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LUNAR VELOCITY IN THE MIDDLE AGES: A COMPARATIVE STUDY

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Angular velocities of the Moon were not measured directly in the period from Ptolemy to Regiomontanus (d. 1476); rather, they were computed on the basis of various geometrical models for its motion. The models described by Ptolemy in the *Almagest* predominated, but there were significant variants as well. Moreover, Ptolemy had two lunar models, and his second model eventually was used as the basis for these computations. Of special interest for us will be the tables constructed for this purpose by various medieval astronomers, and we will examine possible relationships between them. The angular velocities in question are "instantaneous" velocities, rather than mean velocities, despite the absence of a proper definition of such a concept in antiquity and the Middle Ages. These velocities were used in computing the duration of eclipses as well as the time from mean conjunction of the Sun and the Moon to their true conjunction (and correspondingly for opposition). There was much ingenuity in designing the underlying procedures, sometimes with the goal of simplifying the computations (see Chabás and Goldstein 1992). These activities were entirely based on mathematical insights, and observations played no role in them. In the discussion that follows we will concentrate on tables of lunar velocities. Tables of solar velocities are often associated with them, but there is little variation in solar velocity, and it is generally possible to produce the values for it in the texts by a variety of methods[1].

Ptolemy tabulated the mean motions of the Sun and the Moon: the Sun moves about 0;59,8°/d (or 0;2,28°/h); and the Moon moves about 13;10,35°/d (or 0;32,56°/h) in longitude, and 13;3,54°/d (or 0;32,40°/h) in anomaly. Moreover, Ptolemy tabulated the equations of center for the Sun and the Moon. But he did not tabulate their variable instantaneous velocities which were needed in some of his computations: still, there is a rule for computing lunar velocity in the *Almagest*, vi.4, which states that to find the hourly true lunar velocity, "enter the table of lunar anomaly with the [argument of] anomaly at the moment in question and take the corresponding equation; then determine the size of the increment in the

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[1] For a description of Ptolemy's solar and lunar models, see Neugebauer 1969, pp. 192 ff.

equation at that point corresponding to an increment of 1° of [argument of] anomaly. We multiply the increment by the mean motion in anomaly of 0;32,40°/h ... and we add [algebraically] the mean hourly motion in longitude, 0;32,56°/h. The result will be the moon's true motion in longitude in 1 equinoctial hour at that position" (trans. Toomer 1984, p. 282). We can state this algebraically as:

$$v(\alpha) = 0;32,56 + 0;32,40 \cdot \Delta \qquad (1)$$

where v is the "instantaneous" velocity of the Moon corresponding to α, the argument of anomaly, and Δ is the difference in the equation of center. Let us define $c(\alpha)$ to be the function that yields the equation of center. Then

$$\Delta = c(\alpha + 1) - c(\alpha) \qquad (2)$$

These formulas make it possible to tabulate the lunar velocity, and the first table based on them, using Ptolemy's table for the equation of center, is preserved in Arabic in the *Zīj* of al-Battānī (d. 929; see Nallino 1899-1907, ii:88)[2]. In the ninth century there were several translations of Ptolemy's *Almagest* into Arabic and, by the time of al-Battānī, this text was well understood. Al-Battānī wrote an introduction to his tables, but he did not explain how he had computed his table for solar and lunar velocity. It was by no means obvious to modern scholars that he had applied Ptolemy's instructions (see, e.g., Toomer 1968, pp. 82 ff), but recomputation strongly suggests that this was the method he employed. This table was widely copied (usually without any indication of its source), and appears, for example, in the Toledan Tables produced in al-Andalus in the 11th century and preserved in many Latin copies. Note that the solar velocity does not vary very much (from 0;2,23°/h to 0;2,33°/h), and that the lunar velocity varies to a greater extent (0;30,18°/h to 0;36,4°/h): see Table 1.

[2] A *zīj* is a set of astronomical tables with instructions for their use: well over a hundred examples from the Islamic Middle Ages are listed in Kennedy 1956.

