Remarks on Gemma Frisius's *De Radio Astronomico et Geometrico*

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1. INTRODUCTION

A recent article by John Roche [1981] traced the development of the Jacob Staff or the Radius Astronomicus from the time of its invention by Levi ben Gerson in the fourteenth century to the mid-seventeenth century, with special emphasis on its use in England. A few pages there were devoted to Gemma Frisius and the way in which he incorporated some of the design innovations of his predecessors. This paper is intended to extend that discussion and to place Gemma Frisius's contributions in a somewhat broader context.

Reiner Gemma Frisius was born in 1508 in the Dutch province of Friesland, and died in 1555 in Louvain [Haasbroek 1968]. Though he was trained as a physician, his publications treat mathematics, astronomy, and geography. Of special interest is his book entitled *De Radio Astronomico et Geometrico liber* [1545; cf. Jean Bellere's French translation [1581], *l'Usage du Ray Astronomique et Geometrique de Gemma Frison,* published in a collection of French versions of Gemma's works]. I will concentrate on chapter 16 of this influential treatise because of the references to Regiomontanus and Copernicus as well as its subsequent citation by Jean Pena [1557] as part of an argument against the Aristotelian distinction between the sublunary and superlunary realms.

Copernicus is cited here (for his lunar theory that departed significantly from that of Ptolemy) just two years after the publication of *De Revolutio-*

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nibus. Robert Westman [1975, 181] has emphasized that the initial response to Copernicus by astronomers was to see him as a reformer of the Ptolemaic System and as "the admired inventor of new planetary hypotheses" rather than as the founder of a radically new cosmology. Gemma Frisius seems to fit well into this category, for he wrote in a letter to Dantiscus (August 1541): "I am not concerned with the nature of the hypotheses with which he [Copernicus] supports his argument. . . . To me it is unimportant whether he affirms that the earth moves or that it stands immobile, provided that the motions of the stars and the time intervals are precisely determined and are reduced to an extremely accurate calculation" [Hugonnard-Roche 1982, 248-249; cf. McColley 1937].

In order to understand the chapter that is translated below, it is perhaps appropriate to review some of the features of the instrument described in it. The Jacob Staff has the advantage of simplicity in construction, portability, and reasonably good accuracy. In its basic form it consists of two pieces of wood: one is called the Staff or the Radius, hand held such that one end is near the eye and the other end extends along the axis of vision; the other is called the Crosspiece or Transversary, attached at its midpoint perpendicularly to the Radius and allowed to slide along the Radius. The observer then positions the instrument such that a star is visible at each end of the crosspiece: a simple geometric argument leads to the angular separation of the two stars. A difficulty already resolved by Levi ben Gerson, but subsequently rediscovered several times, concerns the apex of the angle between the two stars. Levi demonstrated that this apex lies at the center of the eye about 1 cm from the near end of the radius [Goldstein 1985, 55, 144]. Nevertheless, a number of instrument designers, including Gemma Frisius, ignored this source of error (called the eccentricity) and graduated their instruments from the end of the Radius near the eye. Indeed, Tycho Brahe criticized Gemma's instrument for this reason, among others [Roche 1981, 18; Raeder 1946, 96-97].

Levi ben Gerson's original design, accepted by many of his successors, required that crosspieces of different lengths be used in observing stars of different angular separations such that the crosspiece could be kept at about arm's length. Indeed, the observation is subject to great error when the crosspiece is close to the eye. In effect, this error is due to the sensitivity of the cotangent function to small displacements on the Radius near the eye. In this design the only measurement required is the distance of the crosspiece from the eye, because the length of the crosspiece is known, and

$$d = (a/2) \operatorname{ctn} (\alpha/2) \qquad (1)$$

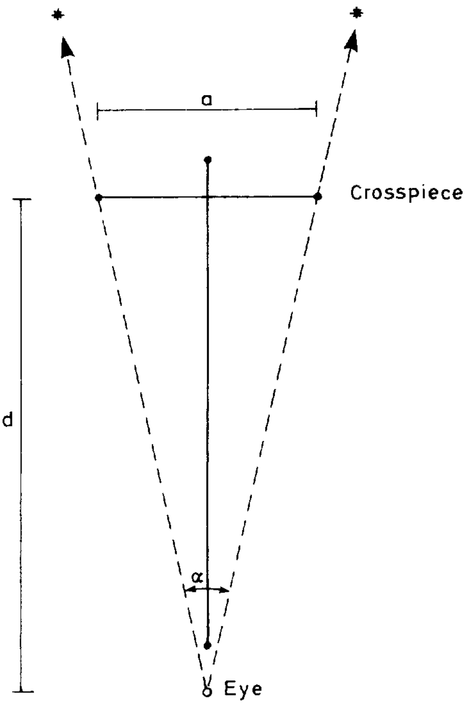

Figure 1. The Jacob Staff (or Radius Astronomicus) according to Levi ben Gerson's design.

