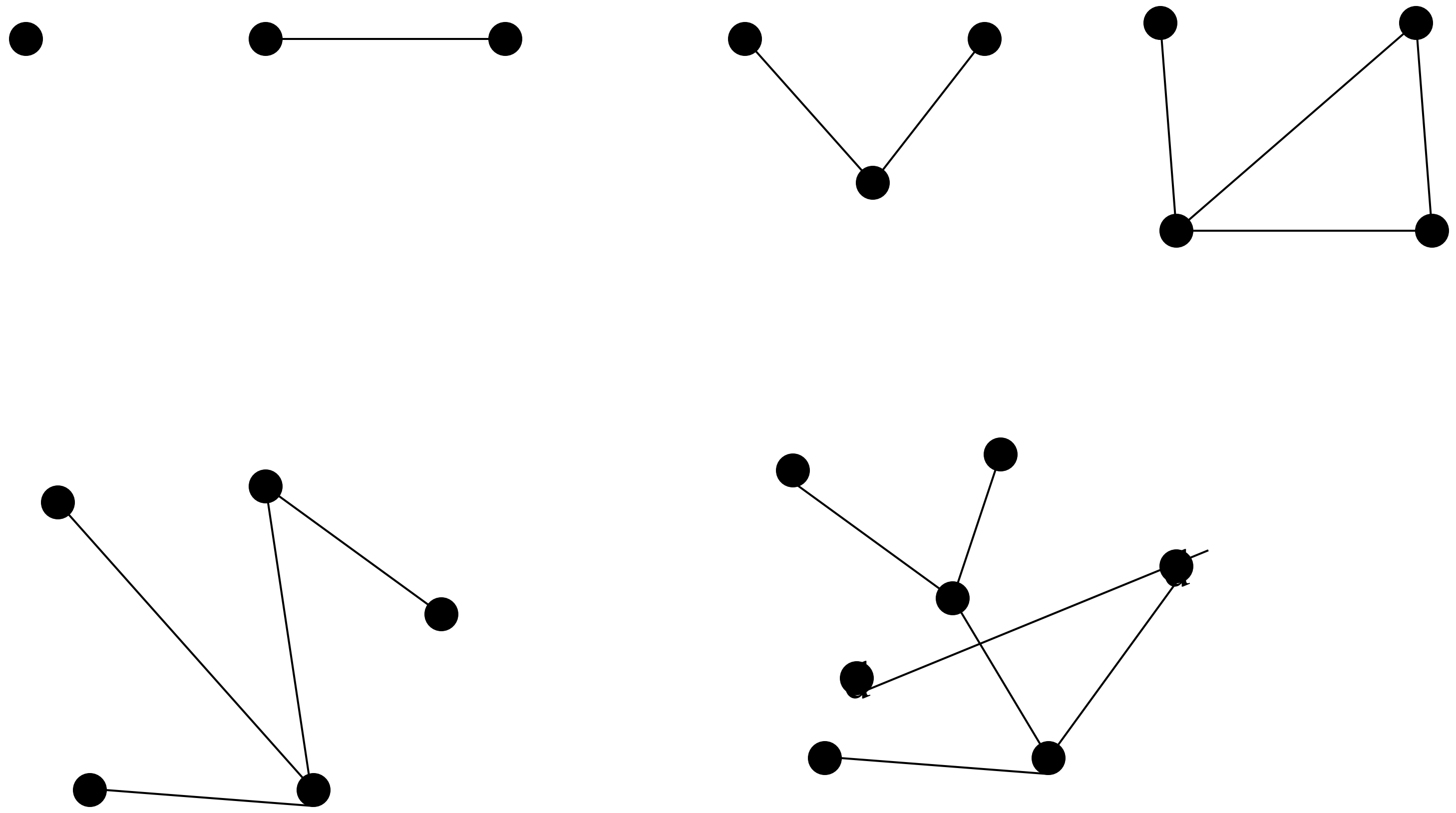
Introduction to Trees



Definition:

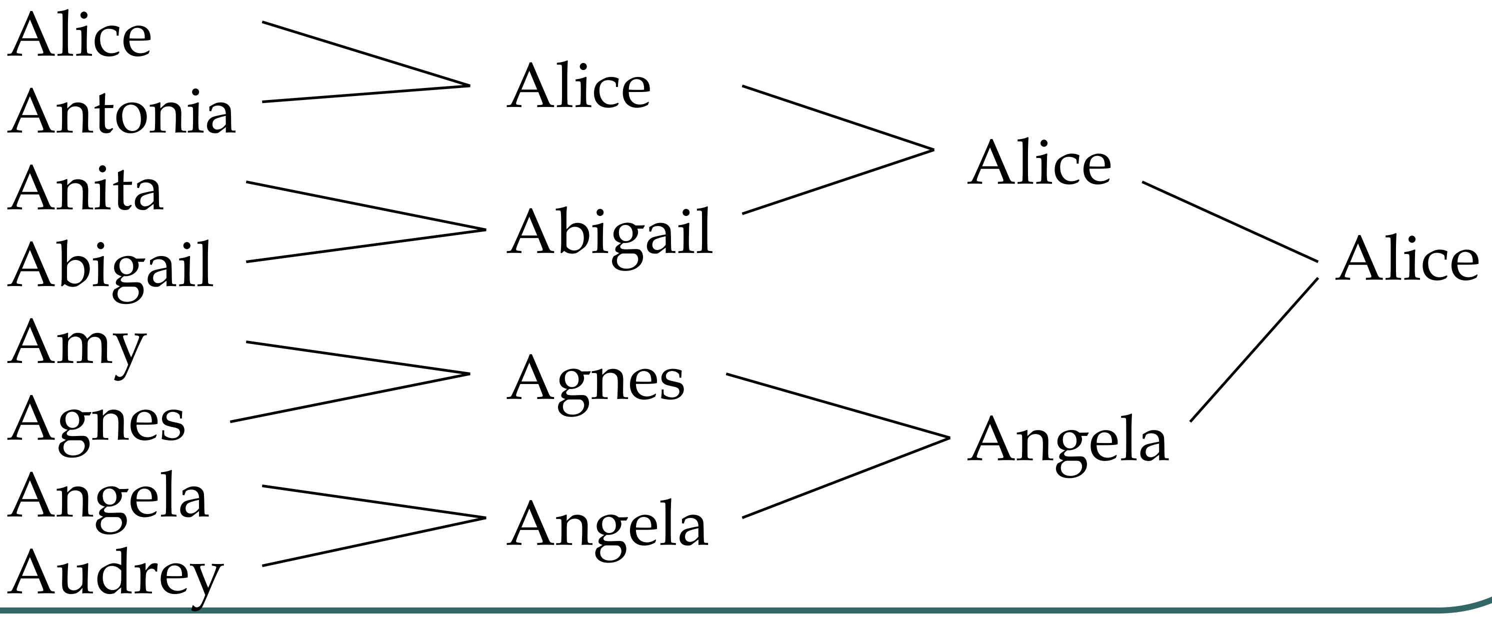
A *tree* is a connected undirected graph with no simple circuits.

: A **circuit** is a path of length >=1 that begins and ends a the same vertex.



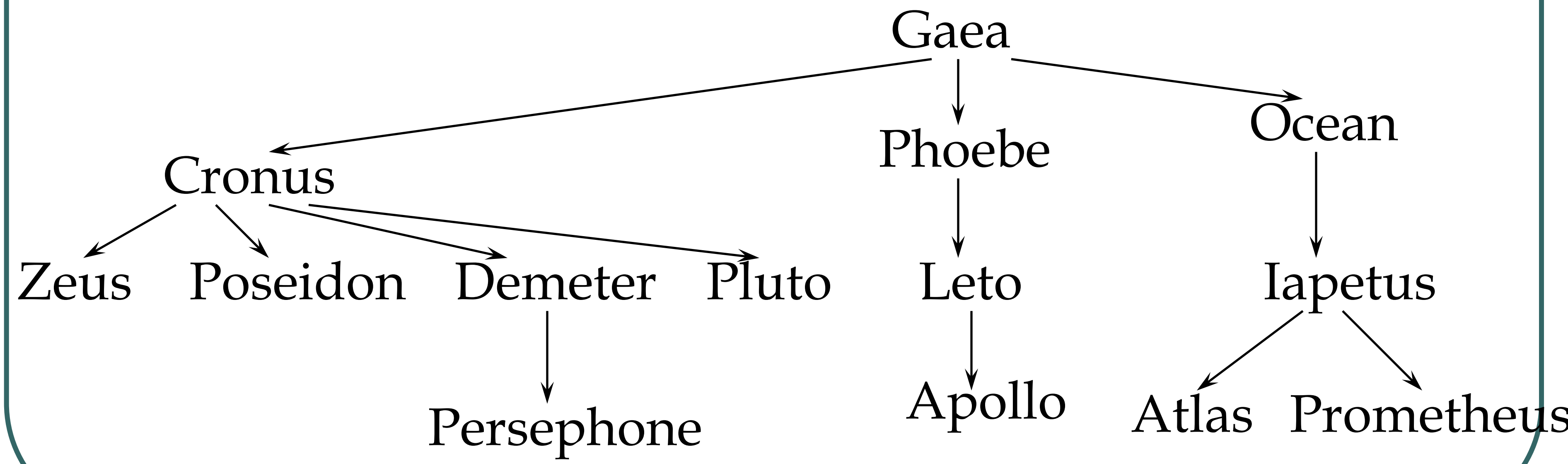
Tournament Trees

A common form of tree used in everyday life is the tournament tree, used to describe the outcome of a series of games, such as a tennis tournament.