Table 1: al-Battānī's lunar velocities

(1) α (°)	(2) v(comp) (°/h)	(3) v(text) (°/h)	(4) T − C (")
0	0;30,18	0;30,18	0
30	0;30,36	0;30,35	−1
60	0;31,25	0;31,25	0
90	0;32,43	0;32,41	−2
120	0;34,14	0;34,14	0
150	0;35,32	0;35,31	−1
180	0;36, 4	0;36, 4	0

The entries in col. 1 are the arguments of lunar anomaly; the entries in col. 2 have been computed with eq. (1) and the entries in al-Battānī's table for the lunar equation; the entries in col. 3 have been copied from al-Battānī's table for lunar velocity (Nallino 1899-1907, ii:88); and the entries in col. 4 are the differences between text and computation as displayed in columns 2 and 3.

Another early Islamic *Zīj* was produced in the ninth century by al-Khwārizmī, but it is only preserved in a Latin version by Adelard of Bath (early 12th century) based on a lost Spanish-Arabic recension by al-Majrīṭī (fl. ca. 1000). These tables were edited by H. Suter (1914; see pp. 175-180), and were the subject of an extensive commentary by O. Neugebauer (1962), but neither of them explained the construction of the table for solar and lunar velocity[3]. In this table the lunar velocity varies from 0;30,12°/h to 0;35,40°/h (in contrast to al-Battānī's 0;30,18°/h and 0;36,4°/h). The formula used is the same as that underlying al-Battānī's table, but the table used for the equation of center was the one in al-

[3] As-Saleh (1970, p. 162) discussed the lunar velocities in the Khwārizmī tables, but did not offer a satisfactory explanation for them. Moreover, as-Saleh (1970, pp. 145-47) included an edition of tables for lunar velocity ascribed to Ḥabash al-Ḥāsib (9th century) that differ from those discussed in this paper. Ḥabash also gave a rule for constructing a table of hourly lunar velocities with a minimum of 0;30 and a maximum of 0;36 (as-Saleh 1970, p. 156), using one of Ptolemy's interpolation functions (*Almagest*, V, 18, col. 7, or VI, 8; Toomer 1984, pp. 265, 308). The same rule appears in Ptolemy's *Handy Tables* with a table of hourly lunar velocities, based on this rule, given at intervals of 6° of lunar anomaly: the entries for 0°, 90°, and 180° are: 0;30,0, 0;32,54, and 0;36,0, respectively (see Stahlman 1959, pp. 109-12, 263). A purely Indian procedure for computing lunar velocity appears in a commentary on the Khwārizmī tables: see Goldstein 1967, pp. 226 f.

Khwārizmī's *Zīj* rather than the one in al-Battānī's *Zīj*. In al-Khwārizmī's *Zīj* the tables for the solar and lunar equation are not based on the *Almagest*, but on Hindu parameters, for Islamic astronomers had access to Indian astronomical texts as well as the Greek tradition. The maximum solar equation for al-Khwārizmī was 2;14° instead of Ptolemy's 2;23°; moroever, the maximum value for al-Khwārizmī corresponds to an argument of 90°, whereas for Ptolemy the maximum corresponds to an argument greater than 90°. Similarly for the Moon: in al-Khwārizmī's *Zīj* the maximum equation of center is 4;56° corresponding to an argument of 90°, whereas for Ptolemy it is 5;1° corresponding to an argument greater than 90°. It seems odd to find a mixture of Hindu parameters and Ptolemaic procedures, but that is characteristic of much of early Islamic astronomy. This alternative table for velocity also appears in the *Zīj* of Ibn al-Kammād (12th century), preserved in a Latin manuscript; in the Tables of Barcelona (14th century: there are extant Hebrew, Latin, and Catalan versions); and in the Tables of Juan Gil of Burgos (14th century: preserved only in a Hebrew manuscript): see Chabás and Goldstein 1994; Chabás 1995. In other words, there were two relevant traditions available to astronomers in the Iberian peninsula in the late Middle Ages.