where a is the length of the crosspiece, d is the distance of the crosspiece from the center of vision, and α is the angular separation of the two stars (see Figure 1). One of Gemma's innovations concerned the design of a single crosspiece that could be used for all angular separations, both great and small. The crosspiece is here fitted with several *pinnules* that can slide along it so that the effective length of the crosspiece can be varied without replacing one crosspiece by another. In the simplest case a pinnule is fitted at each end of the crosspiece such that one star is seen on the inner edge of one pinnule and the other star is seen on the inner edge of the other pinnule (see Figure 2). In Gemma's design the crosspiece is graduated so that one can determine the distance between the two inner edge of the pinnules, and the Radius is also graduated, as before. We can still use formula (1) where both a and d have been measured. Gemma remarks that the pinnules should be set equally distant from the Radius for this formula to be appropriate. In another arrangement one pinnule is set on one side of the crosspiece and another is set at the middle of it (see Figure 3). Then

$$d = b \operatorname{ctn} \beta \qquad (2)$$

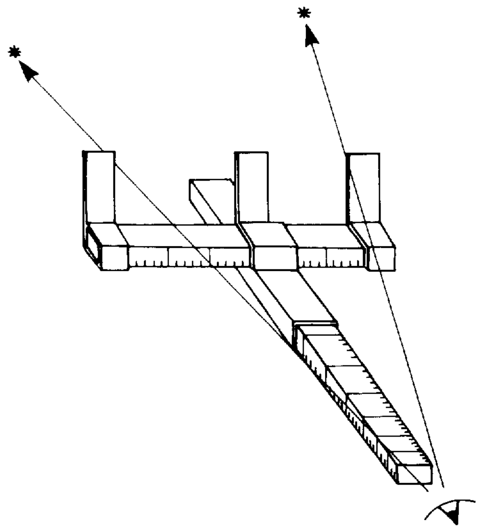

Figure 2. The Radius Astronomicus according to Gemma Frisius's design. Note that the angle between the two stars is bisected by the left edge of the Radius. In Gemma's text the various parts of this instrument are illustrated separately (see the facsimile of them [Roche 1981, 16, 17]), but no figure there corresponds to this one.

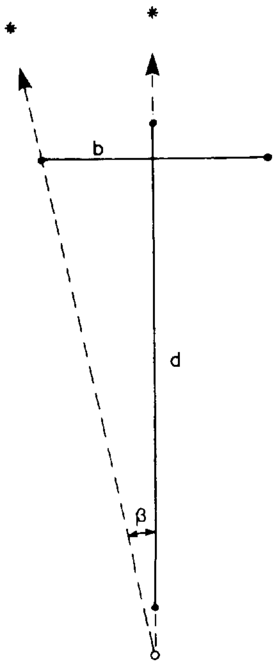

Figure 3. The Radius Astronomicus arranged with one pinnule on the left side and another at the middle of the crosspiece.

where $b = a/2$ and $\beta = \alpha/2$ in formula (1) above, i.e., we measured d and b (the distance between the two pinnules) in order to find β, the angular separation sought. It is also possible, as Gemma tells us, to graduate the Radius such that the angles can be read off it directly without computation. This design already appeared in a work by Werner (1514) and was later criticized by Digges in 1573 [Roche 1981, 13, 20].

2. TRANSLATION

(Note that paragraph numbers have been added to facilitate references in the commentary that follows).

Gemma Frisius's *De Radio Astronomico et Geometrico*

Chapter 16: On the distances of the stars in the sky, and on the apparent diameters of the luminaries.

1. Celestial angular distances are determined in the same way as terrestrial angular distances. The Radius should be set under the eye at the place previously mentioned [i.e., on the cheekbone: cf. chap. 5, ed. 1545, 14a], and the crosspiece at a distance such that the two stars are seen exactly between the middle and the extreme pinnules. Once this is done, the place of the crosspiece on the Radius will indicate the parts of a circle for the [angular] distance between these two stars. But the crosspiece here must be set up carefully such that the two ends are equally distant from the Radius when taking large distances. If the eye must look across the two extreme pinnules because of the large distance between the stars, one must double the degrees found on the radius, as we explained in chapter 14. Moreover, if the distance between the two stars is too small, the crosspiece is fixed in its place and the measurement is then made according to the degrees on the crosspiece, as we indicated in the same chapter 14. The way to use the distance that is found will be explained later.

