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THE PRE-TELESCOPIC TREATMENT OF THE PHASES AND APPARENT SIZE OF VENUS

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Medieval astronomers who sought to establish the physical reality of the Ptolemaic System were faced with two puzzles concerning Venus. If the orb of Venus was entirely below the Sun (in which case the distance from the Earth to Venus would always be less than the distance from the Earth to the Sun), then Venus ought to show crescent phases like those of the Moon. Moreover, according to Ptolemaic theory, the ratio of the maximum to minimum distance of Venus from the Earth was about 7 to 1, and this implied that the apparent size of Venus should vary in the same proportion. But no crescent phase of Venus was observable, and the variation in the apparent size of Venus was barely perceptible. As we shall see, the medieval solution to the absence of phases was to assume that Venus was self-luminous, but the lack of variation in apparent size was not understood until the invention of the telescope in the seventeenth century.

From a modern point of view, the problem for naked-eye observations of Venus is that the variation in its apparent diameter, from about 0;0,10° to about 0;1°, is below the threshold of the resolving power of the eye, which is about 0;1°.[1] Bright sources of light are seen as having a size, and brightness was confused with size before the introduction of the telescope. Indeed, the scale of magnitudes for the brightness of celestial objects goes back to Ptolemy's *Almagest*, and there magnitude certainly was intended to mean the apparent size of the star. The transformation of the traditional estimates of apparent size into the modern scale of stellar magnitudes is beyond the scope of this paper; suffice it to say that a difference of 1 in the scale of magnitudes corresponds to a ratio of brightnesses of about 2.5 to 1.[2] Ptolemy's magnitudes were restricted to integer values from 1 to 6, but this scale has been extended in both directions, including negative magnitudes for such bright objects as Venus, the Moon, and the Sun. In the case of Venus, the brightness shows relatively little variation (less than 1 magnitude) because the effect of the phases is largely cancelled by the enormous variation in its distance from the Earth. According to modern observations, maximum brightness of Venus takes place at about 39° of elongation from the Sun, but this phenomenon was not recognized in the Middle Ages, as far as I know. Of critical importance for the modern understanding of the phases is the phase angle: in the plane triangle formed by the Sun, the Earth, and Venus, the angle at Venus is the phase angle (see Figure 1). At superior conjunction the

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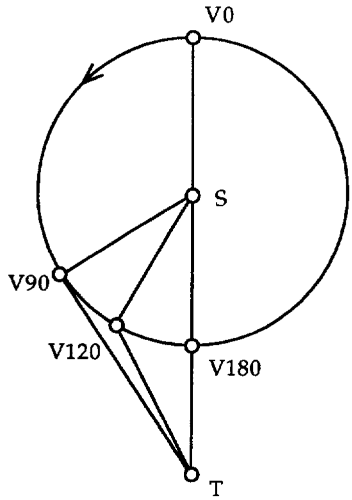

FIG. 1. The elongations and phases of Venus, disregarding its latitude. *S* represents the Sun, *T* the Earth, and *V* Venus. The phase angle is angle *TVS*. *V*0 indicates superior conjunction, *V*90 greatest elongation, *V*120 maximum brightness, and *V*180 inferior conjunction.

phase angle is near 0° and the side of Venus facing us is entirely illuminated; at greatest elongation from the Sun the phase angle is 90°; Venus reaches its maximum brightness when the phase angle is about 120° and its elongation is about 39°; and at inferior conjunction the phase angle is near 180° and hardly any of its side facing us is illuminated. The latitude of Venus at inferior conjunction, in general, has a noticeable effect and keeps Venus from being aligned with the Earth and the Sun; hence, the phase angle at inferior conjunction is not 180° except in those rare cases of a transit of Venus.[3]

For the most part, medieval astronomers chose to ignore these puzzles and they continued to use Ptolemy's model for Venus (with slight modifications), which was adequate for computations of the positions of Venus in longitude.[4] Frequently, it was simply assumed that one would observe a great variation in the size of Venus, but this was based on Ptolemaic theory, rather than on observational reports. Nevertheless, there were many scholars who mentioned this difficulty. One of the most prominent places where it is cited is in Osiander's Introduction to Copernicus's *De revolutionibus*:[5]

> ... but this alone is sufficient if (the models) produce a reckoning that agrees with observations — unless by chance there be someone ignorant of Geometry and Optics to such a degree that he treats the epicycle of Venus as resembling the truth (*pro verisimili*) or supposes that it is responsible (*in causa esse*) for Venus sometimes preceding and sometimes following the Sun by 40 degrees and more. For, who does not see that, when this [i.e., the verisimilitude of the epicyclic model] is posited, it follows necessarily that

> the diameter of this planet appear more than 4 times greater at perigee than at apogee and, moreover, that its very body appear more than 16 times greater, even though the experience of every age opposes this? There are other matters in this branch of knowledge that are no less absurd ....[6]

This argument is only indirectly against Copernicus, whose model for Venus is, in this respect, equivalent to that of Ptolemy, but it challenges directly the interpretation of Ptolemy's model for Venus (and its epicycle in particular) as physically real.

Those who favoured homocentric models for Venus had an easy explanation for the lack of variation in its apparent size — Venus was always at the same distance from the Earth. In the Middle Ages the most important advocate of homocentric models was al-Biṭrūjī (*fl.* 1200) who wrote in Arabic, but whose treatise was available in Latin and Hebrew translations shortly thereafter.[7] Al-Biṭrūjī introduces the phenomena of Venus in the context of determining the order of the planets from the Earth:

> Those whose pretext for opposing the ancients' opinion is that they have not seen Venus or Mercury obscure the Sun under any circumstances, as the Moon does during solar eclipses, would have a sound reason if these two planets were illuminated by another body, as the Moon is illuminated by the Sun. The proof that they [Venus and Mercury] are not illuminated by the Sun and do not receive light from any other source is that their luminosity seems the same whether they are close to the Sun or far from it. But if their light came from the Sun, like the Moon's, Mercury would always appear as a crescent, because it is never very distant from the Sun; and similarly for Venus most of the time. If someone were to say that the distance between them in height makes the surface facing us always appear luminous, it still remains that some of the surface would be without light, and [the planet] would appear elongated.
>
> Moreover, if the Sun were below both of them and they were receiving their light from it, then the higher would receive light from the lower, so that the lower would then be more perfect. This is a repugnant notion and remote from the principles by which things exist. Since they do not obscure the light of the Sun, despite their lying below it and above us, either the rays of the Sun pass through them on account of their translucence, or their light replaces the part of the Sun which they obscure. Since this is the case, their argument is not sound ....[8]

Ptolemy treated the absence of transits differently in his *Planetary hypotheses*, and argued that they may not be visible because of the smallness of the planets or because of their infrequency.[9] In the Middle Ages there were some claims to have seen transits of Venus or Mercury, but they turn out to be, in all likelihood, observations of sunspots rather than of transits.[10] Ptolemy does not raise the question of the self-luminosity of the planets, but many of his successors did. In addition to al-Biṭrūjī, Ibn al-Haytham and al-Bīrūnī (who both lived in the eleventh century) had the following to say on the question of self-luminosity:

> Thus had it [Venus] received the light from the Sun it would have been seen, when it is near to the Moon and both are near to the Sun, in a form other than the form in which it appears when it is far from the Sun. Yet it is not found to be so; but Venus is seen at all times circular, and appears when it is near to the Sun in the same form as it appears when it is far from the Sun. Similarly, Mercury is seen when it is near the Sun in the same form as when distant from the Sun. Thus Venus and Mercury are self-luminous and do not receive light from the Sun.[11]
>
> If they [Venus and Mercury] are not [self-]luminous, there would be a difference in the amount of their light when at their greatest distance from the Sun, and when approaching their disappearances in its rays at conjunction, for indeed they are lower than the Sun, and no such difference is observable.[12]

The same points are discussed in Copernicus's *De revolutionibus*, i, 10:

> Those who put Venus and Mercury beneath the Sun ... do not say that darkness in (these two) planets is somewhat like that of the Moon, but that they shine either by virtue of their own light or because they have been infused in their entire bodies by (the light) of the Sun; and so (they say that) the Sun is not eclipsed because it happens most rarely that (these planets) are interposed in our sight of the Sun ....[13]

Copernicus seems to have 'blended' two different arguments: (1) the self-luminosity of Venus (al-Biṭrūji, etc.); and (2) the infrequency of, and difficulty in perceiving, a transit of Venus (Ptolemy).