There was still another way to combine Hindu parameters with Ptolemaic models: Ibn Muᶜādh (11th century, Spain) used al-Khwārizmī's maximum lunar equation of 4;56° and applied it to Ptolemy's simple lunar model where the maximum equation corresponded to more than 90° (Samsó 1992, p. 157, based on the canons to Ibn Muᶜādh's *Zīj*, chap. 11, published in Latin translation: Nuremberg 1549). The tables in the *Zīj* of Ibn Muᶜādh do not survive, but a Ptolemaic table for the lunar equation with a maximum of 4;56° is preserved in the *Zīj* of Ibn al-Bannā' (d. 1321: see MS Escurial Ar. 909, f. 25r; cf. Vernet 1951), and a maximum of 4;55,59° in the *Zīj* of Ibn al-Raqqām (d. 1315: see MS Kandilli 249, ff. 67v-68r; MS Rabat 260, p. 111): note that Ibn al-Bannā' displays this lunar equation to minutes, whereas Ibn al-Raqqām includes seconds as well. Recently, a manuscript of the *Zīj* by Ibn Isḥāq of Tunis (fl. early 13th century) has been discovered in India, and it contains a table for the lunar equation with a maximum of 4;55,59° (as in Ibn al-Raqqām) corresponding to an argument greater than 90° (see MS Hyderabad, Andra Pradesh State Library 298, table 36)[4]. Of particular interest to us is that Ibn al-Raqqām has a table of lunar velocity (MS Kandilli 249, f. 81r; MS Rabat 260, p. 91) based on formulas (1) and (2), using the entries in his table for the lunar equation [see Appendix]. The minimum value in this table is 0;30,21°/h and the maximum is 0;36,1°/h (in contrast to al-Battānī's 0;30,18°/h, and 0;36,4°/h). Ibn al-Bannā' also has a table for lunar velocity (MS Escurial 909, f. 36v) with the same minimum and maximum values as in the table of Ibn al-Raqqām, but the argument in the table of Ibn al-Bannā' is given at intervals of 6°, rather than at intervals of 1° used by Ibn al-Raqqām. Moreover, Ibn Isḥāq has the same table of lunar velocity that we find in the *Zīj* of Ibn al-Raqqām[5].

Let us now turn to Latin texts that discuss lunar velocity. In the late Middle Ages, the Latin version of the Alfonsine Tables formed the basis for most astronomical activity in Christian Europe. These tables were presumably prepared under King Alfonso X of Castile (reigned: 1252-1284), but only the introductory chapters (or canons) survive of the original Castilian version of the tables, and they contain little in the way of numerical data. The tables themselves are known from the versions produced in Paris, beginning in the 1320s: it has even been argued that the Parisians constructed entirely new tables, and that they ascribed them to Alfonso for some unknown reason (Poulle 1988). While the author of this argument, Emmanuel Poulle, is widely admired for his work on medieval astronomy in general and on the Parisian astronomers in particular, this claim has not been accepted by most of his colleagues[6].

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[4] The discovery of this MS is described in King 1988, pp. 129-132. Cf. King 1977, p. 192; and Kennedy & King 1982, p. 6 n 3.

[5] I am grateful to J. Samsó for informing me that Ibn al-Raqqām's table of lunar velocities also appears in Ibn Isḥāq's *Zīj* (MS Hyderabad, Andra Pradesh State Library 298, Table 93). For the entries displayed in the Appendix, there is exact agreement with one exception: for 90° Ibn Isḥāq has 0;32,40 instead of Ibn al-Raqqām's 0;32,41.

[6] Cf., e.g., Goldstein et al. 1994, for evidence that what seems to be an unusual table in a text by one of the Parisians, John of Lignères, dated 1322, already appears in a 13th century MS from Spain, and that it was one of the original Alfonsine tables. In this case, the description of the table in Latin by John of Lignères is very close to the description in the Castilian canons produced for Alfonso X. Moreover, in many copies of this table there is a column for the lunar velocity at quadrature, based on Ptolemy's second lunar model, that agrees with the description in the Castilian canons (Goldstein et al. 1994, pp. 71-72, 78-79,

We will consider only one point which Poulle introduced to support the originality of the Parisian astronomers of the 14th century (Poulle 1988, p. 101): the table for the lunar equation of center with a maximum of 4;56° for an argument greater than 90° (indicating that the underlying model is Ptolemaic). But we have already noted a number of Islamic astronomers prior to 1320 who had such tables, and so the claim for originality of the Parisian group in this respect cannot be sustained.

