Nuncius, 22 (2007), 261–285

WHAT'S NEW IN PTOLEMY'S *ALMAGEST*?

BERNARD R. GOLDSTEIN

ABSTRACT

Ptolemy's *Almagest* (dating from the 2nd century A.D.) is a remarkable and original astronomical work, despite the general absence of claims for originality. Among Ptolemy's innovations are his dependence on a small number of explicitly dated observations and the derivation from them of the parameters for his geometrical planetary models by mathematical methods that are described in great detail. Moreover, he does not mention the arithmetic schemes of his Babylonian predecessors although he cites their observations. His most important innovation, however, was to base his planetary models and computations on mean motions and anomalies, rejecting the requirement of uniform circular motion. Ptolemy's view of mean motion is seen as a theological commitment. It is further argued that Ptolemy's reliance on Hipparchus in theoretical matters has been exaggerated; in particular, there is no credible evidence that Hipparchus ever appealed to a mean position of a planet.

Keywords: Ptolemy, Ancient Astronomy, Planetary.

INTRODUCTION

Claudius Ptolemy was without doubt the most important astronomer in antiquity. He lived in Alexandria in the 2nd century A.D., but the precise dates of his birth and death are not known. In fact, the traditional estimate of the time when he flourished is based on his dated astronomical observations which range from 127 to 141 A.D.[1] No other biographical informa-

[1] For Ptolemy's observation of Saturn in 127, see *Almagest*, XI.5, and for his observation of

tion is available. In addition to the *Almagest*, his best known work, he composed the *Planetary Hypotheses* (a description of a physical model for the heavens), the *Tetrabiblos* (a compendious account of astrology), the *Handy Tables* (a revised version of the tables in the *Almagest*), the *Planispherium* (a description of an instrument of the same name, based on stereographic projection and the forerunner of the astrolabe), the *Optics* (a study of reflection, refraction, and binocular vision), and so on. But this essay will concentrate on the *Almagest* and demonstrate that it is a truly remarkable and original scientific work.[2] The order of topics is didactic, and the text is self-contained, that is, each part depends on what comes before it and hardly anything outside the text is assumed. The order of discovery is ignored and there are very few claims of originality. It is also important to recognize that the *Almagest* is not a history of astronomy although it has been mined for historical data. Other than the *Almagest*, only minor astronomical works from Greco-Roman antiquity survive. The two earlier extant ancient texts that attempt to deal with a variety of astronomical issues are the so called *Ars Eudoxi* (preserved in a papyrus dating from the first half of the 2nd century B.C.) and the *Introduction to Astronomy* by Geminus (whose date is uncertain – recent scholarship places him in the 1st century B.C.).[3] The contrast, however, between Ptolemy and the authors of these two texts is vast – with respect to the range of topics considered, the attitude to astronomical theory, the treatment of observational data, the level of mathematical sophistication, as well as the response to Babylonian astronomy.

Our focus will be on Ptolemy's methodology, but first we need to discuss some of the topics in the *Almagest* because it is often summarized in a few sentences that do not do justice to its contents. Due to space constraints, only a few themes will be considered here.

Mercury in 141, see *Almagest*, IX.7; GERALD J. TOOMER (tr.), *Ptolemy's Almagest* (New York and Berlin: Springer-Verlag, 1984), pp. 525 and 450. For a list of dated observations in the *Almagest*, see OLAF PEDERSEN, *A Survey of the Almagest* (Odense: Odense University Press, 1974), pp. 408-422.

[2] It has on occasion been argued that Ptolemy borrowed various ideas from his predecessors, but the evidence for borrowing is usually quite weak. It is my position that Ptolemy should be credited with ideas expressed in his works unless there is unambiguous evidence of them in an earlier work. To be sure, if new historical data came to light, it would have to be taken into account.

[3] For a description of the contents of the *Ars Eudoxi*, see OTTO NEUGEBAUER, *A History of Ancient Mathematical Astronomy* (Berlin and New York: Springer-Verlag, 1975), pp. 686-689. GERMAINE AUJAC (ed. and tr.), *Géminos: Introduction aux phénomènes* (Paris: Les Belles Lettres, 1975). On the dating of Geminus, see ALEXANDER JONES, "Geminus and the Isia", *Harvard Studies in Classical Philology*, 1999, *99*: 255-267; and ALAN C. BOWEN, "Geminus and the Length of the Month: The Authenticity of *Intro. Ast.* 8.43-45", *Journal for the History of Astronomy*, 2006, *37*: 193-202.

I. A Brief Survey of the *Almagest*

Book 1 begins with some introductory remarks on basic features of ancient astronomy such as the sphericity of the heavens and of the Earth, and the fact that the planets have a proper motion in the direction contrary to the daily rotation of the heavens. The sphericity of the Earth had long been known, for in *De Caelo* Aristotle gives a proof of it based on the Earth's shadow during lunar eclipses.[4] In chapter 10 of Book 1 Ptolemy begins his presentation of a new quantitative approach to astronomy. A long section is devoted to plane and then spherical trigonometry which includes theorems not otherwise preserved in any extant ancient text and, more importantly, a series of computations that leads to a table of chords for angles at intervals of $^1/_2$° from $^1/_2$° to 180°, where the radius of the circle is 60 and the chords are given to three sexagesimal places [see Table 1].[5] Since a table of chords is equivalent to a table of the sine function (from which the equivalents of cosine and tangent may be derived), Ptolemy is able to solve all plane and spherical triangles by methods that he describes in minute detail. This quantitative approach to trigonometry is unprecedented, that is, there is no credible evidence that anyone before Ptolemy had a table of chords or used it in the ways Ptolemy did.[6] To be sure, Ptolemy does not claim innovation here (or elsewhere, for the most part) but this is part of his style rather than evidence for borrowing. In fact, Ptolemy does not, in general, assign names to theorems or say anything about who was the first to prove a theorem (in contrast to Archimedes who even distinguishes the first to state a theorem from the first to prove it). Finally, in Book 1

---

4 Aristotle, *De caelo*, II.14; William K.C. Guthrie (ed. and tr.), *Aristotle: On the Heavens* (Cambridge, Mass., and London: Harvard University Press, 1939), p. 253. Ptolemy mentions Aristotle in *Almagest*, I.1 (Toomer, *Almagest* (cit. note 1), p. 35), but not in the chapter on the sphericity of the Earth (Toomer, *Almagest* (cit. note 1), pp. 40-41).

5 The introduction of decimal fractions in the West is usually attributed to Simon Stevin (d. 1620), whereas sexagesimal fractions go back to the Babylonians. In standard notation, 1;2,50 means 1 part, 2 minutes, and 50 seconds ($1 + {}^2/_{60} + {}^{50}/_{3600}$). See Neugebauer, *The Exact Sciences in Antiquity* (New York: Dover, 1969), p. 13.

6 On the slender (and not very credible) ancient evidence for Hipparchus's book on chords, see Neugebauer, *A History of Ancient Mathematical Astronomy* (cit. note 3), p. 299. Toomer's reconstruction depends on a number of unwarranted assumptions: see Toomer, "The Chord Table of Hipparchus and the Early History of Greek Trigonometry", *Centaurus*, 1973, *18*: 6-28. Reconstructions of mathematical methods and parameters used by the ancients are fraught with difficulties and hidden assumptions, and should not be considered as certain even when they account for the available data. It is especially unwarranted to assume that methods employed by Ptolemy were available to his predecessors without evidence in texts of their time (or earlier) in support of such claims (and numerical agreement is not sufficient since many procedures can lead to the same numerical result).

Ptolemy describes an instrument (the first of several described in the *Almagest*), called a plinth, a block of stone set in the meridian from which one may determine the noon altitude of the Sun from the shadow of a peg perpendicular to the face of the stone, cast onto a graduated quadrant [see Fig. 1].[7] With this instrument Ptolemy confirms a parameter that he ascribes to Eratosthenes (3rd century B.C.) – namely that the angle between the equator and the zodiacal circle is about 23;51°, where the zodiacal circle (i.e., the ecliptic) is the name given to the apparent path of the Sun in the heavens, disregarding the daily rotation. But Ptolemy first gives this parameter in an unusual form: instead of using degrees and minutes as he does elsewhere, he says that the arc between the noon solar altitudes at winter and summer solstices is equal to 11/83 of a circle. That he puts the parameter in this form may serve as a clue to the way Ptolemy's Greek and Roman predecessors determined it (for they rarely used sexagesimal degrees and minutes), but we will not discuss such conjectures here.[8] And there is no evidence for this fractional value independent of the *Almagest*. So we see that the contents of Book 1 are quite varied – theoretical issues, definitions, trigonometry, the description of an instrument, and the observational basis for a key parameter.

Table 1 - *Ptolemy's Table of Chords. The column labeled "sixtieths" is for the purpose of interpolation*

Arcs	Chords	Sixtieths
½°	0;31,25	0;1,2,50
1°	1; 2,50	0;1,2,50
1½°	1;34,15	0;1,2,50
2°	2; 5,40	0;1,2,50
2½°	2;37, 4	0;1,2,48
3°	3; 8,28	0;1,2,48
...		
179°	119;59,44	0;0,0,25
179½°	119;59,56	0;0,0, 9
180	120; 0, 0	0;0,0, 0

7 On the plinth and its problems, see JOHN P. BRITTON, *Models and Precision: The Quality of Ptolemy's Observations and Parameters* (New York and London: Garland, 1992), pp. 1-11. The fundamental problem is that the graduated face of the plinth falls into shadow at noon; hence the observation has to be made prior to noon, and assumptions about the relation of the observed angle, Z, to the noon altitude of the Sun, Z′, may lead to a false result [see Fig. 1].

8 For a reconstruction of Eratosthenes's determination of twice the obliquity of the ecliptic as 11/83 of a circle see, e.g., BERNARD R. GOLDSTEIN, "The Obliquity of the Ecliptic in Ancient Greek Astronomy", *Archives Internationales d'Histoire des Sciences*, 1983, *33*: 3-14; reprinted in ID., *Theory and Observation in Ancient and Medieval Astronomy* (London: Variorum, 1985), Essay 2.

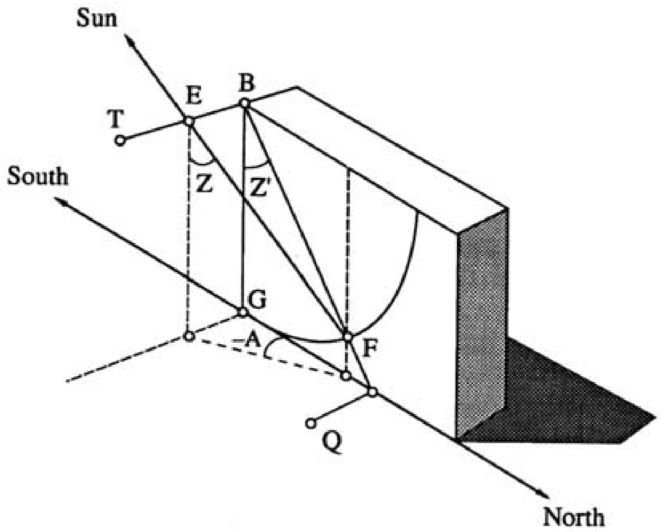

Fig. 1. Ptolemy's plinth, seen from the Northeast, as reconstructed by John P. Britton

Book 2 is devoted to problems in spherical astronomy that mainly relate to the daily rotation. Several chapters lead up to tables of rising times, that is, the time it takes for sections of the ecliptic to rise above the eastern horizon at various geographical latitudes which range from 0° to about 54° and correspond to longest daylight from 12 hours to 17 hours (the tables are arranged at intervals of $\frac{1}{2}$ hour in longest daylight). Ptolemy also shows how to determine the longest daylight for any geographical latitude, applying spherical trigonometry.

After the preliminaries of Books 1 and 2, Book 3 begins with the length of the year and continues with topics related to solar motion. This book is very rich and is worthy of special attention, for most of the essential features of Ptolemaic astronomy can be found in it, but discussion of it is postponed until the survey is completed.

In Book 4 the Moon is considered in ways that are comparable to the treatment of the Sun in Book 3. Ptolemy insists that only observations of lunar eclipses can give precise positional data, since all other observations of the Moon are affected by parallax, that is by the fact that the apparent position of the Moon at any given moment differs for observers at different places on the Earth. The longitude (i.e., the position along the zodiacal circle) determined from the time of the middle of a lunar eclipse is relative to the center of the Earth and this longitude can then be adjusted for an observer at a spot on its surface. Ptolemy's method is to observe the time of the eclipse and then to determine the lunar longitude for that moment by computing the true position of the Sun and adding 180° to it, for the middle of a lunar eclipse takes place when the Sun and the Moon are in opposition, exactly 180° apart. This method is sound but, in principle, any errors in the solar theory may affect the lunar positions. As it happens, the errors in Ptolemy's solar theory were not propagated into his lunar theory (see note 61, below). Ptolemy presents a method for finding the basic parameters of his lunar model from triples of lunar eclipses, using trigonometry. Again, there is no ancient evidence that anyone before Ptolemy invoked this method.

Admittedly, there is a long-standing modern tradition according to which Ptolemy simply took over the methods of Hipparchus who lived about 300 years before Ptolemy and who is repeatedly praised by him.[9] But there are good reasons to doubt this reconstruction, apart from the absence of the relevant works by Hipparchus; indeed, most of the claims about Hipparchus depend on passages in the *Almagest* itself. Hipparchus is not credited with a comprehensive work on astronomy, but Ptolemy cites works by Hipparchus on specific themes. It is not clear if Ptolemy had direct access to these works or if he only had access to summaries of them. Of special interest is Ptolemy's discussion in Book 4, chapter 11. Ptolemy had already presented derivations of the parameters for the lunar model based on a Babylonian lunar eclipse triple going back to the 8th century B.C. and on another lunar eclipse triple that he had observed himself.[10] They both led to the same result, namely, that the lunar eccentricity is about $5\frac{1}{4}$ where the radius of the deferent circle is 60. But in Book 4, chap. 11, Ptolemy reports that Hipparchus had found two results for the basic lunar parameter, equivalent to $4\frac{3}{4}$ and $6\frac{1}{4}$ instead of $5\frac{1}{4}$ (in all cases where the radius of the fundamental or deferent circle is taken to be 60).[11] Hipparchus apparently associated one result with an eccentric model and

---

[9] See, e.g., PAUL TANNERY, *Recherches sur l'histoire de l'astronomie ancienne* (1893; New York: Arno Press, 1976), p. 263. The same view is stated much more harshly in ROBERT R. NEWTON, *The Crime of Claudius Ptolemy* (Baltimore and London: Johns Hopkins University Press, 1977), pp. 364-379, for Newton argues that Ptolemy's contribution to astronomy was negative and that "astronomy would be better off if [the *Almagest*] had never existed". For a response to some of Newton's claims see JAMES EVANS, *The History and Practice of Ancient Astronomy* (New York and Oxford: Oxford University Press, 1998), pp. 273-274. For an assessment of Newton's scholarship on Ptolemy, see my review of NEWTON, "The Crime of Claudius Ptolemy", in *Science*, 1978, *199*: 872.

[10] PTOLEMY, *Almagest*, IV.6; TOOMER, *Almagest* (cit. note 1), pp. 191-192, 198.

[11] According to Ptolemy, Hipparchus gave these ratios as 3144 to $327\frac{2}{3}$ (approximately 60 to 6;15) and as $3122\frac{1}{2}$ to $247\frac{1}{2}$ (approximately 60 to 4;46): PTOLEMY, *Almagest*, IV.11; TOOMER, *Almagest* (cit. note 1), p. 211.

the other with an epicyclic model [see Fig. 2]. But there is no indication that Hipparchus ever decided on a particular lunar parameter, and there is no evidence that he constructed a table for the correction to lunar motion for which a fixed parameter would be required. Ptolemy continues:

> Such a discrepancy cannot, as some think be due to some inconsistency between the two models. Not only have we shown this equivalence by logical argument from the perfect agreement between the phenomena resulting from the two models, but numerically too, if we wanted to carry out the calculations, we would find that the same parameter results from both models, provided we use the same set of data for both, and not, like Hipparchus, different sets [...] We will find that, in the case of those eclipses used by Hipparchus, the oppositions were observed correctly, and are in agreement with our proven models with the lunar motions that we have determined. But Hipparchus's computations of the intervals between the observations must not have been carried out carefully, as we will demonstrate.[12]

Ptolemy then shows, with his methods, that the data ascribed to Hipparchus for the times of the middle of 6 lunar eclipses (two triples of eclipses observed before the time of Hipparchus) lead to Ptolemy's parameter of $5^1/_4$ and not to the results reported in the name of Hipparchus, $4^3/_4$ and $6^1/_4$. Ptolemy says nothing about the method used by Hipparchus – it seems that Ptolemy only had Hipparchus's raw observational data and the resulting parameters. Ptolemy does not locate any specific error committed by Hipparchus and this suggests that his information on Hipparchus was quite limited, either because Hipparchus never elaborated his methods, or because Ptolemy's source did not preserve them. Ptolemy, however, draws a very different conclusion from this confirmation of his parameter for the lunar model:

> Thus we have plainly displayed the unreality of the above discrepancy, and it is clear that we can have even more confidence than before in the correctness of our parameter for the lunar model at opposition, since we have found these very same eclipses agreeing closely with our model.[13]

It seems to me that Ptolemy's confidence is justified since his lunar model with its parameters was able to account for eclipses spread over many centuries: one triple from the 8th century B.C., a second triple from the 4th century B.C., a third triple from the beginning of the 3rd century

---

12 *Ibidem*.

13 PTOLEMY, *Almagest*, IV.11; TOOMER, *Almagest* (cit. note 1), p. 216.

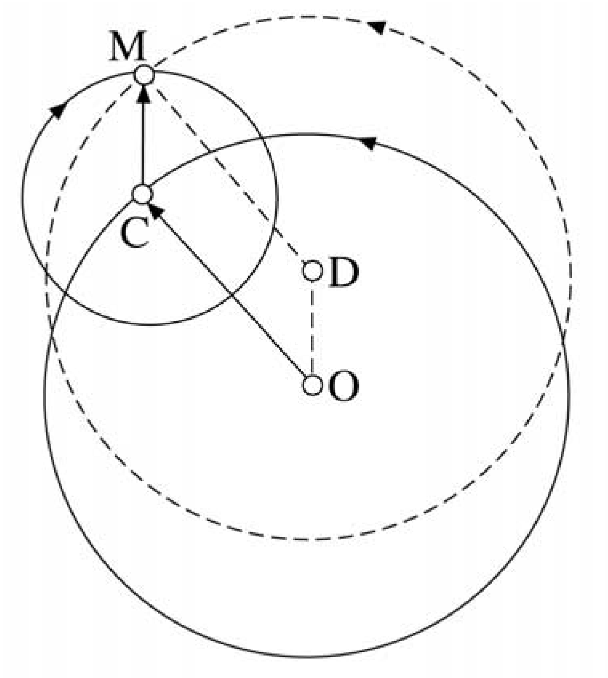

Fig. 2. The Equivalence of the Eccentric and Epicyclic Models. The epicyclic model is illustrated by the solid circle whose center is O (the observer) which carries an epicycle whose center is C on which lies a planet at M. The direction CM is fixed and the direction from the observer to the planet is OM where OM is the vectorial sum of OC and CM. The eccentric model is illustrated by the dashed circle whose center is D such that OD, its eccentricity, lies in a fixed direction that is always parallel to CM. The direction from the observer at O to the planet at M is again OM, but now it is the vectorial sum of OD and DM. The radii of the two circles which rotate at the same rate are equal and parallel, and the eccentricity, OD, is equal and parallel to the radius of the epicycle, CM; hence, the two models are geometrically equivalent

B.C., and a fourth triple observed by Ptolemy himself in the 2nd century A.D.[14]

In Book 5 Ptolemy indicates that the lunar model presented in Book 4 is satisfactory for eclipses and for the Moon at conjunction and opposition with the Sun, but it does not work well at intermediate lunar phases. Ptolemy then modified the model to account for the lunar position at any arbitrary time, and there is no evidence that anyone before Ptolemy had at-

[14] GOLDSTEIN and BOWEN, "The Role of Observations in Ptolemy's Lunar Theories", in *Ancient Astronomy and Celestial Divination*, edited by Noel M. Swerdlow (Cambridge, Mass.: MIT Press, 1999), pp. 341-356: 347.

tempted to do so. This new model is very successful in accounting for lunar positions at all phases but – at phases when eclipses are not possible – it distorts the distance from the Earth to the Moon. Ptolemy was aware of this consequence of his model but says nothing about the difficulty that arises with respect to the apparent variation in the size of the lunar disk.[15]

Book 6 is devoted to the theory of solar and lunar eclipses, while Books 7 and 8 concern the fixed stars, including a catalog of 1022 stars given with their coordinates in longitude and latitude for the year 137 A.D., and separated into 6 classes called *magnitudes* where the original sense was a measure of their relative sizes. Today these magnitudes are understood to be measures of brightness since all these fixed stars are essentially point sources of light.[16]

Books 9 to 13 concern planetary theory, and it is here that Ptolemy introduces the equant into his planetary models, a feature much criticized beginning in the 11th century for failing to conform to the principle of uniform circular motion, a criticism that Ptolemy anticipated and well understood.[17] The equant model is rather successful in accounting for planetary positions and, again, there is no evidence for anyone earlier than Ptolemy using it.[18]

---

[15] In *Almagest* V.13 Ptolemy computes the distances to the Moon measured in terrestrial radii; the maximum is 64;10 and the minimum is 33;33 terrestrial radii (TOOMER, *Almagest* (cit. note 1), p. 251). Cf. NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), pp. 102, 919. This variation in distance implies nearly a twofold variation in the Moon's apparent diameter, contrary to simple observations of the Moon at its various phases. Two astronomers of the 14th century, Levi ben Gerson in southern France and Ibn al-Shāṭir in Syria, presumably independently, found this variation in the apparent lunar diameter troubling and introduced alternative lunar models to eliminate the difficulty. The same point was made by Regiomontanus in the 15th century and he clearly states that, according Ptolemy's model, the Moon should appear twice as large in diameter at quadrature (half-Moon phase) than at opposition (full-Moon phase), contrary to appearances: see GOLDSTEIN, "Theory and Observation in Medieval Astronomy", *Isis*, 1972, *63*: 39-47, p. 46; and IOANNES REGIOMONTANUS, *Epytoma in Almagestum* (Venice: Hamman, 1496), V.22, translated in NOEL M. SWERDLOW, "The Derivation and First Draft of Copernicus's Planetary Theory: A Translation of the *Commentariolus* with Commentary", *Proceedings of the American Philosophical Society*, 1973, *117*: 423-512, p. 462.

[16] For additional details, see GOLDSTEIN, "The Pre-Telescopic Treatment of the Phases and Apparent Size of Venus", *Journal for the History of Astronomy*, 1996, *27*: 1-10, p. 1.

[17] For the critique of Ptolemy's equant model by Ibn al-Haytham (d. *ca.* 1040) see, e.g., GEORGE SALIBA, "Arabic Planetary Theories", in *Encyclopedia of the History of Arabic Science*, edited by Roshdi Rashed (London and New York: Routledge, 1996), *1*: 58-127, pp. 77-78. For Copernicus's view of the equant, see SWERDLOW, "*Commentariolus*" (cit. note 15), pp. 434-435; cf. SWERDLOW and NEUGEBAUER, *Mathematical Astronomy in Copernicus's* De Revolutionibus (New York and Berlin: Springer-Verlag, 1984), pp. 290-291.

[18] For discussion of Ptolemy's planetary models, see ASGER AABOE, *Episodes From the Early History of Astronomy* (New York and Berlin: Springer, 2001), pp. 72-95. For a reconstruction of the path to Ptolemy's equant model, see JAMES EVANS, "On the Function and Probable Origin of

II. Ptolemy's Methodology

Let us now turn to Ptolemy's methodology and the ways it differs from the methodologies of his predecessors, as preserved in texts written prior to Ptolemy. I avoid depending on fragments in texts of late antiquity which ascribe various doctrines to early thinkers whose works are no longer extant because these ascriptions cannot be verified and because, in many cases, at least in astronomy, they can be shown to be anachronistic.[19] This applies particularly to Simplicius who lived in the 6th century A.D.[20] So, the expression "saving the phenomena" first appears in a text by Plutarch (d. *ca.* 125 A.D.) and then in the *Almagest*, and in neither case is it associated with uniform circular motion (despite Simplicius). Plutarch has this to say in his *On the Face in the Disk of the Moon*, 923A:

> Thereupon Lucius laughed and said: Oh sir, just don't bring suit against us for impiety, as Cleanthes thought that the Greeks ought to lay action against Aristarchus the Samian on the ground that he was disturbing the hearth of the universe because he sought to save the phenomena by assuming that the heaven is at rest while the Earth is revolving along the ecliptic and at the same time rotating on its axis.[21]

Here "saving the phenomena" means accounting for the apparent risings and settings of the stars by assuming the daily rotation of the Earth

---

Ptolemy's Equant", *American Journal of Physics*, 1984, *52*: 1080-1089, and Swerdlow, "The Empirical Foundations of Ptolemy's Planetary Theory", *Journal for the History of Astronomy*, 2004, *35*: 249-271. For an alternative reconstruction, see Alexander Jones, "A Route to the Ancient Discovery of Non-unifrom Planetary Motion", *Journal for the History of Astronomy*, 2004, *35*: 375-386.

19 This methodological restraint is here applied more strictly than is the case in most accounts of ancient astronomy. It is often felt that the paucity of ancient evidence "requires" the historian to fill in the gaps by various reconstructions and appeals to later sources. But new historical data often confounds scholarly expectations and generally raises new questions as much as resolving old ones. For example, one could not foresee that Assyrian interest in astronomy in the 7th century B.C. was stimulated by the "ritual of the substitute king", that is, if an omen based on celestial phenomena meant that the king would die, the royal officials appointed someone to be "king" and then killed him (thus fulfilling the omen). For the Assyrian king then astronomy was a matter of life and death: see Francesca Rochberg, *The Heavenly Writing: Divination, Horoscopy, and Astronomy in Mesopotamian Culture* (Cambridge: Cambridge University Press, 2004), pp. 77-78.

20 On the reliability of Simplicius as a source for the history of astronomy, see Bowen, "Simplicius and the Early History of Greek Planetary Theory", *Perspectives on Science*, 2002, *10*: 155-167. See also Id., "Simplicius' Commentary on Aristotle, *De Caelo* 2.10-12: An Annotated Translation (Part 1)", *Sciamvs*, 2003, *4*: 23-58.

21 Harold F. Cherniss (ed. and tr.), *Plutarch's Moralia: Concerning the Face that Appears in the Orb of the Moon* (Cambridge, Mass., and London, 1957), p. 55.

instead of the daily rotation of heaven, and the changes in the seasons by assuming that the Earth revolves around the Sun annually rather than the Sun around the Earth. Nothing is said about the planets or about mathematical models. And it is by no means clear that the original context for this expression had anything to do with astronomy.[22]

As for uniform circular motion – this constraint on astronomy appears in Geminus's *Introduction to Astronomy*:

It is posited for astronomy as a whole that the Sun, Moon, and five planets move at constant speed in a circle, and in a direction opposite to (the daily rotation of) the cosmos. For, the Pythagoreans, who were the first to come to investigations of this sort, hypothesized that the motions of the Sun, Moon, and five planets were circular and smooth. The reason is that they did not admit in things that are divine and eternal such disorder of the sort that sometimes (these things) move more quickly, sometimes more slowly, and that sometimes they stand still – which they in fact call stations in the case of the five planets. For, not even in the case of a man who is well behaved and orderly would one accept such unsmoothness of motion in his modes of progression. For, the necessities of life are often causes of slowness and speed for men: but, in the case of the imperishable nature of the celestial bodies, no cause of speed and slowness can be introduced. For which reason they have proposed the following question: "How can the phenomena be explained by means of circular, smooth motions?"[23]

It is important to notice that ultimately Geminus's argument for uniform circular motion depends on the divine nature of the celestial bodies that are eternal and unchanging. In Book 13 of the *Almagest* Ptolemy then combines the principle of "saving the phenomena" with the nature of the divine, and offers a pragmatic reason for rejecting "simple" models:

Now let no one, considering the complicated nature of our devices, judge such hypotheses [i.e., models] to be over-elaborated. For it is not appropriate to compare human constructions with divine, nor to form one's beliefs about such great things on the basis of very dissimilar analogies. For what could be more dissimilar than the eternal and unchanging with the ever-changing, or that which can be hindered by anything with that which cannot be hindered even by itself? Rather, one should try, as far as possible, to fit the simpler hypotheses to the heavenly motions but, if this does not succeed, [one should apply hypotheses] which

---

[22] See A. MARK SMITH, *Ptolemy's Theory of Visual Perception* (Philadelphia: American Philosophical Society, 1996), p. 19.

[23] AUJAC, *Géminos* (cit. note 3), p. 5. Cf. BOWEN, "The Demarcation of Physical Theory and Astronomy by Geminus and Ptolemy", *Perspectives on Science*, 2007, *15*: 327-358, pp. 331-332.

do fit. For provided that each of the phenomena is duly saved by the hypotheses, why should anyone think it strange that such complications can characterize the motions of the heavens [...]?[24]

For Ptolemy, "saving the phenomena" clearly depends on constructing a geometrical model with parameters such that it can account quantitatively for planetary positions at any arbitrary time. But how can Ptolemy reject the requirement of uniform circular motion if he accepts the nature of the divine as eternal and unchanging? Ptolemy does not address this issue directly, but we can infer his answer from numerous passages in the *Almagest*. "Eternal and unchanging", a phrase that echoes Geminus's expression – "divine and eternal" – for describing the planets, does not mean uniform; rather, it means periodic, that is, the motions return in a cyclical pattern even though they are not uniform. This turns out to be one of Ptolemy's great innovations and it is based on theological considerations.[25] He consistently appeals to mean motion and divergences from it (called anomalies) rather than to uniform motion. Mean motions are derived from periodic returns of the planets by simple division: if there are 8 returns in position in 5 years, the mean motion is 8 times 360° divided by the number of days in 5 years – the result is the mean motion per day.[26] So far this has no geometrical significance and is not dependent on a model – it simply designates a direction and nothing lies in this direction, that is, it is entirely abstract. In fact, it is the deviation (or anomaly) from a mean direction that depends on the model such that the true position is the algebraic sum of the mean direction and the anomaly. There is no evidence for anyone computing planetary positions in this way prior to Ptolemy. In particular, Ptolemy ascribes to Hipparchus true positions at given times, but there is no instance where Ptolemy ascribes a mean position to Hipparchus. Ptolemy's method is so deeply embedded in subsequent astronomy that his way of doing things seems entirely natural to us. Thus, the concern for mean motion is the most significant departure by Ptolemy from the methods of his predecessors.

In his commentary on Aristotle's *De Caelo* Simplicius claimed the following:

---

24 TOOMER, *Almagest* (cit. note 1), p. 600.

25 Nothing is gained if "theological" is replaced by "metaphysical".

26 For a different sense of mean motion as the average of the maximum and minimum values, see BOWEN and GOLDSTEIN, "Geminus and the Concept of Mean Motion in Greco-Latin Astronomy", *Archive for History of Exact Sciences*, 1996, *50*: 157-185, p. 163.

Plato, in imposing the requirement on the motions of the celestial bodies to be circular, smooth, and ordered, set this problem for the mathematicians: what are the hypotheses (i.e., models) that, by smooth, circular, and ordered motions, can save the planetary phenomena?[27]

To me this just seems to be a weaker form of the statements by Geminus and Ptolemy, combining them and eliminating the reference to the divine, while invoking the authority of Plato.[28]

What is the evidence for the methods of Ptolemy's predecessors? Thanks to Alexander Jones, we now have access to Babylonian procedures and tables preserved in Greek papyri – so there is no longer any doubt that Greeks in late antiquity fully understood Babylonian astronomy.[29] Hence, the Babylonian concept of quantitative periodicity was available at the time of Ptolemy – but not mean motions and anomalies applied to geometrical models. The Babylonian schemes were very successful in accounting for planetary positions at any arbitrary time, but they depended on a set of ar-

---

27 The text of Simplicius appears in AUJAC (ed. and tr.), *Autolycos de Pitane: La sphère en mouvement* (Paris: Les Belles Lettres, 1979), p. 160. Simplicius claims to have depended on a lost work by Sosigenes (2nd century A.D.) who in turn depended on a lost work by Eudemus (4th century B.C.): such claims need to be evaluated on the basis on extant contemporary documents. For anachronisms in Simplicius's accounts, see BOWEN, "Simplicius" (cit. note 20), pp. 157-158.

28 For an assessment of Plato's limited role in the history of astronomy, see JOHN L.E. DREYER, *A History of Astronomy from Thales to Kepler* (1906; New York: Dover, 1953), pp. 85-86. For accounts that exaggerate the role of Plato (based on passages in Simplicius) see, e.g., PEDERSEN and MOGENS PIHL, *Early Physics and Astronomy: A Historical Introduction* (New York: American Elsevier, 1974), pp. 27, 69, and 99. I am grateful to Alan C. Bowen for pointing out that the Greek term translated "save" in the quotation is *diasôzein* (*lit.*: to preserve), rather than *sôzein* (to save).

29 JONES, *Astronomical Papyri from Oxyrhynchus* (Philadelphia: American Philosophical Society, 1999). See also BRITTON and JONES, "A New Babylonian Planetary Model in a Greek Source", *Archive for History of Exact Sciences*, 2000, *54*: 349-373. Although Indian planetary models seem to derive from Greek sources, support has been largely lacking in ancient Greek texts, including the papyri described by Jones: see JONES (*ibid.*, p. 16): "There are strong reasons for believing that the indubitable Greek antecedents of the Indian kinematic models were untinctured with Ptolemy's influence and hence perhaps earlier than Ptolemy... One would therefore expect to find in the documentary material from Roman Egypt kinematic tables other than Ptolemy's employing the segregation of mean motions and anomaly. See also DAVID PINGREE, "The Recovery of Early Greek Astronomy from India", *Journal for the History of Astronomy*, 1976, *7*: 109-123; none of the extant Indian texts on which Pingree relied was composed prior to the 3rd century A.D. An eccentric-epicyclic model is perhaps described by PLINY (*Hist. nat.*, II.63-71), but the passage is confused and he made no attempt to compute planetary positions with this model (HARRIS RACKHAM (ed. and tr.), *Pliny: Natural History* (Cambridge, Mass.: Harvard University Press, 1938-1963), 10 vols, vol. 1, pp. 210-217); cf. NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), pp. 802-808, which includes comparisons of Pliny's text with that in P. Mich. 149. See also AABOE, "On a Greek Qualitative Planetary Model of the Epicyclic Variety", *Centaurus*, 1963, *9*: 1-10.

ithmetic rules, not on geometry. The Babylonian schemes impressed the Greeks, and Geminus tried to recast the arithmetic rules into geometry, without much success.[30] Ptolemy changed the program: he did not wish to account for Babylonian schemes geometrically; rather, he wished to account for the positions of the celestial bodies directly. Thus, Ptolemy never discusses a Babylonian scheme, not even to reject it; for Ptolemy, the Babylonians were reliable observers and so he cites their observations and the period relations they established, but nothing else. The Babylonians (and their Greek followers) produced ephemerides, that is, tables which list times and the corresponding planetary positions in chronological order.[31] Ptolemy, however, inserted auxiliary tables in the *Almagest* from which a planetary position at any given time in the past or the future could be calculated by simple arithmetic rules without the computer having to consider the underlying geometrical model. So Ptolemy had the best of both worlds – arithmetic methods for computing planetary positions based on geometrical models.

Another innovation in methodology is Ptolemy's explicit dependence on a small number of dated observations. For the moment there is no need to consider the accuracy of these observations compared with modern recomputation. First and foremost are Ptolemy's own observations which cover each category of astronomical observations. Here Ptolemy seeks to establish the reliability of other observers by comparing their results with those he obtained from his own observations. For the Babylonians the reliability of their systematic observational reports, made on an almost daily basis for about 7 centuries, presumably depended on the authority of the bureaucracy which sponsored them.[32] For Greeks and Romans, however, the reliability of a report generally depended on the status of the observer. For example, Pliny (1st century A.D.) only gives astronomical observations by prominent military leaders and famous astronomers, that is, for him reliability is related to personal authority.[33] For Ptolemy his own observa-

30 BOWEN and GOLDSTEIN, "Geminus" (cit. note 26), pp. 167-171, 181.

31 NEUGEBAUER, *Astronomical Cuneiform Texts* (London: Lund Humphries, 1955).

32 ABRAHAM J. SACHS and HERMANN HUNGER, *Astronomical Diaries and Related Texts from Babylonia*, 5 vols. (Vienna: Verlag der Österreichischen Akademie der Wissenschaften, 1988-2001); FRANCESCA ROCHBERG, "The Cultural Locus of Astronomy in Late Babylonia", in *Die Rolle der Astronomie in den Kulturen Mesopotamiens*, edited by Hannes G. Galter (Graz: Graz-Kult, 1993), pp. 31-46; ROCHBERG, *The Heavenly Writing* (cit. note 19), pp. 232-236.

33 PLINY, *Hist. nat.* II.53-57, 180; RACKHAM, *Pliny* (cit. note 29), vol. 1, pp. 203-207, 313. Cf. BOWEN, "The Art of the Commander and the Emergence of Predictive Astronomy", in *Science and Mathematics in Ancient Greek Culture*, edited by CHRISTOPHER J. TUPLIN and TRACEY E. RIHLL (Oxford and New York: Oxford University Press, 2002), pp. 76-111: 105-109.

tions were certain because of his direct experience and his control of the observational procedures. And his confidence in his theories was enhanced (not established) by the agreement of ancient observations with his theory.[34] Ptolemy is careful (in so far as possible) to name the observers in the case of Greeks and Romans because of the importance he gives to depending on reliable observers, but the same is not true for those Greeks and Romans who produced theories and they are not named by him.[35] One might think that Hipparchus is an exception, for Ptolemy appeals to Hipparchus for observations and refers to several of his theoretical claims. But when Ptolemy mentions Hipparchus's theories, it is often to criticize them.[36]

So there are two considerations here: (1) the reliability of observations and (2) the need for only a small number of observations to fix a particular parameter. The second consideration leads us to another innovation by Ptolemy. The parameters in the *Almagest* are explicitly derived from specific dated observations. This was not true for the Babylonians who do not discuss the way their parameters were derived from the observations they are known to have made. And the Greeks before Ptolemy tended to borrow Babylonian parameters that, according to Geminus, were established by observations "from ancient times", without offering any specifics.[37] According to Proclus, on the authority of Iamblicus citing Hipparchus, the Assyrians had made observations for 270,000 years![38] Hence, it was rather bold of Ptolemy to depart from the view that astronomical parameters were based on a vast number of observations over a long period of time, replacing it with the view that a few selected, reliable observations form a better basis for astronomical theory.[39]

---

[34] GOLDSTEIN and BOWEN, "The Role of Observations" (cit. note 14), pp. 347-348.

[35] To be sure, Ptolemy mentions some observations by unnamed Greeks, but these exceptions do not challenge the rule that Ptolemy gives the names of those Greeks that were reported to him. Menelaus (ca. 100 A.D.) is cited in the *Almagest* as observer, but not for his theorems on spherical geometry which are reported anonymously by Ptolemy in *Almagest*, I.13. Cf. NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), pp. 26-29.

[36] See Ptolemy's criticism of Hipparchus's treatment of parallax: "Hipparchus attempted to correct this [kind] of inaccuracy too, but it is apparent that he attacked the problem in a very careless and irrational way" (*Almagest*, V.19; TOOMER, *Almagest* (cit. note 1), p. 268). According to TOOMER (*ibid.*, p. 268 n. 82), Hipparchus's procedure has not been successfully reconstructed.

[37] AUJAC, *Géminos* (cit. note 3), p. 94.

[38] NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), p. 608.

[39] One might argue that Hipparchus determined the lunar eccentricity from a small number of observations, but Ptolemy's report in *Almagest* IV.11 (discussed above) is the only information we have and it is difficult to know exactly how Hipparchus understood his procedure or

III. Characteristic Features of the *Almagest*

Here are some of the characteristic features of the *Almagest* that set it apart from any other extant ancient Greek astronomical text:

(1) numerical methods and trigonometry;
(2) descriptions of all observational instruments used by Ptolemy to make his observations;
(3) proofs of all mathematical theorems required in Ptolemaic astronomy;
(4) appeal to a small number of specific dated observations, including those by Ptolemy himself, taken to be reliable;
(5) explicit derivations of parameters from the stated observations;
(6) appeal to mean motions and anomalies in determining the true positions of the planets;
(7) explicit methods for making auxiliary numerical tables based on the models with their parameters in order to compute the positions of planets at any time, past or future.

We can add a few details based on topics treated in Books 1 and 3. In Book 1 Ptolemy seeks the chord of 1° from which he can construct the rest of the table of chords. In modern terms, crd $1^\circ = 2 \sin (1/2^\circ)$, and the length of this chord cannot be represented by a ratio of integers. Ptolemy, however, proves that, to three sexagesimal places, Crd 1°≤1;2,50 and that Crd 1°≥1;2,50 (where the radius of the circle, R, is 60, that is, Crd x = R · crd x, and Sin x = R · sin x). Hence, he concludes that he can take Crd 1° = 1;2,50.[40] In contrast, Archimedes says that π lies between $3\,{}^{10}/_{71}$ and $3\,{}^{10}/_{70}$, in effect bracketing the interval in which this value lies.[41] For computational purposes this makes for great difficulties which are generally avoided by taking $\pi = 3\,{}^{1}/_{7}$.[42] Ptolemy's approach to approximation is pervasive throughout the *Almagest* and it makes the construction of all sorts of numerical tables relatively simple, both for the person constructing the table as well as for the person using it. That is, the tables approximate the geometrical model to a certain degree of accuracy which is all that is needed in the practice of astronomy. In contrast to Ptolemy, there is no

---

whether it was to be applied generally. In particular, it is not clear how Hipparchus dealt with his divergent results.

40 For details, see Aaboe, *Episodes From the Early History of Mathematics* (Washington: Mathematical Association of America, 1964), pp. 101-125.

41 Thomas L. Heath, *The Works of Archimedes* (1897; New York: Dover, 1950), p. 98.

42 Ptolemy's value for π is 3;8,30 which he takes to be the mean between the two values given by Archimedes: *Almagest* VI.7; Toomer, *Almagest* (cit. note 1), p. 302.

ancient evidence that Hipparchus constructed numerical tables despite the reports that he had lists of various kinds.[43]

It is now appropriate to look more closely at Book 3, on the solar motion, to illustrate Ptolemy's procedures and to contrast them with what is known about his predecessors. Ptolemy begins by pointing out that the phenomena of the Moon, the fixed stars, and the planets cannot be understood without first treating the Sun. Chapter 1 is devoted to a discussion of the length of the solar year. Ptolemy's first distinction is between the tropical year, the return of the Sun to the same equinox, and the sidereal year, the return of the Sun with respect to one of the fixed stars: the former is a little less than 365¼d and the latter is a little more than 365¼d. For Ptolemy the year properly understood is the tropical year, and the sphere of the fixed stars rotates slowly about the poles of the ecliptic in the direction of increasing longitude with a motion known as *precession*. To show that the length of the tropical year is constant over time, Ptolemy presents a set of observations and refers to Hipparchus's treatise, "On the displacement of the solstitial and equinoctial points". Ptolemy then tells us about his use of an old instrument for observing equinoxes in Alexandria and the errors it seems to produce.[44] After listing a series of equinox observations, