PRELIMINARY REMARKS ON JUDAH BEN VERGA'S CONTRIBUTIONS TO ASTRONOMY

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Little is known about the life of Judah ben Verga other than that he was an astronomer in Lisbon, writing in Hebrew, who flourished from about 1455 to 1480. The best account of Ben Verga's life has recently been published by Prof. Tzvi Langermann of Bar Ilan University in Tel Aviv whose work is mainly based on unpublished Hebrew manuscripts, many of which were identified by him for the first time. In several treatises Ben Verga records astronomical observations he made in Lisbon in 1456 and 1457 and, in the introduction (or canons) to his astronomical tables, he refers to a lunar eclipse to take place in the future on March 22, 1475, as well as to a solar eclipse to take place on July 29, 1478. Ben Verga cites a few predecessors who

¹ Y. Tzvi Langermann, "Science in the Jewish Communities of the Iberian Peninsula: An Interim Report", in: *idem, The Jews and the Sciences in the Middle Ages* (Aldershot, 1999), essay I, pp. 19-25.

² St. Petersburg, Russian Academy of Sciences, MS C-076, f. 63a (solar eclipse), f. 63b (lunar eclipse).

wrote in Hebrew or Arabic, but not any who wrote in Latin, Spanish, or Portuguese. Also, I have found no evidence of an interest in astronomical navigation (despite some modern claims to the contrary),3 or of any relationship with contemporary Portuguese scholars. Abraham Zacut of Salamanca mentions Ben Verga only twice - concerning geographical and chronological matters - in his Great Composition (ha-Hibbur ha-gadol), composed in Hebrew and completed in 1478.4 Moreover, Ben Verga's name also appears in the 1496 edition of Zacut's tables where there is a Latin dedication to an unnamed bishop (copied, for the most part, from a dedication by Regiomontanus to a bishop in Hungary, published in 1490) that was added by the printer, Samuel d'Ortas.⁵ In fact, Zacut also cites Iudah Ben Asher of Burgos (d. 1391) in both sentences where Judah Ben Verga is mentioned; it would have made more sense in the dedication to refer to Ben Asher whose name occurs frequently in a variety of astronomical contexts in Zacut's Great Composition.6 In any event, as far as I have been able

³ See, e.g., A. Fontoura da Costa, *A marinharia dos descobrimentos*, 4th ed. (Lisbon, 1983), pp. 80-81.

⁴ F. Cantera Burgos, "El judío salmantino Abraham Zacut", Revista de la Academia de Ciencias Exactas, Físicas y Naturales, Madrid, 1931, 27:63–398, espec. pp. 153, 156 (cf. index of names on p. 237).

⁵ A. Zacut, Tabule tabularum coelestium motuum astronomi zacuti (Leiria, 1496), f. 2r; J. Chabás and B. R. Goldstein, Astronomy in the Iberian Peninsula: Abraham Zacut and the Transition from Manuscript to Print, in: Transactions of the American Philosophical Society, vol. 90.2 (Philadelphia, 2000), pp. 90-95.

⁶ On Judah ben Asher, see B. R. Goldstein, "Abraham Zacut and the Medieval Astronomical Tradition", *Journal for the History of Astronomy*, 1998, 29:177-86, espec. pp. 179-81.

to determine, Ben Verga is mentioned nowhere else by a contemporary (or near contemporary) astronomer.

In this paper the focus will be on Ben Verga's astronomical tables together with the canons to them. But first it will be useful to say something about the history and the contents of astronomical tables in the Middle Ages.

There were two principal sources of medieval astronomy that came together in Baghdad in the ninth century. The first to arrive there (in the late eighth century) came from India as a set of rules with no explanations. These rules were turned into tables, and the most important representative of this tradition is the zij (i.e., a set of astronomical tables) by al-Khwārizmī (ca. 820). This zij was written in Arabic and used the Persian calendar, but no copy of the original version is extant. Rather, one has a Latin translation made in Spain in the twelfth century based on a revised version by Maslama al-Majrīṭī (ca. 1000) that uses the Hijra calendar (instead of the Persian calendar). This text was very popular in medieval Spain, in the Muslim, Jewish, and Christian communities.

The second source was the Ptolemaic tradition, largely based on the *Almagest*, composed in Greek in the second century in Alexandria, that came to dominate medieval astronomy. An important representative of this tradition is the zij of al-Battānī (ca. 880), composed in Arabic and sur-

⁷ H. Suter, Die astronomischen Tafeln des Muhammad ibn Mūsā al-Khwārizmī (Copenhagen, 1914); O. Neugebauer, The Astronomical Tables of al-Khwārizmī (Copenhagen, 1962).

viving in Arabic, Hebrew, Latin, and Castilian versions.⁸ Again, this was a popular text in Spain, in the Muslim, Jewish, and Christian communities.⁹ In 1956, E. S. Kennedy listed well over 100 zijes from the medieval Islamic world, and many more have come to light subsequently.¹⁰ Other zijes have been found in Byzantine Greek, Hebrew, Latin, Castilian, and Catalan, all depending to a greater or lesser extent on the antecedent tradition in the Islamic world.¹¹ For present purposes it is best to concentrate on astrono-

⁸ C. A. Nallino, Al-Battānī sive Albatenii Opus Astronomicum, 3 vols. (Milan, 1899-1907).

⁹ B. R. Goldstein, "Astronomy in the Medieval Spanish Jewish Community," in: L. Nauta and A. Vanderjagt (eds.), Between Demonstration and Imagination: Essays in the History of Science and Philosophy Presented to John D. North (Leiden, 1999), pp. 225-41.

¹⁰ E. S. Kennedy, A Survey of Islamic Astronomical Tables, in: Transactions of the American Philosophical Society, vol. 46 (Philadelphia, 1956); cf. idem, "The Astronomical Tables of Ibn al-Raqqām, a Scientist of Granada", Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften, 1997, 11:35-72. See now D. A. King and J. Samsó, with a contribution by B. R. Goldstein, "Astronomical Handbooks and Tables from the Islamic World (750-1900): An Interim Report", Suhayl, 2001, 2:9-105.

¹¹ For Catalan see, e.g., J. Chabás, L'astronomia de Jacob ben David Bonjom (Barcelona, 1992); idem, "Astronomia andalusí en Cataluña: Las Tables de Barcelona", in: J. Casulleras and J. Samsó (eds.), From Baghdad to Barcelona: Studies in the Islamic Exact Sciences in Honour of Prof. Juan Vernet (Barcelona 1996), pp. 477-525. For Byzantine Greek see, e.g., A. Tihon and R. Mercier, Georges Gémiste Pléthon: Manuel d'astronomie (Louvain-la-Neuve, 1998); A. Tihon, Etudes d'astronomie byzantine (Aldershot, 1994); D. Pingree, "The Byzantine Version of the Toledan Tables: The Work of George Lapithes?", Dumbarton Oaks Papers, 1976, 30:87-132; P. Solon, "The Six Wings of Immanuel Bonfils and Michael Chrysokokkes", Centaurus, 1970, 15:1-20.

mical activity in Spain, and so we will ignore the texts produced in Byzantium and elsewhere.

In Latin, one of the most widely copied zijes is called the Toledan Tables which uses the Hijra calendar and, in general, it depends on the zijes of al-Khwārizmī and al--Battānī; according to L. Richter-Bernburg, they were originally compiled in Arabic by Sacid al-Andalusī in the eleventh century and translated into Latin in the twelfth century. The Toledan Tables were used all over Latin Europe, and there were several adapations of them. 12 But the best known medieval zij is the Alfonsine Tables, preserved in a Latin version produced in Paris in the 1320s, based on a Castilian original produced under the patronage of Alfonso X of Castile (d. 1284) by two Jewish astronomers, Isaac ben Sid and Judah ben Moses ha-Kohen. Of the original Castilian version only the canons survive. The Parisian version of the Alfonsine Tables is mainly in the Ptolemaic tradition, and incorporated some material from the Toledan Tables.¹³

The medieval tradition of zijes in Hebrew is not well known. Although there are more than 30, only a few of them have been published or even described in the modern

¹² L. Richter-Bernburg, "Ṣacid, the *Toledan Tables*, and Andalusī Science", *Annals of the New York Academy of Sciences*, 1987, 500:373-401; G. J. Toomer, "A Survey of the Toledan Tables", *Osiris*, 1968, 15:5-174.

¹³ E. Ratdolt (ed.), Tabule astronomice illustrissimi Alfontij regis castelle (Venice, 1483); see also E. Poulle, Les Tables Alphonsines avec les canons de Jean de Saxe (Paris, 1984); J. D. North, "The Alfonsine Tables in England", in: Y. Maeyama and W. G. Salzer (eds.), Prismata: Festschrift für Willy Hartner (Wiesbaden, 1977), pp. 269-301.

secondary literature.¹⁴ But it is clearly important for understanding the zij of Judah ben Verga, one of those that has not been described previously. This tradition began in Spain in the twelfth century with the zij of Abraham Bar Ḥiyya of Barcelona, based on the zij of al-Battānī.¹⁵ The most original zijes in Hebrew were composed by Levi ben Gerson (d. 1344) in Orange, France, and his work was cited by Jews in Spain (both in Castile and Catalonia). His short zij is extant in Hebrew as well as in a Provençal version, and his long astronomical treatise that includes many tables was translated into Latin during his lifetime.¹⁶ In particular, Levi reported many dated astronomical observations that he made himself, and he used them explicitly to construct his theories and the tables based on them.¹⁷ This practice was

¹⁴ B. R. Goldstein, "The Role of Science in the Jewish Community in Fourteenth Century France", Annals of the New York Academy of Sciences, 1978, 314:39-49; idem, "The Survival of Arabic Astronomy in Hebrew", Journal for the History of Arabic Science, 1979, 3:31-39; idem, "Scientific Traditions in Late Medieval Jewish Communities", in: G. Dahan (ed.), Les Juifs au regard de l'histoire: Mélanges en l'honneur de M. Bernhard Blumenkranz (Paris, 1985), pp. 235-47.

¹⁵ J. M. Millás Vallicrosa, La obra Séfer Ḥeshbón mahlekot hakokabim de R. Abraham Bar Ḥiyya ha-Bargeloni (Barcelona, 1959).

¹⁶ B. R. Goldstein, The Astronomical Tables of Levi ben Gerson (New Haven, 1974); J. L. Mancha, "The Latin Translation of Levi ben Gerson's Astronomy", in: G. Freudenthal (ed.), Studies on Gersonides: A Fourteenth-Century Jewish Philosopher-Scientist (Leiden, 1992), pp. 21-46; idem, "The Provençal Version of Levi ben Gerson's Tables for Eclipses", Archives Internationales d'Histoire des Sciences, 1998, 48:269-352.

¹⁷ B. R. Goldstein, "Medieval Observations of Solar and Lunar Eclipses", *Archives Internationales d'Histoire des Sciences*, 1979, 29:101-56; *idem*, "A New Set of Fourteenth Century Planetary Observations", *Proceedings of the American Philosophical Society*, 1988, 132:371-99.

unusual in the Middle Ages, and it is therefore of interest that Ben Verga also cites a few of his own dated observations.

We then come to the extensive tables compiled by Abraham Zacut of Salamanca (1452-1515) in Hebrew, based largely on the Parisian version of the Alfonsine Tables. Zacut also depended on his Jewish predecessors who compiled zijes in Hebrew, and they are explicitly mentioned. Two versions of these tables were published in Leiria, Portugal, in 1496, by the printer, Samuel d'Ortas, who otherwise is only known to have published Hebrew texts. This printing house produced one version of Zacut's zij with canons in Latin, and another with canons in Castilian (the headings for the tables in both versions are in Latin).18 It had been thought that the printer was Abraham, the son of Samuel d'Ortas, but Cecil Roth argued persuasively that Samuel was the printer, and his sons (including Abraham) were merely his assistants.¹⁹ These versions of Zacut's zii (usually cited as his Almanach Perpetuum) were the only representatives of Hebrew zijes to be printed while they were still of current scientific interest. Unfortunately, there is no general survey of zijes in Latin or in Hebrew comparable to Kennedy's pioneering study of zijes in the Islamic world.

In general, zijes contain a variety of tables. The first group is for converting dates between the calendars used by each community, and for indicating the difference in days between various epochs. The second group concerns trigo-

¹⁸ Chabás and Goldstein (cit., n. 5), pp. 95-98.

¹⁹ C. Roth, "Who Printed Zacuto's Tables?", Sefarad, 1954, 14:122–125.

nometric functions, beginning with a sine table, for solving problems in plane and spherical trigonometry. Related to them are tables for specific functions such as a table for the declination of the Sun as a function of its longitude, and tables for right and oblique ascensions for each degree on the ecliptic. Usually, there is a table (or a group of tables) for determining the boundaries or cusps of the twelve astrological houses for each degree of the ecliptic when it crosses the eastern horizon. There were several methods for determining the astrological houses (all of which are entirely mathematical), and the tables for them were not generally distinguished from the astronomical part of a zij.20 The next group of tables concern planetary motions (including tables for the Sun and the Moon): mean motions, corrections to mean motions, and velocities. Then comes a group of tables for computing eclipses: tables for mean conjunctions and oppositions of the Sun and Moon (or syzygies), corrections for finding true syzygy from mean syzygy, corrections for parallax, and magnitudes and durations of solar and lunar eclipses. In addition, some zijes have tables for the colors of eclipses.²¹ Also, we often find a geographical list with the coordinates of cities, and a star list with the coordinates of fixed stars. Associated with the fixed stars is often a table for the trepidation of the equinoxes, for it was believed by many medieval astronomers that the fixed stars had a non-

²⁰ J. D. North, Horoscopes and History (London, 1986).

²¹ See, e.g., Ratdolt (cit., n. 13), f. 18v; J. M. Millás Vallicrosa, Las Tablas Astronómicas del Rey Don Pedro el Ceremonioso (Madrid-Barcelona, 1962), p. 238; J. Chabás and B. R. Goldstein, "Andalusian Astronomy: al-Zīj al-Muqtabis of Ibn al-Kammād", Archive for History of Exact Sciences, 48 (1994). pp. 1-41, espec. pp. 18-19.

uniform motion with respect to the equinoxes.²² All zijes do not have every one of these groups of tables, but they do have several of them. In almanacs, e.g., the *Almanach Perpetuum* of Zacut, there are lists of true planetary positions at intervals of a few days, and lists of this kind are sometimes incorporated in a zij. Moreover, zijes frequently contain a variety of astrological tables, for astrologers who sought this information were among the 'consumers' whose needs had to be addressed.²³

We may now apply these categories to the zij of Ben Verga. There are two extant copies of the tables (Paris, Bibliothèque Nationale de France [BNF], MS Heb. 1085, ff. 86b-98a [P]; and Oxford, Bodleian Library, MS Poc. 368, ff. 222b-236a [Ox]); and one copy of the canons (St. Petersburg, Russian Academy of Sciences, MS Heb. C-076, ff. 57a-65a [R]). Surprisingly, the two copies of the tables do not agree on technical terminology in all cases. For example, for the Moon, the Paris copy uses the term *levana*, whereas the Oxford copy uses the term *ha-yareaḥ*. Although most of the tables in the two copies are identical with minor textual variants, there are some tables in one copy but not

²² See, e.g., R. Mercier, "Accession and Recession: Reconstruction of the Parameters", in: Casulleras and Samsó (*cit.*, n. 11), pp. 299–347; M. Comes, "Accession and Recession Theory in al-Andalus and the North of Africa", in: Casulleras and Samsó (*cit.*, n. 11), pp. 349-64; B. R. Goldstein, "Historical Perspectives on Copernicus's Account of Precession", *Journal for the History of Astronomy*, 1994, 25:189-97. Note that the modern expression, "trepidation of the equinoxes", corresponds to "the motion in access and recess" in medieval texts.

²³ See, e.g., Suter (*cit.*, n. 7), pp. 194-231. Cf. Neugebauer (*cit.*, n. 7), pp. 128-32.

the other. The canons explain neither how the tables were computed nor how the parameters underlying them were derived, but only indicate how to use them. This was a common, but not universal, practice in the Middle Ages. Ben Verga's name does not appear in either the tables or the canons and, while Lisbon is only mentioned once in the tables (Ox, f. 232a), it is often cited in the canons. The tables have no title, but in the canons it is given as *Ordinances of the Heavens* (*Huqot shamayim*: cf. Job 38:33); it may be of interest that a similar title was used by Judah ben Asher for his astronomical work (*Ordinances of the Heavens and the Earth*: Jer 33:25).²⁴ The identification of the author as Ben Verga derives from the references to an astronomical observation in 1456 reported in the canons in the first person:

Know: I observed (ciyyanti) at Lisbon with the large instrument I had that was divided at intervals of 10 minutes [of arc], and I found that the autumnal equinox (tequfat tishri) of year [5]217 [A.M.] took place on Tuesday, the first day of [the festival of] Sukkot, 13 complete days of September having passed of the year 1456 according to the Christians, and another 7 hours after noon. (R, f. 64a)

This same observation is reported by Ben Verga in his treatise on the *Horizontal Instrument* (see, e.g., Paris, BNF, MS Heb. 1031, ff. 155b-163a). In that treatise, specifically ascribed to Ben Verga, the author remarks that he resides in

²⁴ Vatican, MS 384, f. 285b:21; cf. Zacut, *Ḥibbur*, in Cantera (cit., n. 4), p. 340.

Lisbon (Paris, MS Heb. 1031, f. 155b). In fact, no other Jewish astronomer writing in Hebrew is known to have been active in Lisbon at that time.

In the tables, Ben Verga does not mention any of his predecessors or contemporaries, and in the canons he mentions only a few of them: Ptolemy, Azarquiel, and Abraham Ibn Ezra. In his commentary on the astronomical treatise by al-Farghani (ninth century), he mentions some other Jewish predecessors, all of whom were active in the fourteenth century: Levi ben Gerson, Moses Narboni (d. 1362), Jacob ben David Bonjorn, also known as Jacob Poel (fl. 1360), and Judah ben Asher.²⁵ It is surprising that he does not mention Isaac ben Sid, the principal compiler of the Alfonsine Tables, who was the most prominent Jewish astronomer in Spain in the thirteenth century.²⁶ Moreover, he does not mention the Alfonsine Tables that were the most widely copied and studied in Christian Europe, but there is internal evidence to suggest that he might have depended on them to some extent. Ben Verga does not refer to Abraham Zacut (born 1452) who was a generation younger, although there was some overlap in the times when they were active. On the other hand, Zacut cites Ben Verga, and their respective tables have certain matters in common; we will discuss a few of them below. In contrast to Ben Verga, Zacut's tables

²⁵ Langermann (cit., n. 1), p. 23.