2. Now we turn to the apparent diameter of the Moon or the Sun. Ptolemy wrote in Book 5 of the *Almagest* that some had tried to determine it by aquatic means, i.e., with the use of a water clock, but he says these observations are inaccurate and ought to be rejected. There is no greater certainty in the method described by Macrobius in *Scipio's Dream,* book 1, chap. 20, for all these things are subject to many errors. Therefore, Ptolemy preferred the dioptra of Hipparchus, a rule of 4 cubits with two pinnules. But our Radius successfully replaces it, for our [instrument] can be used equally well to measure the smallest magnitudes such as the parts of a lunar eclipse as well as large magnitudes. Therefore, if one wishes to determine the apparent diameter of the Sun or the Moon, one should fix the

crosspiece in its place, and bring the extreme pinnule towards the middle pinnule on the crosspiece such that the two limbs [*lit.:* ends] of its diameter [*lit.:* size] are visible between the edges of the two pinnules. In this way one may find the diameter that is sought to one minute [of arc] by means of the distance between the two pinnules marked on the crosspiece.

3. As for the Sun, I think it best to find its size at sunrise or sunset or when there is a small transparent cloud blocking it that weakens somewhat the brightness of its light. If those who imagine the heavens to be concentric had often been accustomed to this observation, they would never have accepted these dreams instead of most certain experience, and would not have doubted the doctrines underlying the most exact computation of eclipses. For it is obviously seen that the Sun always appears smaller at summer solstice than at any other time of year, and greatest at winter solstice.

4. Whether the Sun is carried on an eccentric circle or an epicycle is not a matter to dispute here. At the least, it is clear that its motion appears nonuniform at the center of the world by a perceptible amount because of the variation in its distance. This must be known for the observation [*read:* computation] of eclipses. Thus, we measured the diameter of the Sun on 27 October 1544, a little before sunset, and found it to be about 33 minutes, and at the same time we measured the apparent diameter of the Moon and found it to be about 31 minutes. By experience that is certain, these diameters vary according to certain periods. For the size of the Sun appears least near summer solstice when it reaches its highest altitude. But the diameter of the Moon appears very small in the superior part of its epicycle, i.e., when its motion is slowest.

5. I cannot believe that the eccentric [sphere] of the Moon is as Ptolemy described it, and following him all the others except for Copernicus alone. He, like a second Ptolemy, contradicts this ancient theory with strong reasons, and with extraordinary brevity demonstrates a new one. Although Johannes Regiomontanus, in his *Epitome,* book 5, chap. 22, noticed that there was something wrong here; nevertheless, he did not wish to change anything, only warning the reader of his surprise. Certainly he was right to be surprised that such great masters had accepted theses contrary to clear experience. For it follows from the ancient and commonly accepted theory of the Moon that at half-Moon [phase] it ought to be double its size at full-Moon [phase] or in conjunction with the Sun, altogether against experience.

6. After we had seen the most excellent works of Copernicus, we often found the same thing with the help of the Radius, and even before the work of Copernicus was produced we were quite surprised many times by the

same thing. So, on 15 December 1542 in the evening, we measured the diameter of the Moon and it was only 30 minutes. But according to the ancient theories, it should have been a little less than 50 minutes, i.e., almost double its size. For the Moon at that time was near the part of its circle closest to the earth, its distance being only 39 parts, whereas at other times its distance is 65 parts. Therefore, by the rules of perspective, it should appear larger.

7. But here I have to laugh at the arrogance of those who, to add confidence *(fides)* to their inventions, and to acquire autthority, completely deny that the sizes of the Sun and the Moon vary, as when they try to affirm that their motions take place in concentric circles, while ridiculing and scorning these experiences that easily destroy the foundations [of their arguments]. Nevertheless, in order that one not think they had not seen these things, they also add some new cause, such as the non-uniform [density] of the air which causes images *(simulacra)* of things [to appear].