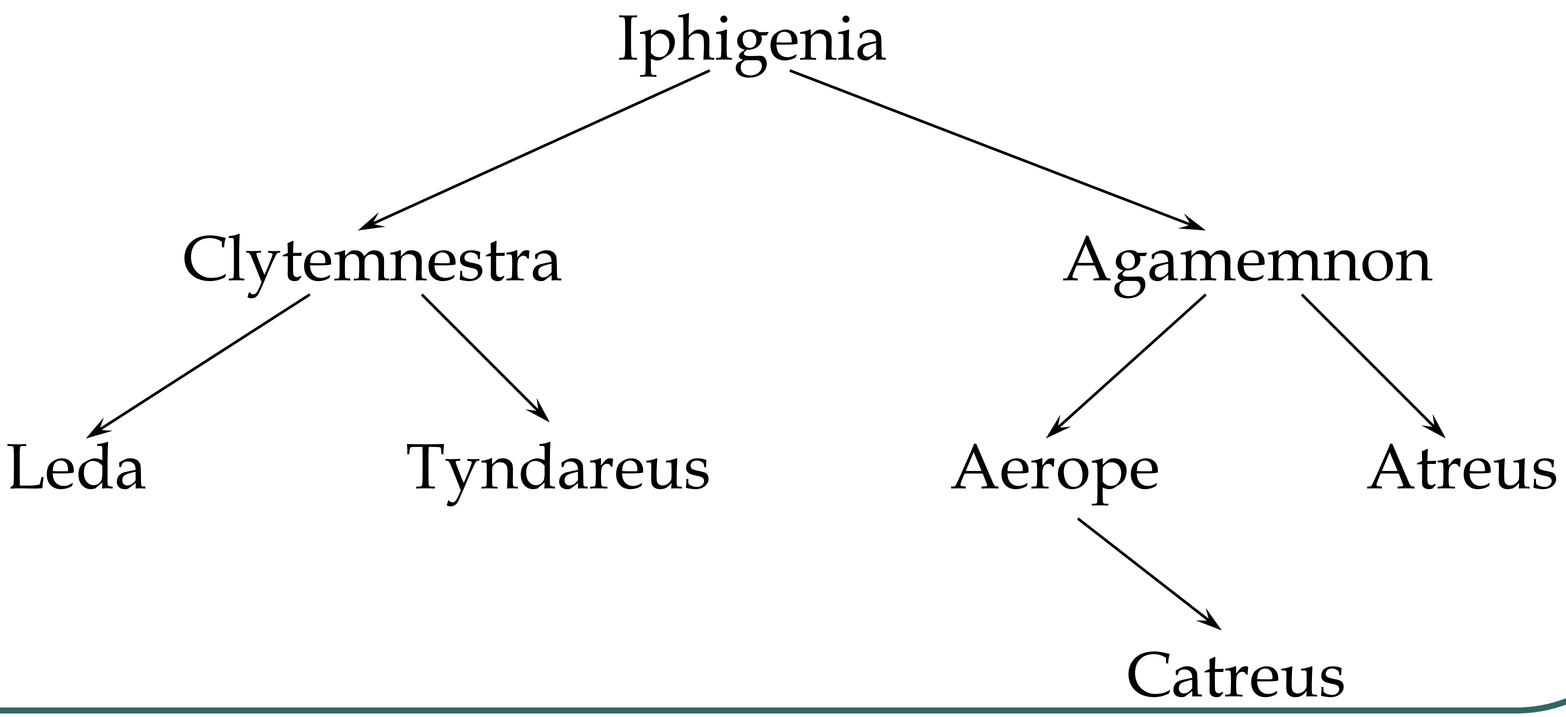
A Family Tree

Much of the tree terminology derives from family trees.



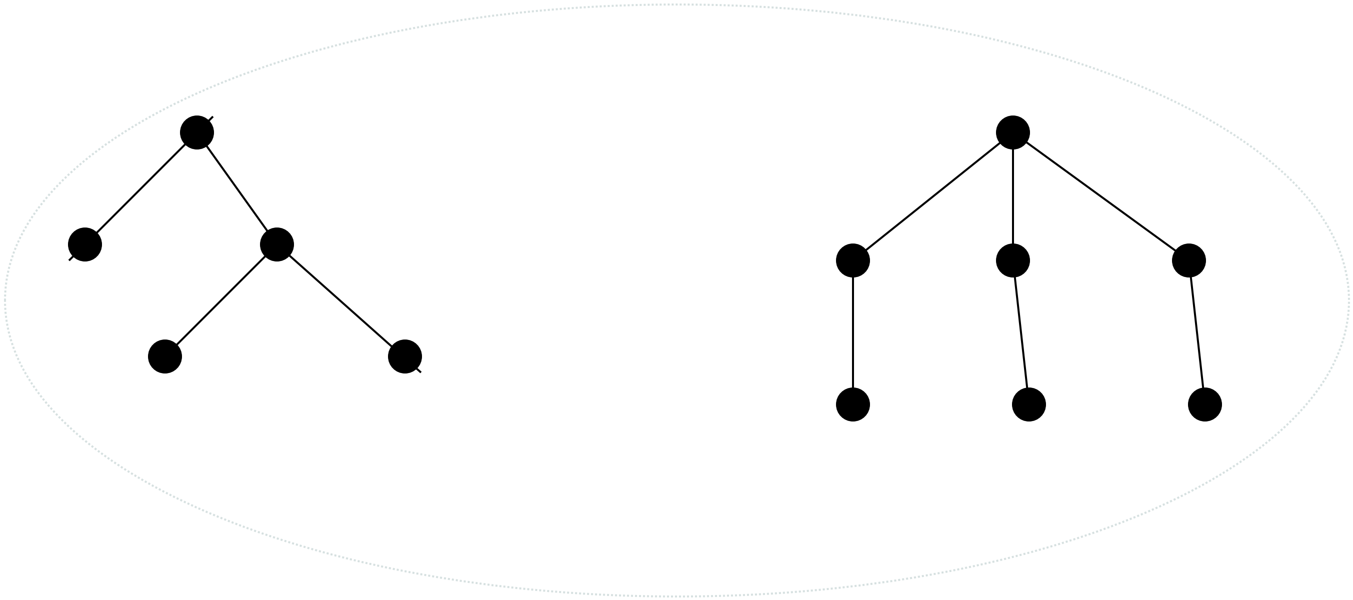
Ancestor Tree

An inverted family tree. Important point - it is a *binary* tree.



Forest

Graphs containing no simple circuits that are not connected, but each connected component is a tree.