²⁶ N. Roth, Jewish Collaborators in Alfonso's Scientific Work", in: R. I. Burns (ed.), *Emperor of Cultures: Alfonso X the Learned of Castile and His Thirteenth Century Renaissance* (Philadelphia, 1990), pp. 59-71. Isaac ben Sid is also mentioned by Judah ben Asher in the canons to his zij: Vatican, MS Heb. 384, 285b:2.

are heavily dependent on the Parisian version of the Alfonsine Tables.

The two manuscripts of Ben Verga's zij have a total of 30 distinct tables: 26 of them are preserved in both manuscripts, with one additional table in the Paris manuscript, and 3 additional tables in the Oxford manuscript. They can be separated into a number of categories, and here I will restrict my attention mainly to his planetary mean motion tables, and his planetary correction tables. Let me note in passing that there are no tables for daily positions of the Sun, in contrast to the almanac of Abraham Zacut, among others.

There are mean motion tables for the Sun, the Moon, the lunar anomaly, the lunar node, and the five visible planets, plus a table for the mean elongation of the Sun from the lunar node, and a table for mean conjunctions of the Sun and the Moon. In this category the unusual tables are those for the solar elongation from the lunar node, and for the mean conjunctions of the Sun and the Moon. The purpose of the table for the solar elongation from the lunar node is to help in deciding when it is appropriate to compute a forthcoming solar or lunar eclipse, for an eclipse of either kind can only occur when the Sun is near a lunar node (one of the points where the lunar orb intersects the ecliptic, i.e., the apparent path of the Sun). Despite its usefulness, the only examples of which I am aware of similar tables are in the zij of Abraham Zacut and in the anonymous Tabule Verificate for Salamanca, produced in the 1460s.²⁷

²⁷ Chabás and Goldstein (at., n. 5), pp. 29-30, 118.

All of Ben Verga's mean motion tables first give positions at 28-year intervals, beginning with 1384 for the tables of mean motion for the Sun, the Moon, the lunar anomaly, the lunar node, and the five planets; but 1400 for the table for the solar elongation from the lunar node, and the table for mean conjunctions of the Sun and the Moon. It is certainly uncommon for the epochs of mean motion to differ in the same zij, but there is nothing in his canons or the tables to call attention to this peculiarity.

There are several subtables for the mean motions: the motion in Julian years from 1 to 28, the motion in months from January to December, and the motion in days from 1 to 30. In all cases the differences in motion between the epochs at 28-year intervals agree with the motion in the subtable for 28 years, and so they provide no additional information for computing the mean motion per day. When the motion in 28 Julian years is divided by the number of days in 28 Julian years, we arrive at the mean motion per day that characterizes the table, and it can then be compared with the corresponding mean motions in other zijes. In this case, the results indicate that the parameters in Ptolemy's Almagest are closest to those of Ben Verga for the mean motions of Saturn, Jupiter, Mars, and Mercury; the Toledan Tables are closest for Venus; and the Alfonsine Tables are closest for the Moon and the lunar node. For the Sun and for lunar anomaly, Ben Verga's parameters are very close to those of Levi ben Gerson. While two mean motion parameters are close to those in the Parisian version of the Alfonsine Tables, the structure of Ben Verga's tables is unrelated to them. So it is possible that this closeness of the parameters is due to a common source. It needs to be

emphasized, however, that changes in parameters did not usually result from new observations, for the necessary dated observations are very few and far between in medieval astronomical sources; indeed, most medieval astronomers do not report any observational activity of their own.

The table for mean conjunctions is also unusual. It yields the date and time of mean conjunction of the Sun and the Moon, as well as providing the arguments of the Sun and Moon to be used in a subsequent double argument table for finding the time from mean to true conjunction. In the canons there are worked examples using this unusual table which supports the claim that the canons belong with these tables even though they are preserved in different manuscripts. As with the other mean motion tables, it is arranged for epochs at 28-year intervals, Julian years from 1 to 28, and months from January to December. The entries in the column labeled "days of conjunction" yield the number of days since the most recent mean conjunction, i.e., after casting out multiples of the length of the mean synodic month of about 29d 12;44h. The entries in the other two columns, labeled "days of the Sun" and "days of the Moon" yield the number of days since the Sun was at its apogee (the point on its orb farthest from the Earth), and number of days since the Moon was at the apogee of its epicycle. These arguments for the Sun and the Moon are in days, hours, and minutes, whereas usually they are given in degrees of arc. The reason for tabulating these quantities is that the time from mean to true conjunction is a function of both solar and lunar velocities, and they in turn depend on the arc between the apogees (where velocity is least) and the position of the luminary on the appropriate orb.

There is no other known example of a table with precisely this structure, but two columns in Ben Verga's subtable for months are also found in the Tabule Verificate for Salamanca. Moreover, a related table in the Tabule Verificate (Table TV 1) is based on a parameter very close to the one in Ben Verga's table, although the presentation is entirely different. In Table TV 1 the tabulated magnitude is the period of time between the beginning of the year and the first mean conjunction of the Sun and the Moon for each of 77 consecutive years, beginning in 1461.28 For example, the entry in that table for year 31 is 19d 12;26h, and the entry for year 3 is 10d 2;28h; hence the difference is 9d 9;58h, whereas in Ben Verga's table the entry for 28 years of "the days of conjunction" is 9d 9;57h. These are different roundings of the same parameter which we compute as follows, noting that there are a little more than 346 synodic months in 28 years:

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1m = 29;31,50,8d \approx 29d 12;44h

1y = 365;15d = 365\frac{1}{4}d

28y \cdot 365;15d = 10227d

346m \cdot 29;31,50,8d = 10217;35,6,8d

28y - 346m = 9;24,53,52d = 9d 9;57,32,48h \approx 9d

9;58h
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These similarities with the *Tabule Verificate* suggest some connection between Ben Verga and Salamanca even though he says nothing about it.

²⁸ Chabás and Goldstein (*cit.*, n. 5), pp. 24f; cf. Madrid, MS 3385, f. 104r.

It is by no means clear if Ben Verga computed the entries in all his tables himself, or if he copied some of them from an earlier unidentified zij (as is clearly the case with many compilers of zijes). The use of two different epochs, 1384 and 1400, both well before the time he was active, seems to support the suggestion that he was not the computer of all of them.

A second category of tables in Ben Verga's zij is for finding the correction between the mean position of a planet and its true position, again a function of two variables. In its original geometric form as Ptolemy defined the planetary motions in his models, the solution requires a considerable amount of trigonometric computations. However, Ptolemy cleverly decided to approximate the exact solution by introducing tables with relatively few entries that only require interpolation and some simple multiplication of entries in different columns. As a result, a computer has only to follow a few arithmetic procedures without having to draw a figure for the model for the particular circumstances at the time in question, or having to solve any triangles by means of trigonometry.

Ptolemy's planetary correction tables, with minor variants, were often imitated in the corresponding tables of medieval zijes: e.g., the Alfonsine Tables have planetary correction tables in much the same form as Ptolemy had them but for some small changes in parameters. However, Ben Verga did not follow that pattern, for he introduced double argument tables to represent Ptolemy's models for each planet, thus requiring only interpolation while eliminating the need to multiply entries from different columns. This fits in with a general trend in medieval zijes

to make tables "user-friendly", by introducing ingenious methods of approximation and by shifting most of the computation from the user to the compiler of the tables. Again, one needs to bear in mind that observation played no role in changing the presentation of these tables. The goal was not to produce a new planetary model to account for the motions in the heavens, but a simpler way to obtain planetary positions according to Ptolemy's models. Since this is a "rule", of course there are some exceptional instances of new models introduced by medieval astronomers. But this happened rarely, and in many such cases the new models were intended to produce the same positions, very nearly, as were produced by Ptolemy's models.²⁹ Ben Verga's solution is ingenious but not as "user-friendly" as some of the solutions introduced by others. For example, in the table for Saturn, one variable (in the headings for the columns) is the mean motion of Saturn at intervals of 30° beginning with its apogee at 252° of longitude, and the other (for the rows) is the mean motion of the Sun at intervals of 14 days from 0 days to 364 days, with an added row for 365 days. The entries were computed according to Ptolemy's model with Ben Verga's mean motions for the Sun and for Saturn.

²⁹ For example, the planetary models of Naṣīr al-Dīn al-Ṭūsī (d. 1274) were intended to produce positions that agree with Ptolemy's model: E. S. Kennedy, "Late Medieval Planetary Theory", *Isis*, 1966, 57:365-78, espec. p. 366; reprinted in: *idem*, *Studies in the Islamic Exact Sciences* (Beirut, 1983), pp. 84-97. On al-Ṭūsī, see now: F. J. Ragep, *Naṣīr al-Dīn al-Ṭūsī's* Memoir on Astronomy (New York, 1993). On the other hand, Levi ben Gerson's lunar model was intended to account for Levi's own observations, and not to reproduce positions that follow from Ptolemy's lunar model: see Goldstein, (*cit.*, n. 16), pp. 53-74.

A worked example using this table is found in the canons, and it also serves to confirm the relationship between the canons and the tables. But the canons do not indicate how the entries in this table were computed.

In the Ptolemaic geometric model for solar motion, the correction from mean to true Sun only depends on one variable, and so Ben Verga's table for it is different from his planetary correction tables, although he made it look like them. Also, the underlying parameter for the maximum correction (that takes place at a little more than 90° from the solar apogee) is 1;53°, a value probably rounded from 1;52,44° associated with Ibn al-Kammād (early twelfth century). Ptolemy's corresponding parameter is 2;23°, and Ben Verga uses it for a different table (preserved only in MS Ox) that has the heading: "The solar correction according to Ptolemy". Ibn al-Kammād is not mentioned, and perhaps Ben Verga associated this table with Azarquiel who was ultimately responsible for it.

Ben Verga's other tables are not particularly unusual, although his avoidance of anything astrological is worthy of mention. The geographical list in the Paris manuscript has places only in Spain and Portugal, and the coordinates for Lisbon are: latitude 39;38°, longitude 22;54°. In Zacut's list, the latitude of Lisbon is 39;35°, and the longitude is 22;54° in close agreement with our text. On the other hand, in Ben Verga's treatise on the horizontal instrument, the coordinates of Lisbon are given as about latitude 39°, and longitude 5° from the west. Similarly, in the editio princeps of the Alfonsine Tables (1483) the longitude is 5° (but the latitude is now 41°). In the Oxford copy of Ben Verga's tables there is a heading in which the longitude of Lisbon is

23° (Ox, f. 232a). The values 5° and 23° for the longitude of Lisbon would seem to be incompatible, but there were several conventions in the Middle Ages for the prime meridian, and one of them, called "the meridian of water", was about $17\frac{1}{2}$ ° to the west of the meridian defined by the "Fortunate Islands" (i.e., the Canary Islands) that was Ptolemy's prime meridian. So 5° for the longitude of Lisbon is defined with respect to the Fortunate Islands, and 22;54° with respect to the "meridian of water".³⁰

While in medieval Islam there are many scientific discussions on finding the qibla, i.e., the direction for praying towards Mecca, there is surprisingly little that is comparable in the Jewish tradition. Al-Bīrūnī, a well informed Muslim scholar of the eleventh century, tells us that "the Jews also need to determine a direction [of prayer], because they turn in their prayers to the Temple in Jerusalem." Except for one Yemenite Jewish text of the fifteenth century identified by Langermann, the only scientific discussion in the Middle Ages of the direction of prayer towards Jerusalem is in Ben Verga's Treatise on the horizontal instrument. Ben Verga states the problem as finding the direction for praying towards Jerusalem from any city in the world, and uses Lisbon as an example. For the coordinates of Jerusalem he cites Pto-

³⁰ M. Comes, "The 'Meridian of Water' in the Tables of Geographical Coordinates of al-Andalus and North Africa", *Journal for History of Arabic Science*, 1992-94, 10:41-51.

³¹ J. Ali, Al-Bī rūnī's Determination of the Coordinates of Cities (Beirut, 1967), p. 176.

³² Paris, BNF, MS Heb. 1031, ff. 155b-163a; cf. Langermann (cit., n. 1), p. 19 n. 51; idem, The Jews of Yemen and the Exact Sciences (Jerusalem, 1987), p. 21 [in Hebrew].

lemy's Geography (or Cosmographia) which, he says, gives longitude 66;30° and latitude 33;30°. By way of contrast, the editio princeps of this treatise, published in 1475, assigns to Jerusalem longitude 66°, and latitude 31½,3°.33 To be sure, Ben Verga may have used a manuscript of Ptolemy's text with different entries from those in the editio princeps. But the coordinates of Jerusalem in a manuscript of Ptolemy's Cosmographia (Salamanca, MS 2586, f. 50r), dated 1456, are the same as those in the printed edition.34

Ben Verga cites the Babylonian Talmud to justify his interest in this question. In this discussion from late antiquity we are told the following:

Our rabbis taught: A blind man or one who cannot tell the cardinal points should direct his heart (while praying) towards his Father in Heaven ... If he is standing outside the Land of Israel, he should turn his heart (mentally) toward the Land of Israel. ... Consequently, someone to the east (of Jerusalem) should

³³ I have checked a copy at Carnegie Mellon University Library (Pittsburgh, USA) of the *editio princeps* of Ptolemy's *Cosmographia* (Vicenza, 1475), f. B5r.

³⁴ Salamanca, MS 2586, is described in: G. Beaujouan, Manuscrits scientifiques médiévaux de l'Université de Salamanque et de ses «Colegios Mayores» (Bordeaux, 1962), p. 161. The same coordinates of Jerusalem are also found in another copy of Ptolemy's text: Salamanca, MS 2495, f. 100v. I am most grateful to Dr. Bertha Gutiérrez for providing me with the data from these manuscripts; cf. Langermann (cit., n. 1), p. 20. For discussion of the evidence that manuscripts of Ptolemy's Geography circulated in Portugal prior to the time of Ben Verga, see W. G. L. Randles, "The Alleged Nautical Schol Founded in the Fifteenth Century at Sagres by Prince Henry of Portugal, Called the 'Navigator'", Imago Mundi, 1993, 45:20-28.

turn his face towards the west; someone to the west (of Jerusalem) should turn his face to the east; someone to the south (of Jerusalem) should turn his face to the north; and someone to the north (of Jerusalem) should turn his face to the south. In this way all Israel will be turning their hearts towards the same place.³⁵

Ben Verga comments that the same rule should apply to someone who is not in one of the cardinal directions from Jerusalem, and hence it is necessary to determine the direction of prayer towards Jerusalem. However, he says nothing about Muslim practice or about the mathematical treatment of this problem by Muslim scholars. In fact, the problem is one of spherical trigonometry for which Muslim scholars came up with a variety of solutions, but Ben Verga omits such details.36 In discussions of the qibla it is usual to give a value for the circumference of the Earth and, for Ben Verga, it is 24,000 miles, equivalent to 66²/₃ miles per degree. This value was widely cited in the Middle Ages, based on Ptolemy's claim that the circumference of the Earth is 180,000 stades, together with an erroneous conversion factor of 71/2 stades per mile.37 Despite his interest in the size of the Earth, Ben Verga has no discussion of navigation.

³⁵ M. Simon, Hebrew-English Edition of the Bablyonian Talmud: Berakoth (London, 1965), f. 30a.

³⁶ D. A. King, World-Maps for Finding the Direction and Distance to Mecca (Leiden, 1999).

³⁷ For Ben Verga's discussion, see Paris, BNF, MS Heb. 1031, f. 160b; see also E. S. Kennedy, *A Commentary upon Bī rūnī's Kitāb Taḥdī d al-Amākin* (Beirut, 1973), p. 131.

The most notable Jewish scholar in Portugal at the time of Ben Verga was Isaac Abravanel (1437-1508), but in the 1460s he was mainly engaged in business affairs and in the 1470s he became a protégé of the House of Braganza.38 As far as I can tell, Abravanel does not mention Ben Verga in any of his voluminous works (at least, I have found no reference in the secondary literature on Abravanel to Ben Verga). Abravanel's interest in astronomy seems to have been limited to astrological issues,39 and the absence of astrology in Ben Verga may have set them apart. I have not found a polemic against astrology in the works of Ben Verga, but it is likely that he opposed it on religious grounds, following the opinion of Maimonides (d. 1204), the great Jewish philosopher and legal scholar. The relationship of astronomy and astrology in the Middle Ages is a separate topic and deserves extensive treatment, but this is not the occasion for it.40

³⁸ B. Netanyahu, Don Isaac Abravanel: Statesman & Philosopher (Philadelphia, 1968), pp. 18-25; R. Goetschel, Isaac Abravanel: conseiller des princes et philosophe (Paris, 1996), pp. 16-19.

³⁹ For discussion of Abravanel's positive assessment of astrology, see Netanyahu (*cit.*, n. 38), pp. 118-21; for his attempts to use astrology to confirm his calculations for the date of Messianic fulfillment, see Goetschel (*cit.*, n. 38), pp. 153-56.

⁴⁰ For the views of Maimonides, see his "Letter to Yemen" and his "Letter on Astrology", translated in: I. Twersky, *A Maimonides Reader* (Philadelphia, 1972), pp. 437-73. For discussion, see, e.g., G. Freudenthal, "Maimonides' Stance on Astrology in Context: Cosmology, Physics, Medicine and providence", in: F. Rosner and S. S. Kottek (eds.), *Moses Maimonides: Physician, Scientist, and Philosopher* (Northvale and London, 1993), pp. 77-90, 244-49.

In sum, based on an analysis of his zij, one can conclude that Ben Verga was a competent astronomer by the standards of his day, and that his tables were cleverly arranged. But the intellectual context in which he worked and the sources that were available to him have not yet been established. Perhaps documents will soon be discovered in Portugal that illuminate these issues.

Added in proof. See now: B.R. Goldstein, "The Astronomical Tables of Judah ben Verga", Suhayl, 2001, 2:227-289.

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