Some of those who opposed homocentric models appealed to the variation in the size of Venus. The best known example of this argument is that given by Simplicius (sixth century) against the homocentric models described by Aristotle in *Metaphysics* Λ 8. According to Simplicius, both Mars and Venus appear very much greater "in the middle of their retrogradations" than elsewhere, and he adds that for this reason homocentric models were abandoned shortly after the time of Aristotle. Now, Mars does vary considerably in brightness (which, before the telescope, would be interpreted as a change in apparent size), but Venus varies very little in brightness. Hence, as will be argued in detail elsewhere,[14] this observational claim is to be understood as a 'reconstruction' based on a consequence of Ptolemaic theory that was well known at the time of Simplicius. For, according to Ptolemaic theory for both Venus and Mars, the ratio of maximum to minimum distance is about 7 to 1 and, in the absence of knowledge of the phases of Venus, it would be assumed that Venus's apparent diameter varied inversely with its distance. This kind of 'reconstruction' is evident in a number of ancient Greek astronomical texts, and it persisted in the Middle Ages.[15]

Al-Biṭrūjī's homocentric models were also criticized for their inability to account for the variation in the size of Venus. In the phenomena of astronomy listed by Bernard of Verdun (*fl.* 1300) we find:

> This is likewise evident in the cases of Mercury and especially Venus. For, when they begin to appear in direct motion, that is, in withdrawing from the Sun following the succession of the [zodiacal] signs, they seem to be smaller in size than when they begin to appear in retrograde motion, that is, in lengthening [their distance] from the Sun contrary to the succession of the [zodiacal] signs. Therefore, this must happen by virtue of the fact that they are sometimes more distant and sometimes closer to us as observers.[16]

The appearance of Venus after a period of invisibility near superior conjunction takes place while the planet is in direct motion, whereas its appearance after a period of invisibility near inferior conjunction takes place while the planet is retrograde. Hence, the claim is that the planets Venus and Mercury are farther from the Earth at superior conjunction than at inferior conjunction. Though stated as a phenomenon, it is most likely a 'reconstruction' based on Ptolemaic theory. Mercury was rarely observed by astronomers in the Middle Ages, and with no specifics offered, this report has little credibility. Moreover, such competent observers as al-Bīrūnī and, as we shall see, Levi ben Gerson and Galileo, failed to see this difference in apparent size. Bernard of Verdun was more careful than Simplicius, for he was aware that Venus is invisible "in the middle of its retrogradations", and therefore refers to the first visibilities of Venus after superior and inferior conjunctions, rather than to the conjunctions themselves where the distances take on their extremal values.[17]

In addition to discussions of the relative sizes of Venus under different conditions, there were also claims about its absolute angular diameter. In the *Planetary hypotheses*, Ptolemy reports in the name of Hipparchus that the apparent diameter of Venus is $\frac{1}{10}$ that of the Sun, i.e., it is about 0;3°.[18] Ptolemy adds that, although Hipparchus failed to specify the distance of Venus for this apparent diameter, mean distance is to be understood. This parameter was rarely challenged in the Middle Ages, but some dissent was heard: al-ᶜUrḍī (thirteenth century) noticed that if Venus was 0;3° at its mean distance, then near inferior conjunction it would appear to be about $\frac{2}{5}$ the size of the Sun, contrary to the appearances.[19] This seems to be a theoretical argument, for there is no indication that al-ᶜUrḍī undertook any measurements. In fact, al-ᶜUrḍī merely interpreted the report in the *Planetary hypotheses* as meaning that Venus's apparent diameter was 0;3° at minimum distance rather than at mean distance.

Levi ben Gerson (d. 1344) was particularly concerned with the apparent sizes of the planets as among the phenomena that an astronomical theory was to explain, and he rejected Ptolemy's second lunar model because it brought the Moon too close to the Earth, doubling its apparent size, contrary to the appearances.[20] For Venus he remarks:

> It also became clear to us on the basis of observation that the apparent size of the diameter of Venus is greater at greatest elongation from the Sun than at 0° or 180° of anomaly; on the other hand, we did not observe it to be greater at 180° of anomaly than at 0° of anomaly. All this is at variance with what follows from Ptolemy's model, for according to it the diameter of Venus should appear to be greater at 180° of anomaly than at 0° of anomaly by more than 6 times. We also observed diligently seeking to find the apparent size of Venus at each time relative to the apparent size of the fixed stars of first or second magnitude, and in general to determine the variations in the apparent sizes of the planets. For Venus we could determine this by its appearance with the Sun during the day, because when Venus is at its greatest elongation from the Sun, you can see it in the afternoon sunlight. But when it is closer than 20°, it cannot be seen in sunlight. You can also verify this by observing the size of the diameter of Venus in these two places and by observing it at 0° and 180° of anomaly. Another way to verify this is by noting its rays that enter the window of the instrument that we described earlier [i.e., the camera obscura], and this should be done when the light of the Moon is not shining and it is pitch dark.[21]

The previous passage, to which Levi alludes, describes an experiment involving a combination of a Jacob Staff and camera obscura to find the apparent sizes of Venus and Jupiter, but he does not present his results in this passage. Levi was aware that the angular size of the luminary is related to the difference between the size of the image and the size of the hole divided by the distance from the hole to the image.