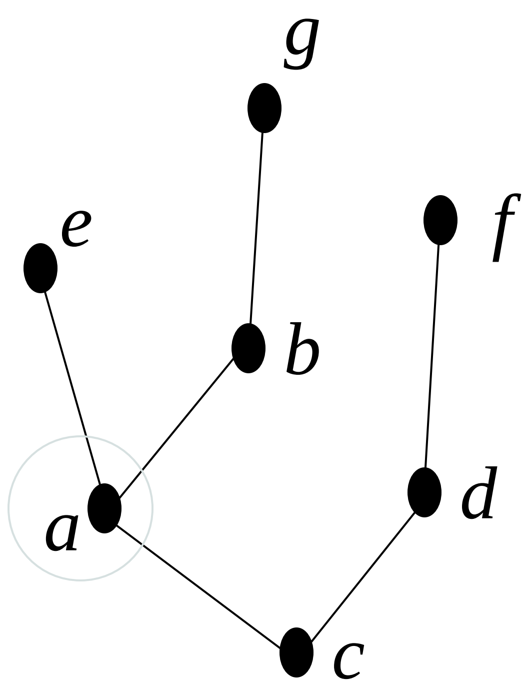
Theorem

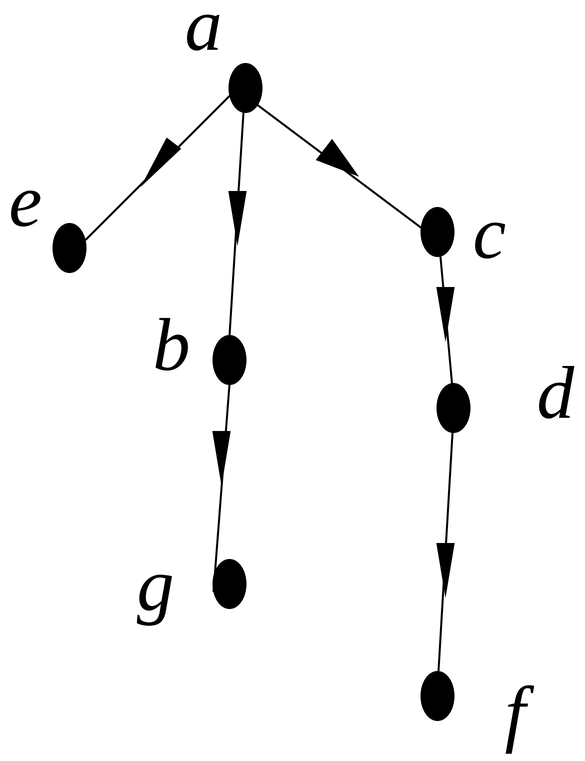
An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