There has been no proper census of the manuscripts of the Alfonsine tables of which a great number of copies survive, and there are also several printed editions, beginning in 1483; but the variety of tables in the manuscripts has not been adequately appreciated. Indeed, though Poulle gave precedence to the *editio princeps* of 1483 in his own edition of the Alfonsine tables (1984), it really has the same status as that of any manuscript, for it was not a critical edition by any reasonable standard. In the Alfonsine corpus, i.e., those texts associated with the Alfonsine tables, there are several tables for lunar velocity. The *editio princeps* does not include the usual table of al-Battānī, but has a table with a minimum of 0;30,21°/d for argument 1° and a maximum of 0;36,25°/d for argument 180° (see Goldstein 1980): the entries from argument 1° to 159° agree well with computations based on the table for the lunar equation of center in the Alfonsine tables and do not differ appreciably from those in the corresponding tables in the *Zīj*es of Ibn Isḥāq and Ibn al-Raqqām. But the last 20° depart increasingly from computations based on the table for the lunar equation in the Alfonsine tables, and no satisfactory explanation has been proposed. On the other hand, J. L. Mancha has found another table in Latin for lunar velocity (MS Paris B.N. lat. 7288, f. 58v, associated with instructions for eclipse computations and a worked example for 1460), where the argument is given at 15° intervals. In this table the minimum velocity is 0;30,20°/h for argument 0° and the maximum is 0;36,1°/h for argument 180°; these values agree quite closely with the comparable entries in the tables of Ibn Isḥāq and Ibn al-Raqqām. Clearly, the ideas that lie behind the tables in

88, 92). For another recent account of the Alfonsine tables, see North 1992. See also Goldstein 1994 for a discussion of the Alfonsine theory of precession.

Latin had already been explored in Arabic[7].

From what has been reported up to now it would seem that there was little originality among astronomers in the Latin Christian world. But there are other materials in the Alfonsine corpus that lead us to a very different conclusion. To understand these advances we have to consider Ptolemy's second model for lunar motion (see Neugebauer 1969, pp. 194-6)[8]. Ptolemy reports that his simple model works perfectly well at syzygy (i.e., conjunction and opposition of the Sun and the Moon), but that elsewhere a more complicated model is needed. In this model the epicycle is attached to a "crank" that varies the epicyclic distance from the observer; a new argument is introduced, namely the double elongation (where elongation refers to the angular distance between the Sun and the Moon); and the epicyclic apogee oscillates so that a distinction is made between the mean argument of anomaly and the true argument of anomaly. Since the second model reduces to the first model at syzygy, it was thought by many that the second model could be ignored in computing the lunar velocity at syzygy (see, e.g., Toomer 1984, p. 282 n 15). However, a careful analysis, using modern mathematical techniques, has demonstated that the second lunar model would have a noticeable effect on the lunar velocities at syzygy (Pedersen 1974, p. 226). It can now be said that several medieval astronomers were also aware of the need to take Ptolemy's second lunar model into account, for there are tables in the Alfonsine corpus ascribed to John of Lignères, John of Genoa, and John of Montfort (all active in the 1330s) that differ substantially from the tables we have discussed above (see Goldstein 1992). According to John of Genoa the minimum lunar velocity is 0;29,37,13°/h, and the maximum is 0;36,58,54°/h (there is a special problem with the maximum value: see below). The entries in this table can be derived by the following formula:

$$v(\alpha') = 0;32,56 + 0;41,49 \cdot \Delta \qquad (3)$$

[7] Samsó (1994, *Addenda*, p. 3) argued that the Alfonsine table of solar declination derives from Ibn Isḥāq, and this new evidence of similarity suggests a broader dependence.

[8] There is no consensus on the numbering of Ptolemy's models: what I call Ptolemy's second lunar model is the final version as illustrated in Neugebauer 1969, p. 196. This model serves as the basis for the tables in *Almagest,* V, 8.

where α' is the true argument of lunar anomaly. This formula differs from eq. (1), for instead of the mean argument of lunar anomaly, we use the true argument of lunar anomaly, and instead of the mean hourly motion in anomaly of 0;32,40°, we use 0;41,49° which represents the corrected hourly lunar motion in anomaly at syzygy. This formula first appears, as far as I know, in the *Epitome of the Almagest*, vi.4, by Regiomontanus, with no explanation. In a comment on this formula, it has been suggested that the coefficient 0;41,49 was computed as follows (see Swerdlow and Neugebauer 1984, pp. 274-6):