8. Whoever wishes to destroy this impression *(phantasia)* can do so easily with the Radius: let him measure the diameter of the Moon emerging from the horizon when it is full, in southern signs [of the zodiac] or at any other time, and then measure its diameter on the same night when it reaches culmination. When he finds that the diameter does not differ by even a minute from that found at first, he can surely and without doubt believe that the density of the air in no way changes the size of the stars. For although the luminaries seem larger near the horizon, when they are measured with an instrument no difference is perceived. Though it is true that images of things which appear in air that is denser seem larger, in fact they do not become larger as one can see from ordinary experience. For, though the distances between stars near the horizon appear to be greater than when they are high in the sky, nevertheless, when they are measured with the Radius, they do not differ at all.

3. COMMENTARY

§ 1. The content of this paragraph has been discussed in the Introduction: it is only necessary to add that the determination of terrestrial angular distances is the subject of Gemma's chapter 14, and that the instructions there are the same as those here.

§ 2. Gemma emphasizes that his instrument can be used to measure small angles such as the apparent diameter of the Sun and the Moon as well as large angles. In this context he compares his instrument with the dioptra of Hipparchus, mentioned in Ptolemy's *Almagest,* which was used to measure the angular diameters of the luminaries. This dioptra consisted of a graduated rod (or Radius) with a sliding plate on it: the plate is so placed that it exactly covers the object when the eye is at one end of the rod (see Proclus's description, below). Apparently, Gemma was unaware of Archimedes's dioptra for measuring the solar disk described in his *Sandreckoner* (the editio princeps of the Greek text with the Latin translation by Jacobus Cremonensis was published in 1544, i.e., one year before Gemma's treatise; see also [Shapiro 1975]). It is noteworthy that Hamellius's commentary [1557] on the *Sandreckoner* includes a comparison of Archimedes's dioptra with the *"Radius Astronomicus seu baculus"* (fol. 17v), and Commandino's commentary [1558] on the *Sandreckoner* includes a chapter from Levi ben Gerson's *Astronomy* concerning the Jacob Staff explicity ascribed by him to Levi. Commandino adds that Levi's instrument is not entirely dissimilar to that of Archimedes (fol. 60v). It seems plausible that both Hamellius and Commandino were inspired by Gemma's text, rather than by some knowledge of each other's work. This passage in Hamellius's commentary has not been cited in discussions of the Radius Astronomicus, as far as I can determine, except in my edition of Levi ben Gerson's *Astronomy* [Goldstein 1985, 146]. In that work Levi ben Gerson describes a modified version of this instrument (combining it with a camera obscura) for measuring the angular diameters of the luminaries [Goldstein 1985, 69-71; cf. Straker 1981, 269-271].

The passage in the *Almagest* (V. 14) cited here by Gemma is the following:

> Of the various methods used to solve this problem, we have rejected those claiming to measure the luminaries by measuring [the flow of] water or by the time [the bodies] take to rise at the equinox since such methods cannot provide an accurate result for the matter at hand. Instead, we constructed the kind of dioptra which Hipparchus described which uses a four-cubit rod ... [trans. Toomer 1984, 252].

For a detailed description of Hipparchus's dioptra we depend on Proclus *(Hypotyposis Astronomicarum Positionum* IV. 71-81, 87-99):

> Let *AB* be the beam with *A* the side from which sightings are taken and at which is fixed plate *DG* (see Figure 4). Let *EZ* be the other plate, the one which moves along the whole length of the beam and contains the two above mentioned openings on a perpendicular. Let *E* be the opening at the base similarly placed to opening *D;* and *Z* the other opening near the top.
>
> The sighting instrument may be used and set up as follows: At the time

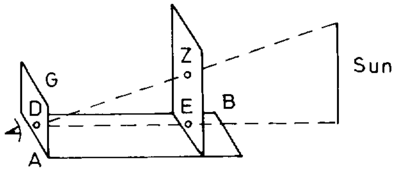

Figure 4. Hipparchus's dioptra according to Proclus. This figure is slightly modified from the one found in [Cohen and Drabkin 1958, 141].

> of the rising or setting of the sun, place the sighting piece in a plane parallel to the horizon where the view of the horizon is as clear and unobstructed as possible. Let the observer stand at the immovable plate, the sun being on the side of the movable plate. The latter is moved back and forth until it is possible to see the lower part of the sun's disk through openings *D* and *E* in the two plates, and the upper part through openings *D* and *Z*. In this way the observer obtains a view of both ends of the apparent diameter of the sun and can determine angle *EDZ* ... [trans. in Cohen and Drabkin 1958, 141].