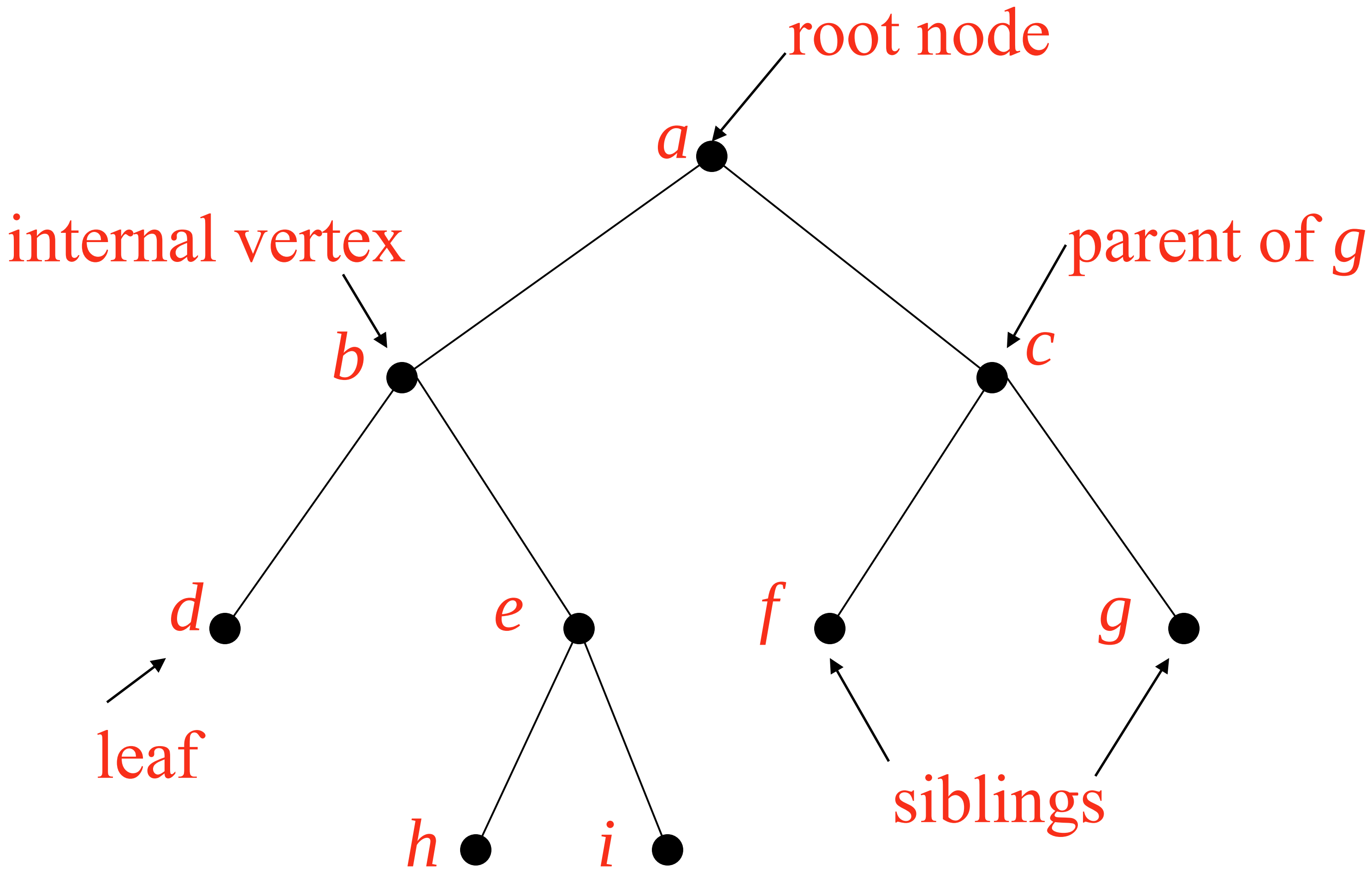
Rooted Trees

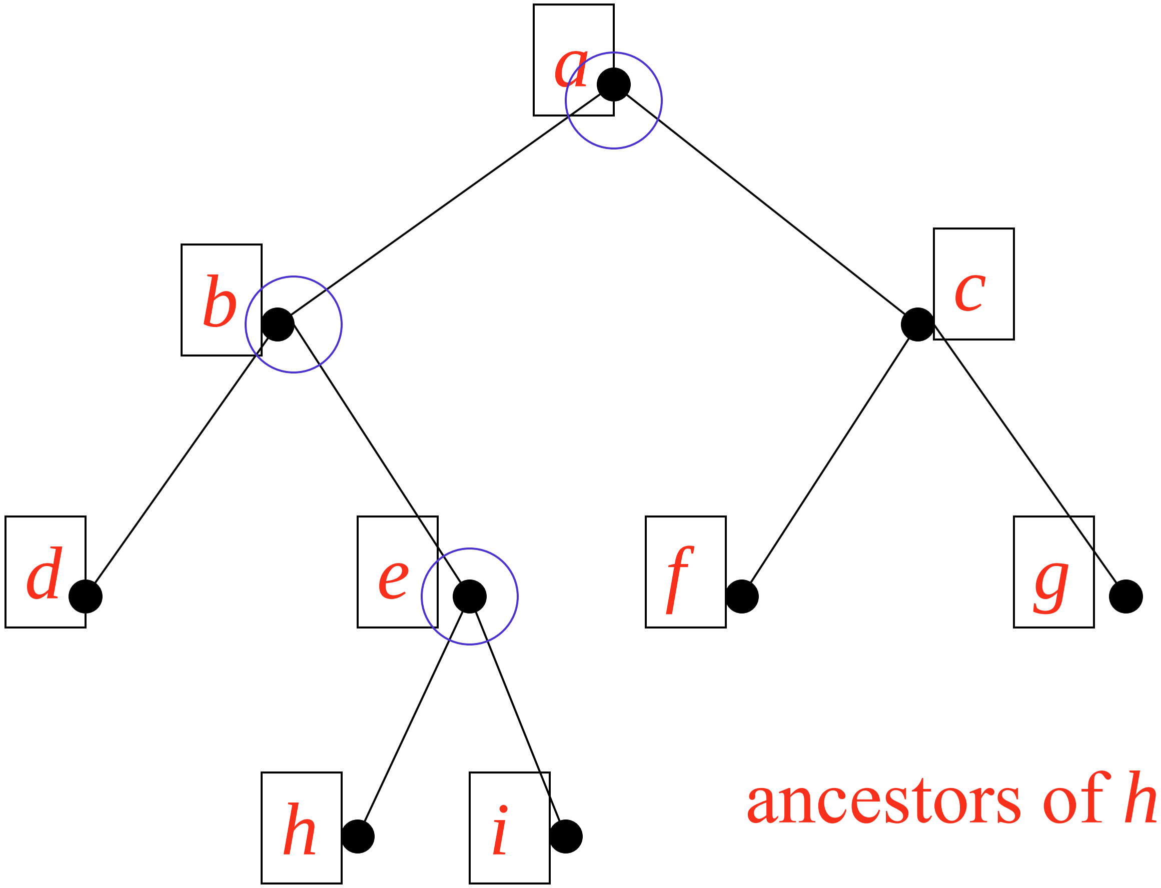
Once a vertex of a tree has been designated as the *root* of the tree, it is possible to assign direction to each of the edges.