> For Venus and Jupiter, despite the difficulty due to the weakness of their light, we will describe [an experiment to be undertaken] in utter darkness when the light of the remaining stars is dim [*lit.* absent], i.e., in the dark of night, in which their light enters through a window without mixing with the light of another luminary ....[22]

Levi seems to have made a great effort to determine the variation in the apparent diameter of Venus, comparing its size with that of bright stars, and seeking its image in a camera obscura — a method that several modern authors have recommended.[23] He failed to find any difference in the apparent size of Venus near inferior and superior conjunctions, while claiming that its greatest apparent size was at greatest elongation (about 47° from the Sun). According to modern theory, maximum brightness occurs about 35 days after inferior conjunction, and greatest elongation about 35 days after that (give or take a few days). So, one might argue that Levi is just as far off in the other direction as those who took maximum size to take place at inferior conjunction. But those who took maximum size at inferior conjunction seem to have depended on Ptolemaic theory, recast as an observation, whereas Levi gives the impression of reporting what he had seen. There was no previous significance to an elongation of about 39°, and he was equally unaware of any. But greatest elongation was a known phenomenon of Venus, and it would have been natural to observe Venus in that circumstance.[24]

Levi also measured the apparent sizes of the Sun at apogee and perigee with the same instrument on specified dates in 1334, and his results were that the apparent diameter of the Sun varied from 0;27,51° to 0;30°, from which he derived a solar eccentricity of 2;14, where the radius of the deferent is 60.[25] His derivation of the solar eccentricity from observations of the apparent solar diameter convinced Levi of the reality of the Sun's variation in distance from the Earth. He was also convinced by his observations of Venus that it too varied in distance from the Earth. He then considered these two instances of eccentricity as strong arguments against the homocentric theory of al-Biṭrūjī.[26]

In another chapter Levi considered the phases of Venus, and presented a mathematical argument based on the phase angle, the only instance of the phase angle for Venus in a pre-modern text of which I am aware (see Appendix for a complete translation of Chapter 54). His conclusion is not unexpected, namely, that the absence of visible phases for Venus means that it is self-luminous, but the method is of interest, for it gives a precise sense to the phases that ought to have been seen, if Venus received its light from the Sun. Moreover, he indicates that the distance of the planet from the Earth also has an effect on the 'defect', i.e., the part of the planet facing us that would be unilluminated (see Appendix, [16–17]). In the figure that accompanies this chapter, the body of the Sun and the body of the planet are both represented, but usually in Ptolemaic theory the mean Sun rather than the true Sun appears with a planet. In other words, the reasoning here is physical.

Finally, we come to Galileo (d. 1642), who observed Venus with a telescope, and commented on the visibility of Venus without it. In *Il saggiatore* (*The assayer*) he maintained that "... the disc of Venus shows itself in its two conjunctions and elongations from the Sun to vary but little in magnitude",[27] and in his *Dialogo*:

> Then when it [Venus] is beneath the Sun and very close to us, [the area of] its disc ought to appear to us a little less than forty times as large as when it is beyond the Sun and near conjunction. Yet the difference is almost imperceptible.[28]

Moreover, with the telescope he found that the size of Venus did vary by more than 6 to 1 (from about 0;1° to about 0;0,9°), as both Ptolemaic and Copernican theory required, thus removing the 'embarrassment' of this missing phenomenon.[29] But the phases of Venus revealed by the telescope implied that Venus is sometimes more distant than the Sun and sometimes closer than it to the Earth, whereas the Ptolemaic theory had Venus always closer to the Earth than the Sun. Hence, although phases of Venus were consistent with Ptolemaic theory, the set of phases that are actually observed were not.[30]

In sum, we see the power of a successful theory, for Ptolemy's theory of Venus was successful in accounting for the positions of this planet. Its shortcoming was that it failed to account for the relatively slight variation in its apparent diameter, but most astronomers seem to have taken this as a minor flaw that in no way put in question the theory as a whole. The absence of phases was not seen as a fundamental challenge to the theory, for it was thought to be compatible with self-luminosity. But observations with the telescope left no doubt that the phases were real. Indeed, the most serious problem in the theory of Venus had been due to a vast over-estimate of its apparent size — a parameter that had hardly been disputed in the time from Ptolemy to the early seventeenth century.

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APPENDIX: LEVI BEN GERSON'S *ASTRONOMY*, CHAP. 54.
(MS Paris, heb. 724, 103b:17–104a:11; MS Paris, heb. 725, 79a:1–24)

[1] It is appropriate for us to investigate at this place whether all the planets receive light from the Sun, or whether this applies uniquely to the Moon among the planets. [2] We say that the claim (*ha-ma'mar*) that all the planets receive their light from the Sun is patently false. [3] For if it is so assumed, it would follow necessarily that all planets would only appear as a completed disk (*ᶜiggul*) when they are in the same direction (*nekhoḥiyim*) as the Sun, for then the portion facing the Sun is the portion of them that we ought to see. [4] But elsewhere the portion that we ought to see of them, i.e., the portion that faces us, is not the same as the portion that faces the Sun. [5] Therefore, it [the planet] ought to be seen as an incomplete disk, and this defect (*ḥisaron*) ought to be most noticeable for the planets that are below the Sun when they are close to the Sun which is the case for Venus and Mercury — if they are indeed below the Sun. [6] According to this assumption, they should always appear crescent-shaped (*yarḥit*), for they do not depart from the Sun very much. [7] In the case of the planets that are above [the Sun], it is appropriate that this defect be noticeable when they are at the place where the line from the centre of the planet to the centre of the Sun is perpendicular to the line from the centre of the Sun to the centre of the Earth, [8] for at that place a sensible part of the portion of the planet that faces the Sun will be hidden [from the observer]. [9] But none of this is seen for any of [the planets].

[10] We shall establish the truth of what we just mentioned by the following figure [see Figure 2]. [11] Let the circle [representing] the body of the planet in

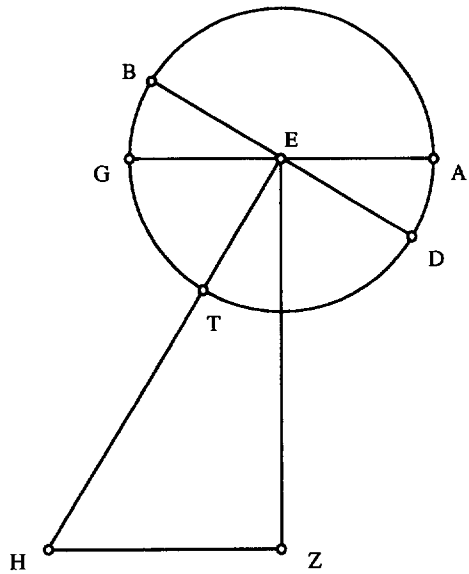

FIG. 2. With MS Paris, heb. 725, 79a; in MS Paris, heb. 724, 103b, the angle at *Z* is acute, and *AEG* is still perpendicular to *EZ*. Note that the Sun is at *Z*, the Earth at *H*, and the centre of the planet at *E*. In modern terms, the phase angle is angle *ZEH*.

the plane of the triangle formed by the three aforementioned centres be *ABGTD* whose centre is *E*. [12] The centre of the Sun is at point *Z*, and the centre of the Earth is at point *H*; we draw lines *EZ*, *EH*, *ZH*. [13] We draw straight line *AEG* perpendicular to line *ZE*, and we draw straight line *BED* perpendicular to line *HE*. [14] It is clear that semicircle *DTB* of that circle faces us, and that a part of the portion that faces the Sun, namely arc *AD*, is missing from [that semicircle]. [15] Therefore, it follows that the planet would be seen as an incomplete disk, because a portion of it, arc *BG*, does not face the Sun and its light does not fall upon it; therefore we would not see [that portion of] it. [16] It should be clear to you from the figure that as angle *ZEH* increases, arc *AD* increases, because angle *AED* is always equal to angle *ZEH*.* [17] If you investigate this, you will find that, for a superior planet, the closer it is to the centre of the Earth, the greater will be angle *ZEH* in this illustration, and for this reason, this defect should be more noticeable. [19] But this is all false because we always see the planets as complete disks except for the Moon; hence, it is clear that no planet receives its light from the Sun except the Moon, and this is what we intended to demonstrate here. [End chap. 54]

*MS Paris, heb. 724, mg., adds: Angle *AED* plus angle *DEZ* equal a right angle, and angle *ZEH* plus angle *DEZ* equal a right angle. If angle *DEZ* is subtracted from both sums, there remain angles *AED* and *ZEH* that are equal to one another, for they are both complements of the same angle, *DEZ*.

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1. D. J. Price, "Precision instruments: To 1500", in *A history of technology*, ed. by C. Singer, E. J. Holmyard, and A. R. Hall (7 vols, New York and London, 1954–78), iii, 582–619 (espec. p. 583).
2. See W. M. Smart, *Text-book on spherical astronomy*, 5th edn (Cambridge, 1962), 414.
3. The brightness of Venus is measured in negative magnitudes that range from –3.9 to –4.7 (where –4.7 is of course at the brighter end of the scale); this is a small variation in brightness, and is difficult to detect with the naked eye: *cf.* D. J. Price, "Contra-Copernicus: A critical re-estimation of the mathematical planetary theory of Ptolemy, Copernicus, and Kepler", in *Critical problems in the history of science*, ed. by M. Clagett (Madison, 1959), 197–221 (espec. pp. 212–14). The phases vary over half the synodic period of Venus, i.e., from superior conjunction to inferior conjunction, and this is about 292 days. I am informed by Dr Leroy E. Doggett of the U.S. Naval Observatory that the formula in actual use for the *Astronomical almanac* is the following:

$$V = -4.4 + 5 \log (d^*r) + 0.09\,(i/100) + 2.39\,(i/100)^2 - 0.65\,(i/100)^3$$

where V is the magnitude of Venus, d is the geocentric distance of Venus in astronomical units, r is the heliocentric distance of Venus, and i is the phase angle in degrees. Dr Doggett adds that "The constants were determined from observations of phases in the range 0.9° – 170.7°. Outside this range (i.e., close to conjunction), Earth-based photometric observations are not possible" (private communication, 11 July 1995). *Cf.* A. Danjon, *Astronomie générale*, 2nd edn (Paris, 1959), 377–9.

In 1994 (*The astronomical almanac for the year 1994* (Washington and London, 1993), A4, A8, E60-62; with additional information provided by Dr Doggett), at the limit of visibility near superior conjunction (elongation near 0°; phase angle 1.2°), the magnitude of Venus was -3.9; and at the limit of visibility near inferior conjunction (elongation 7°; phase angle 170.7°), the magnitude was -4.0. At greatest elongation (about 47°; phase angle 90°) its magnitude was -4.3; at maximum brightness (elongation about 39°; phase angle 120°) its magnitude was -4.7. Note that Venus is not visible to the naked eye at superior conjunction, but its magnitude stays the same for a long time around superior conjunction; and it is also not visible for somewhere between 2 and 19 days near inferior conjunction, i.e., for 1 to 10 days before and after it (depending on the season at which it takes place and the geographical latitude): see P. Huber, *Astronomical dating of Babylon I and Ur III* (Malibu, 1982), 11. For telescopic views of Venus at inferior conjunction, maximum brightness, and greatest elongation, see D. Byrd, "Eye on the sky", *Astronomy*, xvi, no. 7 (July 1988), 54–56. For the elongation corresponding to maximum brightness, see J. Herschel, *Outlines of astronomy* (London, 1849), 279; L. Rudaux and G. de Vaucouleurs, *Larousse encyclopedia of astronomy* (New York, 1959), 187; G. C. Flammarion and A. Danjon, *The Flammarion book of astronomy* (New York, 1964), 279. The time interval from greatest elongation to maximum brightness is about 35 days and the time interval from maximum brightness to inferior conjunction is also about 35 days (these intervals may vary by a few days): *cf.* G. Müller, "Helligkeitsbestimmungen der grossen Planeten und einiger Asteroiden", *Publicationen des Astrophysikalischen Observatoriums zu Potsdam*, viii (1893), 193–371 (espec. p. 324); M. Danloux-Dumesnils, *Eléments d'astronomie fondamentale* (Paris, 1985), 232–3.
4. *Cf.* B. R. Goldstein and F. W. Sawyer III, "Remarks on Ptolemy's equant model in Islamic astronomy", in *Prismata: Festschrift für Willy Hartner*, ed. by Y. Maeyama and W. G. Salzer (Wiesbaden, 1977), 165–81.
5. Though this introduction was written by a Lutheran theologian, Andreas Osiander, for a while it was thought to be by Copernicus. Kepler was the first to call attention to the true identity of its author. See N. Jardine, *The birth of history and philosophy of science* (Cambridge, 1984), 150–4; B. Wrightsman, "Andreas Osiander's contribution to the Copernican achievement", in *The Copernican achievement*, ed. by R. S. Westman (Berkeley and Los Angeles, 1975), 213–43.