Note that in this dioptra there are holes in the plates: this is also a feature of one of the modified versions of Levi ben Gerson's Staff [Goldstein 1985, 154; for the advantage of having a pierced plate near the eye, see Roche 1981]. However, Gemma's Radius is fitted with pinnules (or plates) that are not pierced.

The reference to Macrobius (early fifth century, A. D.) concerns a passage in his commentary on *Scipio's Dream,* I.20:

> On the day of the equinox before sunrise place in an exactly level position a stone vessel that has been hollowed until its cavity forms a perfect hemisphere [to serve as a sundial]. On the bottom there must be lines representing the twelve hours of the day which the shadow cast by a stylus will mark off as the sun passes along in the sky ... Shortly before sunrise let the observer fix his eyes upon this bowl - be sure it is in a perfectly level position - and then when the first ray of the sun at its top just steals over the horizon has cast a shadow of the top of the stylus upon the edge of the bowl, let him carefully mark the place where the shadow first fell. Then he must watch until the moment when the full orb comes into view so that the bottom seems to be just resting on the horizon. Again he must mark the point which the shadow has reached in the bowl. The measurement between the two shadow marks, corresponding to the full orb or diameter of the sun, will be found to be a ninth part of the space in the bowl between the line marking sunrise and the line marking the first hour [trans. Stahl 1952, 173; cf. Dreyer 1953, 187; and Heath 1913, 311-312].

This method is notoriously inaccurate because of the difficulty in determining the appropriate moments.

§ 3. Gemma does not name those who imagine the celestial orbs to be concentric, but presumably he is referring to the adherents of Aristotle's modification of Eudoxus's homocentric spheres [cf. Heath 1913, 193ff; Goldstein and Bowen 1983]. An influential medieval system of concentric planetary models was invented by al-Biṭrūjī (Spain, ca. 1200): a printed edition of the Latin version translated from a Hebrew rendering of the original Arabic appeared in Venice 1531 [Goldstein 1971, 3], and al-Biṭrūjī (or Alpetragius) is cited by Copernicus in his discussion of the order of the planets in *De Revolutionibus,* I.10. Other homocentric models were developed in the sixteenth century, notably those of Amico and Fracastoro [Dreyer 1953, 296-304; cf. Swerdlow 1972].

In the *Almagest* Ptolemy does not distinguish the sizes of the Sun at different times of the year, but in his *Planetary Hypotheses* we do find a variation in the solar distance and hence in the apparent solar diameter [Goldstein 1967, 9]. This aspect of the Ptolemaic System based on the *Planetary Hypotheses* was well known in the Middle Ages and the Renaissance (for Levi ben Gerson's measurements of the seasonal variation in the solar diameter, see [Goldstein 1985, 98, 183-184]).

§ 4. On the matter of eccentric versus epicyclic models: Ptolemy showed that they are geometrically equivalent, but Levi ben Gerson and Amico both argued that they lead to different physical consequences for the lunar orb; namely, the epicyclic model implies that we should see both faces of the Moon, contrary to experience, whereas the eccentric model implies that we should always see the same face of the Moon [Goldstein 1985, 14, 193].

Despite what Gemma says here, the variation in solar diameter has little effect on solar eclipses. The observation cited is dated 27 October 1544 in the French version and *sexto Kalend. Novemb.* 1544 in the Latin edition. The solar diameter was found to be about 0;33° whereas Ptolemy's value for it was 0;31,20° [cf. *Almagest,* V.15]. Of greater interest is Gemma's measurement of the lunar diameter for, on the basis of such observations, he affirms the superiority of Copernicus's lunar theory over that of Ptolemy (see § 6, below).

§ 5. Here we learn that Ptolemy's lunar theory leads us to expect that the diameter of the Moon at half-Moon phase be twice that at full-Moon phase (or, equivalently, that the areas are in the ratio of four to one), contrary to experience [cf. Goldstein 1967, 7, 11]. In this regard he cites a passage from Regiomontanus's *Epitome,* V.22:

But it is surprising *(mirum)* that the Moon does not appear so great at quadrature, when it is at the perigee of its epicycle, whereas if the entire disk were visible, it should appear four times its apparent size at opposition, when it is in the apogee of the epicycle.