Rooted Trees

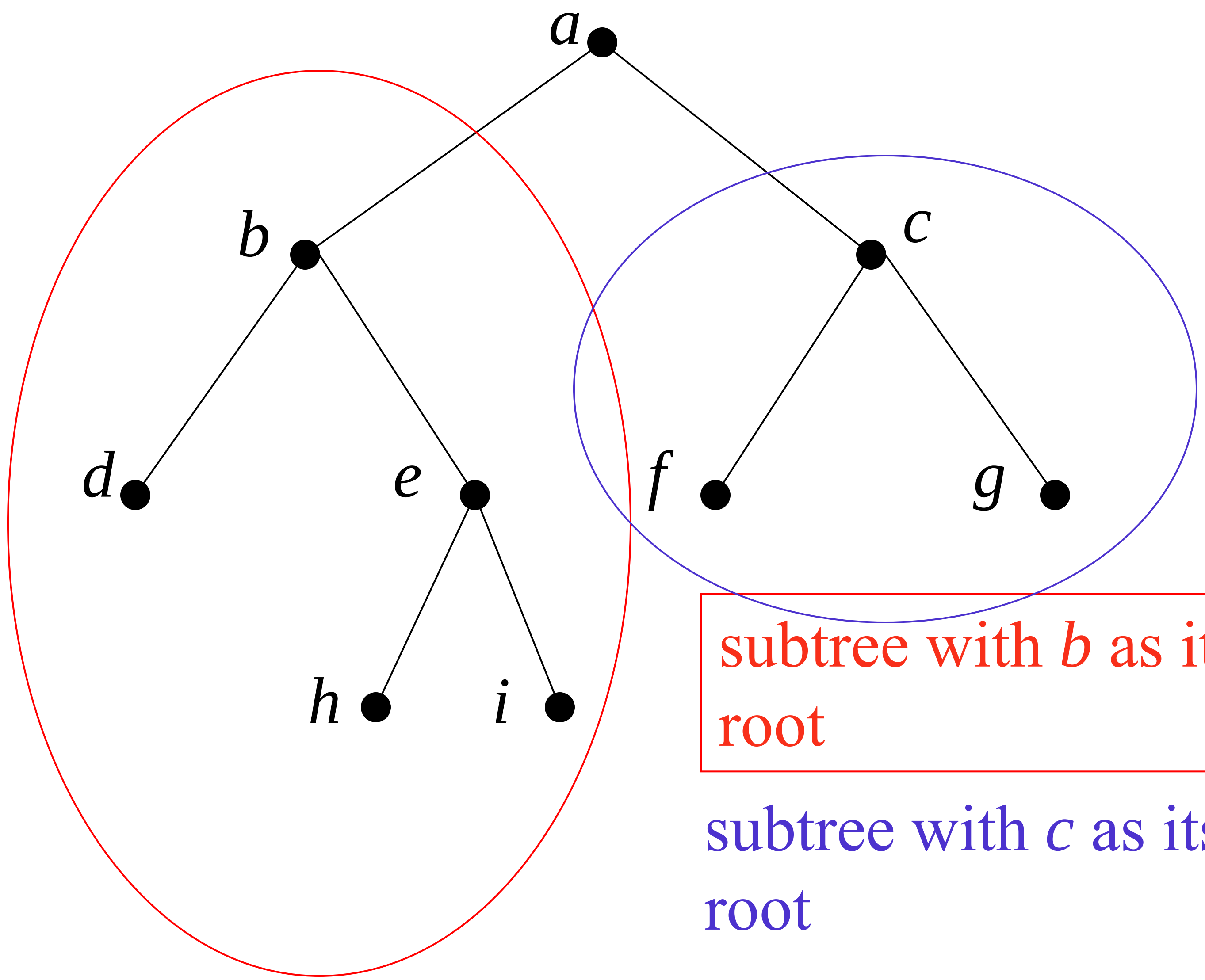





root node
a
internal vertex
parent of g
b
c
d
e
f
g
leaf
siblings
h
i



ancestors of *h* and *i*

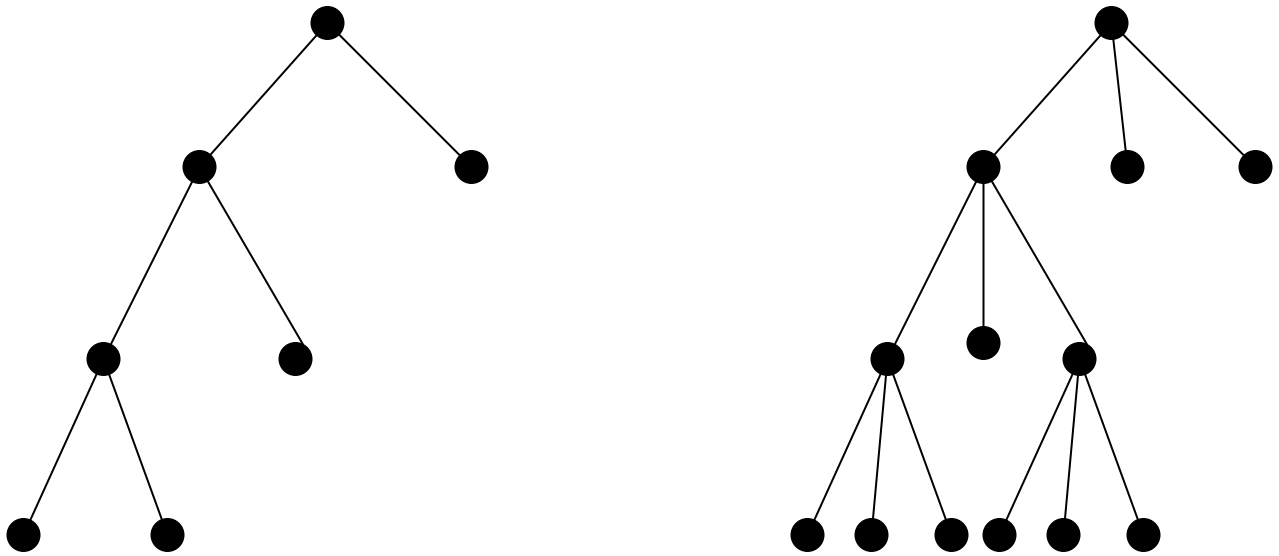
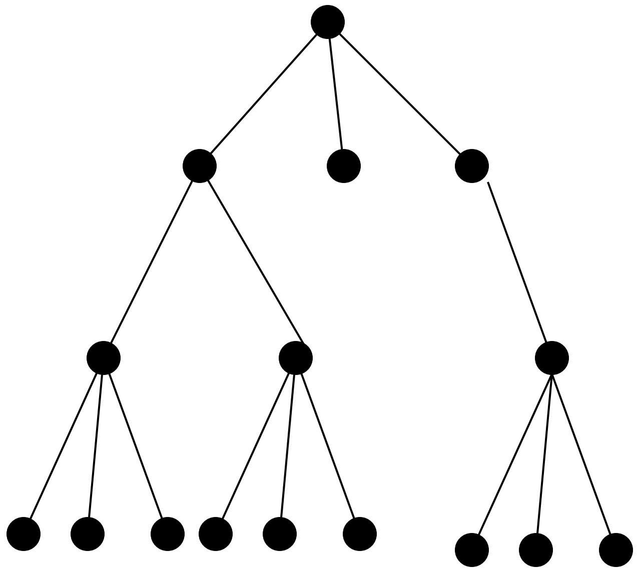


subtree with *b* as its root

subtree with *c* as its root

m-ary trees

A rooted tree is called an *m-ary tree* if every internal vertex has no more than *m* children. The tree is called a *full m-ary tree* if every internal vertex has exactly *m* children. An *m*-ary tree with *m*=2 is called a *binary tree*.