6. N. Copernicus, *De revolutionibus orbium coelestium* (Nuremberg, 1543), fol. i-*v*. Copernicus (*De revolutionibus*, i, 10, fol. 8*r*) gives the ratio of greatest to least geocentric distance for Venus as 6 to 1. Osiander's implied ratio of 4 to 1 for Venus's greatest to least distance is peculiar: according to Ptolemy's *Planetary hypotheses*, this ratio is 104 to 16 although, with the parameters in the *Almagest*, it is 104;25 to 15;35 which is closer to 7 to 1 than to 6 to 1; see also ref. 28, below. Osiander, of course, assumed that the apparent diameter of a planet varies inversely with its distance from the observer, and that this followed from a knowledge of optics.
7. F. J. Carmody, *Al-Biṭrūjī : De motibus celorum* (Berkeley and Los Angeles, 1952); B. R. Goldstein, *Al-Biṭrūjī: On the principles of astronomy* (2 vols, New Haven, 1971).
8. Goldstein, *op. cit.* (ref. 7), i, 125.
9. B. R. Goldstein, *The Arabic version of Ptolemy's* Planetary Hypotheses (Philadelphia, 1967), 7: "The occultation of a large body [the Sun] may not be perceptible on account of the remainder of the solar body which would still be exposed, for when the Moon eclipses part of the Sun, equal to, or even greater than, the diameter of one of the planets, the eclipse is not perceptible. Moreover, such events could only take place at long intervals ...."
10. B. R. Goldstein, "Some medieval reports of Venus and Mercury transits", *Centaurus*, xiv (1969), 49–59; reprinted in B. R. Goldstein, *Theory and observation in ancient and medieval astronomy* (London, 1985).
11. W. Arafat and H. J. J. Winter, "The light of the stars: A short discourse by Ibn al-Haytham", *The British journal for the history of science*, v (1971), 282–8 (espec. p. 287).
12. Al-Bīrūnī, *The book of instruction in the art of astrology*, ed. and transl. by R. R. Wright (London, 1934), para. 156.
13. Copernicus, *op. cit.* (ref. 6), fol. 8*r*; for prior discussions in Latin, see R. Ariew, "The phases of Venus before 1610", *Studies in the history and philosophy of science*, xviii (1987), 91–92.
14. See A. C. Bowen and B. R. Goldstein, *Homocentrics and history: Aristotle's testimony about Eudoxus and Callippus*, Chap. 1: "The question of evidence" (in preparation).
15. *Cf.* A. C. Bowen and B. R. Goldstein, "Geminus and the concept of mean motion in Greco-Latin astronomy", in *Archive for history of exact sciences* (in press).
16. Bernardus de Virduno, *Tractatus super totam astrologiam*, ed. by P. Hartmann (Werl, 1961), 68: "Similiter in Mercurio et maxime in Venere hoc apparet. Cum enim incipiunt apparere procedendo, id est a Sole secundum successionem signorum, recedendo apparent minoris quantitatis quam cum apparere incipiunt retrocedendo, id est a Sole contra successionem signorum protendendo. Ergo hoc necesse est contingere eo, quod aliquando sunt remotiora aliquando propinquiora obtutibus nobis." The translation is based on a slight emendation, reading *obtutis nobis* and construing *obtutus* (*obtueo*) analogously to *exercitus* (*exerceo*) in classical usage, since *obtutus*, at least in classical times, was apparently limited to acts of observation and not extended to the agents. *Cf.* P. Duhem, *Le système du monde* (10 vols, Paris, 1954), iii, 445. Duhem misinterprets the passage; according to him the claim would be that the apparent diameter of Venus is greater at superior conjunction than at inferior conjunction. See also J. L. E. Dreyer, "Mediaeval astronomy", in *Toward modern science*, ed. by R. M. Palter (2 vols, New York, 1961), i, 235–56 (espec. p. 251).
17. The same argument is made in a text by an unknown author, previously thought to be Regiomontanus. This text was probably written in Vienna in the early fifteenth century, and the author was heavily indebted to Guido de Marchia (early fourteenth century), a Franciscan as was Bernard of Verdun. See M. Shank, "The 'Notes on al-Biṭrūjī' attributed to Regiomontanus: Second thoughts", *Journal for the history of astronomy*, xxiii (1992), 15–30.
18. Goldstein, *op. cit.* (ref. 9), 8.
19. B. R. Goldstein and N. Swerdlow, "Planetary distances and sizes in an anonymous Arabic treatise preserved in Bodleian Ms. March 621", *Centaurus*, xv (1970–71), 135–70 (espec. p. 150); reprinted in Goldstein, *op. cit.* (ref. 10). A similar remark appears in a letter from Regiomontanus (d. 1476) to Bianchini (d. after 1469) concerning outstanding problems in astronomy. Regiomontanus noted that "the surface of Venus ought to appear to our sight sometimes as 1, but sometimes as 45, which has never become known to anyone observing. Further, its apparent