$$0;41,49 = 0;32,40 + 1;1 \times 0;9 \qquad (4)$$

where 0;32,40° is the mean hourly motion in anomaly, 1;1° is the mean hourly motion in double elongation, and 0;9° is the difference in the equations for 0° and 1° due to the double elongation. Despite the absence of direct evidence, it seems that this formula was known more than a century before Regiomontanus. There are some indications in the manuscripts that these astronomers in the 14th century knew what they were doing, but no formula was given (cf. Goldstein 1992, p. 4 n 4). Using this formula we get excellent agreement with John of Genoa's table to within 1 or 2 in the third sexagesimal place, but for a few problematic cases, including the maximum entry (0;36,58,54°/h instead of the recomputed value of 0;36,53,20°/h). However, in another version of this table, ascribed to John of Lignères, in which only two sexagesimal places are displayed, the maximum is 0;36,53°/h as expected according to the recomputation (see Goldstein 1992, p. 13). The Alfonsine corpus also includes tables for lunar velocity in degrees per sixtieth of a day (°/mn)[9]. In the table ascribed to John of Genoa the minimum is 0;11,50,53°/mn (= 0;29,37,12°/h) and the maximum is 0;14,47,33°/mn (= 0;36,58,52°/h): these values do not differ significantly from John of Genoa's stated values for lunar velocity in °/h. But in the table ascribed to John of Montfort the minimum is 0;11,51,9,11°/mn (= 0;29,37,53°/h) and the maximum is 0;14,44,47,8°/mn (= 0;36,51,58°/h).

[9] See Goldstein 1992, pp. 11-14. There is also a table for lunar velocity in these units in a *Zīj* ascribed to Ḥabash al-Ḥāsib (see as-Saleh 1970, p. 145) where the minimum is 0;12,6,3°/mn (= 0;30,15,7°/h) and the maximum is 0;14,28,35°/mn (= 0;36,11,27°/h). For a discussion of Ḥabash's tables for the lunar equations, see Salam and Kennedy 1967.

There is some evidence that the table for lunar velocity by John of Genoa was used by later astronomers. In a table for finding the time from mean to true syzygy by Nicholaus de Heybech (ca. 1400), the computations for determining the entries require values for the minimum and maximum lunar velocities: it has been argued that these velocities were taken from John of Genoa's table as preserved in the extant copies (Chabás and Goldstein 1992, p. 274). So this suggests that the maximum entry in John of Genoa's table is not simply a copyist's error, but may reflect a computational error by the author.

There are also comparable tables for lunar velocity in some medieval Hebrew texts. The *Astronomy* of Levi ben Gerson (d. 1344, southern France: see Goldstein 1974, and Goldstein 1985) contains many tables, and among them are two tables for lunar velocity (Goldstein 1974, p. 182); but again there is no explanation of the procedure used to compute their entries. In the first table the minimum is 0;30,18°/h and the maximum is 0;36,4°/h (as in al-Battānī). Elsewhere in this table, however, there are small discrepancies between the values given by Levi and al-Battānī: I have argued previously that in computing these velocities Levi used his own table for the lunar equation of center (based on his new model for lunar motion) which differs slightly from those of Ptolemy and al-Battānī for syzygy (Goldstein 1974, p. 113; Goldstein 1992, p. 9). The second table, where the minimum lunar velocity is 0;29,35°/h and the maximum is 0;36,56°/h, is more unusual: its entries can be derived by Regiomontanus's formula using the entries in al-Battānī's table for the lunar equations. This method yields results that are somewhat closer to the text than recomputing with Regiomontanus's formula and the entries in Levi's table for the lunar equations, based on his new model for lunar motion (Goldstein 1992, p. 10).

It is surprising that, despite all the activity in producing tables for lunar velocity [see Table 2], there was little discussion of the motives to change the tables or of the procedures for determining the entries in these tables. Still, it seems reasonable to conclude that this activity was motivated by mathematical ingenuity rather than generated by analyzing observations, for most of the changes are quite small and have little effect. So, for example, replacing Ptolemy's maximum lunar equation of 5;1° with 4;56° reflects a literary heritage that ultimately goes back to Hindu sources rather than a serious attempt to adjust a parameter to a set of

Table 2: A Summary of Medieval Tables of Lunar Velocity

The tables for lunar velocity listed here have all been published previously except for No. 3, No. 4 (which appears in Appendix 1), and No. 6.