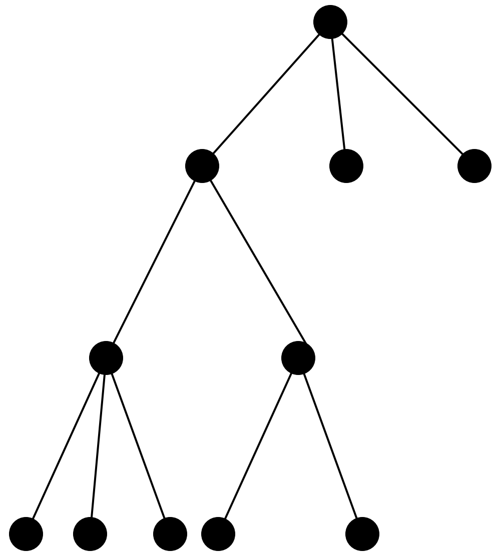



Ordered Rooted Tree

An ordered rooted tree is a rooted tree where the children of each internal vertex are ordered. Ordered trees are drawn so that the children of each internal vertex are shown in order from left to right.

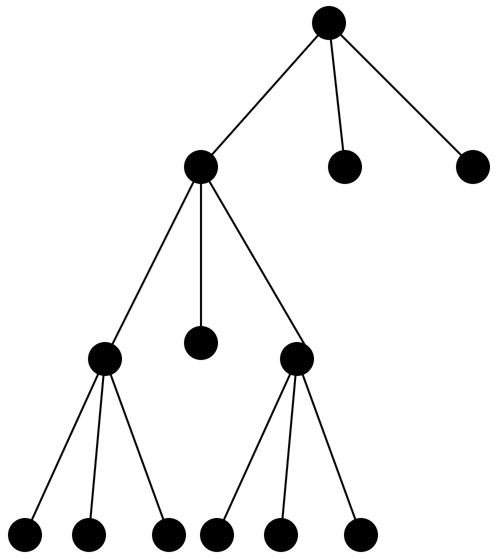
Properties of Trees

A tree with n vertices has n-1 edges.



Properties of Trees

A full m-ary tree with i internal vertices contains $n = mi+1$ vertices.



Properties of Trees

A full m-ary tree with

(i) n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves.

(ii) i internal vertices has $n = mi + 1$ vertices and $l = (m-1)i + 1$ leaves.

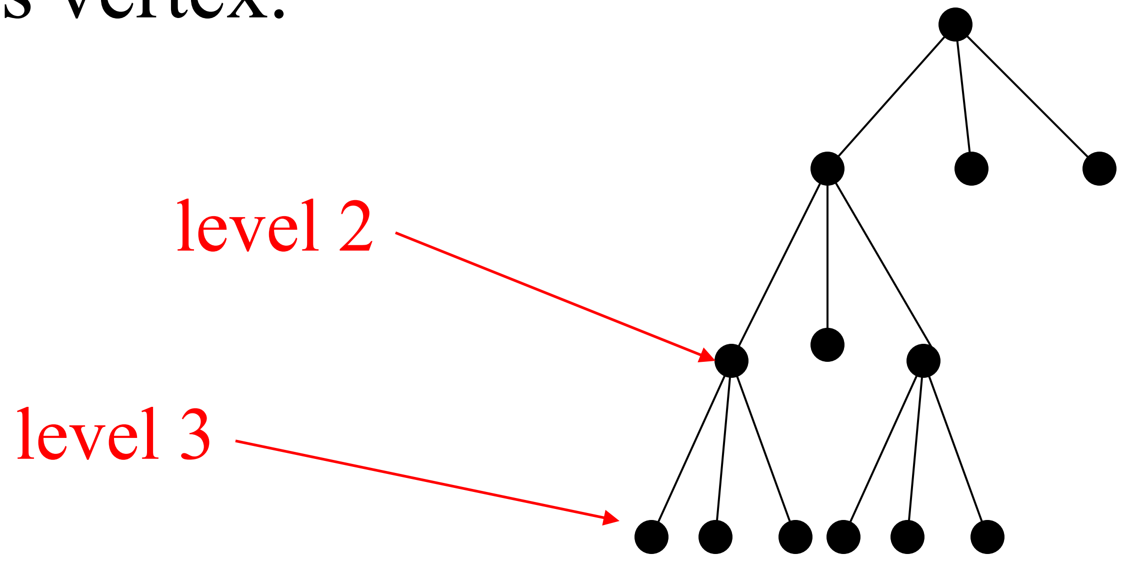
(iii) l leaves has $n = (ml - 1)/(m-1)$ vertices and $i = (l-1)/(m-1)$ internal vertices.

Proof

- We know $n = mi+1$ (previous theorem) and $n = l+i$,
- n – no. vertices
- i – no. internal vertices
- l – no. leaves
- For example, $i = (n-1)/m$

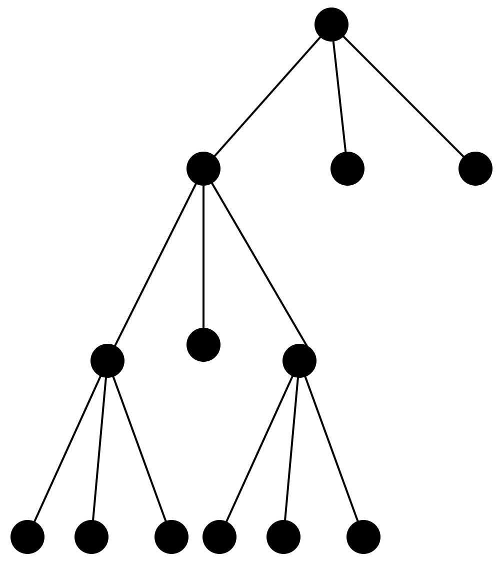
Properties of Trees

The level of a vertex *v* in a rooted tree is the length of the unique path from the root to this vertex.



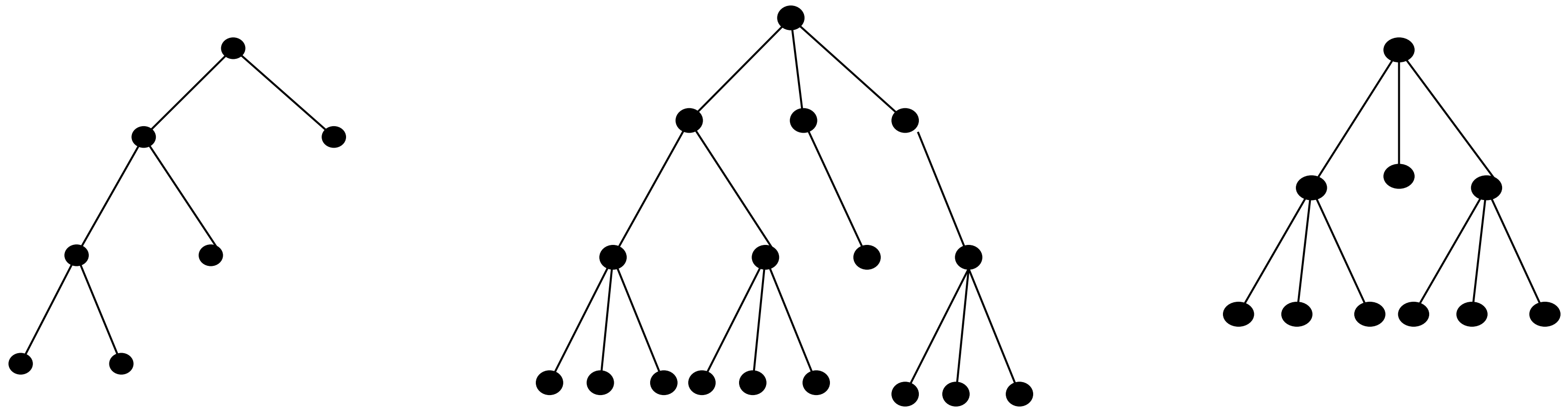
Properties of Trees

The *height* of a rooted tree is the maximum of the levels of vertices.



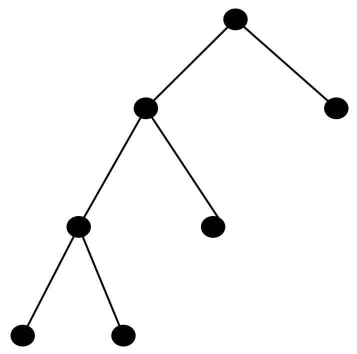
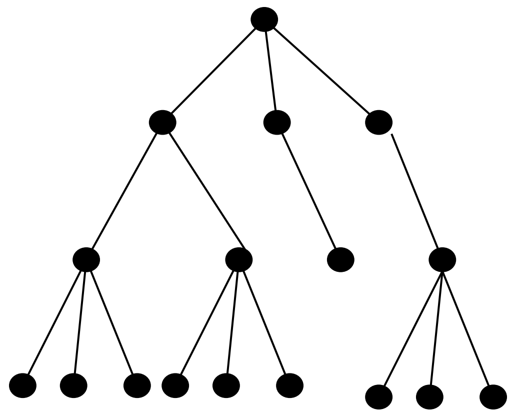
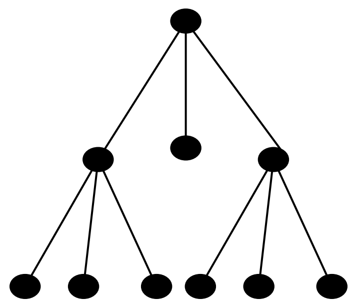
Properties of Trees

A rooted *m*-ary tree of height *h* is called *balanced* if all leaves are at levels *h* or *h*-1.



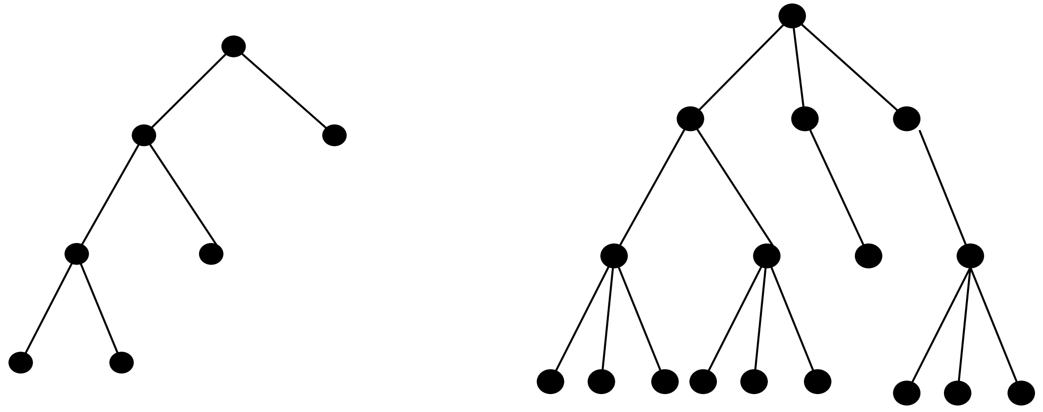
Properties of Trees

There are at most m^h leaves in an m-ary tree of height h.

Properties of Trees

If an m-ary tree of height h has l leaves, then $h \geq \lceil \log_m l \rceil$



Proof

- From previous theorem:

$$l \le m^h \Rightarrow \log_m l \le h \Rightarrow \lceil \log_m l \rceil \le h$$