This remark appears almost *verbatim* in Copernicus's *Commentariolus* without citing the source [Swerdlow 1973, 461-462; cf. *De Revolutionibus*, IV.2]. The *Commentariolus* was written no later than 1515 [Swerdlow 1973, 431] and not published until 1882. It did circulate to some extent in manuscript, but there is no certain evidence that Gemma had seen it. In any event, no figures are given in it for the lunar distance. However, in the first account of the Copernican System to be printed, namely Rheticus's *Narratio Prima* (Danzig 1540), we find this passage with the appropriate reference to Regiomontanus's *Epitome* [Hugonnard-Roche 1982, 105, 160-161]. Though it is possible that Gemma was independently attracted to this criticism of Ptolemy (the text of the *Epitome* was widely known at the time), I think it more likely that he depended on Rheticus.

It has been asserted that Gemma knew the text of the *Commentariolus*, based on a passage in Gemma's letter of 1541 to Dantiscus [Waterbolk 1974, 228], but the reference is ambiguous and, in a note to the French translation of this letter, we are told that Gemma was alluding to Rheticus's *Narratio Prima* [Hugonnard-Roche 1982, 248-249], which to me seems more likely. In any event, Waterbolk refers neither to Gemma's direct citations of Copernicus in *De Radio Astronomico*, nor to his use of the passage from Regiomontanus's *Epitome*. There is also the possibility that Gemma learned of Copernicus's views through Dantiscus as early as the 1530's [Hugonnard-Roche 1982, 249].

It is surprising that this defect in Ptolemy's lunar theory was not noted until the fourteenth century when both Levi ben Gerson and Ibn al-Shāṭir attempted independently to repair it [Goldstein 1974, 24-25; Goldstein 1985, 105, 187; Roberts 1957].

§ 6. Here Gemma cites his observation of the lunar diameter on 15 December 1542 to demonstrate that he was aware of this difficulty in Ptolemy's lunar theory before the publication of *De Revolutionibus* in 1543, leading us to believe that this was an independent discovery. However, the citation from Regiomontanus (discussed above, § 5), that seems to derive from Rheticus, strongly suggests such dependence, albeit indirect.

§ 7. The last two paragraphs deal with a theory that Gemma opposes in which it is assumed that the density of the air varies. No proponent of this theory is named, but the view seems to derive from a passage in Aristotle's *Meteorology* I.5 where certain atmospheric phenomena are explained by light penetrating through layers of air of different densities. It was not until the seventeenth century that the density of air was found to vary with its altitude [Goldstein 1976]. For most medieval astronomers air had a uniform density which differed from that of the superlunary world: see, for example, the view of Ibn Mu^{c}ādh [Goldstein 1977].

§ 8. Gemma now describes an experiment with the use of the Radius to prove that the lunar diameter at full-Moon does not vary with its altitude. The phenomenon in question is known as the "Moon Illusion", i.e., that the full-Moon at the horizon appears to the naked eye to be much larger than when it is high in the sky, but that when it is measured no difference is discerned. In the *Almagest* I.3 [trans. Toomer 1984, 39] Ptolemy suggests that the Sun may appear larger on the horizon than when it is high in the sky because of the magnifying effect of the atmosphere and its moisture; this view was invoked to explain the Moon Illusion until well into the seventeenth century, despite alternative explanations as early as Ptolemy's *Optics* [ed. Lejeune 1956, 116] later taken up by Ibn al-Haytham and others [Reimann 1902; Enright 1975].

The final remark in the chapter, that the distance between stars is unaffected by their altitude, implies that atmospheric refraction does not exist. Ptolemy had already described atmospheric refraction in his *Optics* [cf. Cohen and Drabkin 1958, 281]: this phenomenon was widely known in the Middle Ages and the Renaissance despite the absence of attempts to measure its effect [cf. Beaver 1970, 40]. Gemma's remark was cited by Jean Pena (d. 1558) to prove that air reaches from our eyes to the fixed stars without any change in density, and hence the Aristotelian distinction between the sublunary and superlunary domains had to be abandoned [Pena 1557, foll. aaiij/r, bb/v; Rochas 1912, 220, 227; cf. Barker 1985]. Tycho Brahe was most displeased to learn that Pena had preceded him in arriving at the view that there are no Aristotelian solid spheres in the heavens. For, whereas Brahe had reached this conclusion based on careful observations of the new star of 1572 and the comet of 1577, Pena had relied upon a false argument, namely the absence of atmospheric refraction [Brahe 1916, vol. 3: 154-155]. The dissolution of the Aristotelian spheres was clearly an important event in the scientific revolution and, in a curious way, Gemma Frisius may have played a role in it [cf. Donahue 1975].

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