No.	Text/Author	Minimum	Maximum
1.	al-Battānī	0;30,18°/h	0;36, 4°/h
2.	al-Khwārizmī	0;30,12	0;35,40
3.	Ibn Isḥāq	0;30,21	0;36, 1
4.	Ibn al-Raqqām	0;30,21	0;36, 1
5.	Alfonsine tables (ed. 1483)	0;30,21	0;36,25
6.	Paris, BN lat. 7288	0;30,20	0;36, 1
7.	John of Genoa	0;29,37,13	0;36,58,54
8.	John of Lignères	0;29,37	0;36,53
9.	Levi ben Gerson (1)	0;30,18	0;36, 4
10.	Levi ben Gerson (2)	0;29,35	0;36,56
11.	John of Montfort*	0;11,51,9,11°/mn	0;14,44,47,8°/mn

*John of Montfort's velocities are given in degrees per 60th of a day (°/mn).

observations. The use of Ptolemy's second lunar model (rather than his simple lunar model) for computing lunar velocities reflects a better mathematical understanding of the way his model worked rather than the need to account for new observational data. In sum, it is important to recognize that the force behind scientific change is often a tension perceived between two textual traditions, or new mathematical insights applied to already existing models.

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Appendix: Ibn al-Raqqām's lunar velocities

The entries in col. 2, c(α), and in col. 4, v(text), have been taken from the Arabic MSS of Ibn al-Raqqām (R: MS Rabat 260; and K: MS Kandilli 249). The entries in col. (3) have been computed using eq. (1) and the values in col. 2. The entries in col. 5, "T − C", are the differences between text and computation. An asterisk indicates that there is a note below. In all the cases shown here, the entries in col. 4 are the same in R and K. Note that in the MSS there is no entry for argument 0°; the minimum velocity corresponds to argument 1°.

(1) α (°)	(2) c(α) (°)	(3) v(comp) (°/h)	(4) v(text) (°/h)	(5) T − C (")
0	0; 0, 0	0;30,21	0;30,21	0
1	−0; 4,45			
10	−0;47,17	0;30,23	0;30,23	0
11	−0;51,58			
20	−1;33,28	0;30,28	0;30,27	−1
21	−1;37,59			
30	−2;17,25	0;30,38	0;30,37	−1
31	−2;21,39			
40	−2;58, 3	0;30,51	0;30,51	0
41	−3; 1,52			
50	−3;34,15*	0;31, 7	0;31, 7	0
51	−3;37,35*			
60	−4; 4,57	0;31,27	0;31,27	0
61	−4; 7,41			
70	−4;29,20	0;31,49	0;31,49	0
71	−4;31,23			
80	−4;46,14	0;32,11	0;32,14	+3
81	−4;47,36			

(1) α (°)	(2) c(α) (°)	(3) v(comp) (°/h)	(4) v(text) (°/h)	(5) T − C (")
90	−4;55, 0	0;32,42	0;32,41	−1
91	−4;55,26			
100	−4;55, 1	0;33,12	0;33, 8	−4
101	−4;54,32			
110	−4;45,42	0;33,44	0;33,40	−4
111	−4;44,13			
120	−4;27,18*	0;34,13	0;34,14	+1
121	−4;24,57			
130	−3;59,44	0;34,41	0;34,43	+2
131	−3;56,31			
140	−3;23,30	0;35, 7	0;35, 9	+2
141	−3;19,29			
150	−2;39,53	0;35,30	0;35,30	0
151**	−2;35,11			
160	−1;50,12	0;35,47	0;35,47	0
161	−1;44,58			
170	−0;56,14	0;35,58	0;35,57	−1
171	−0;50,40			
180	0; 0, 0	0;36, 1	0;36, 1	0
181	0; 5,39			

Notes:

*ad 50: with K; R reads: 3;34,14
*ad 51: with K; R reads: 3;37,55
*ad 120: R and K both read: 4;27,38 (The emendation is based on nearby line-by-line differences; 38 and 18 are easily confused in Arabic.)

**ad 151 to 180: This column for c(α) is labelled "5 [signs]". In the sub-column for seconds in R, "corrected" readings have been written over the original entries, making some of them illegible -- others seem to be corrupt, judging by the entries in K that yield good values for *v*. Only the entries in K for c(α) from 151 to 180 are displayed here.

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