diameter will sometimes be 0;12,30°, that is, two-fifths the apparent diameter of the Moon, which certainly has never been perceived in the heavens." Although Regiomontanus recognized these problems, he offered no solution to them. See N. M. Swerdlow, "Regiomontanus on the critical problems of astronomy", in *Nature, experiment, and the sciences*, ed. by T. H. Levere and W. R. Shea (Dordrecht and Boston, 1990), 165-95, espec. p. 173. On 45 to 1 as the ratio for the apparent area of Venus's disk at least and greatest distances, see ref. 28, below.

20. B. R. Goldstein, "Levi ben Gerson's contributions to astronomy", in *Studies on Gersonides: A fourteenth-century Jewish philosopher-scientist*, ed. by G. Freudenthal (Leiden, 1992), 3–19 (espec. p. 9).
21. B. R. Goldstein, *The astronomy of Levi ben Gerson (1288–1344)* (New York and Berlin, 1985), 105.
22. Goldstein, *op. cit.* (ref. 21), 69. Levi described the use of this instrument for determining the apparent size of the Moon in his *Astronomy*, chap. 75 (MS Paris, heb. 724, 146a:24–146b:1, and MS Paris, heb. 725, 113b:14–24). For a list of 45 dated planetary observations by Levi ben Gerson, see B. R. Goldstein, "A new set of fourteenth century planetary observations", *Proceedings of the American Philosophical Society*, cxxxii (1988), 371–99.
23. *Cf.* W. H. Steavenson, "Shadows cast by Venus", *Journal for the British Astronomical Association*, lxvi (1956), 264–5; Herschel, *op. cit.* (ref. 3), 272.
24. Most of the observations of Venus reported in the *Almagest* were taken when Venus was at its maximum elongation: see *Almagest* x; G. J. Toomer, *Ptolemy's Almagest* (New York, Berlin, 1984), index, *s. v.* Venus, observations of.
25. Levi ben Gerson's *Astronomy*, chap. 56 (MS Paris heb. 724, 104b:4–8, 105a:13, and MS Paris heb. 725, 79b:13–15, 80a:13–14). The Latin version of this chapter, with English translation and commentary, was published as an Appendix to J. L. Mancha, "Astronomical use of pinhole images in William of Saint-Cloud's *Almanach planetarum* (1292)", *Archive for history of exact sciences*, xliii (1992), 275–98. Levi later revised his value for the solar eccentricity on the basis of an examination of eclipse observations: see B. R. Goldstein, "Medieval observations of solar and lunar eclipses", *Archives internationales d'histoire des sciences*, xxix (1979), 101–56 (espec. p. 104); reprinted in Goldstein, *op. cit.* (ref. 10).
26. See Levi ben Gerson's *Astronomy*, chap. 44 (MS Paris heb. 724, 84a:20–22, and MS Paris heb. 725, 62a:16–18). In chap. 15 (Goldstein, *op. cit.* (ref. 21), 98), Levi argued that "if you find that the sizes of the [Sun] are equal [at apogee and perigee], you can conclude that the solar sphere is not eccentric to the centre of the world as Ptolemy assumed; but if you find that the sizes are different, you can conclude that its centre is eccentric to the centre of the world".
27. S. Drake and C. D. O'Malley, *The controversy on the comets of 1618* (Philadelphia, 1960), 184.
28. S. Drake, *Galileo: Dialogue concerning the two chief systems of the world* (Berkeley and Los Angeles, 1962), 334; *cf.* Ariew, *op. cit.* (ref. 13). In contrast to Osiander, for whom the ratio of the area of Venus's disk at least and greatest distances was more than 16 to 1, Galileo has a little less than 40 to 1 which implies a ratio of greatest to least distance between 6 to 1 and 7 to 1.
29. *Cf.* A. Van Helden, *Measuring the universe* (Chicago, 1985), 71.
30. Van Helden, *op. cit.* (ref. 29), 68.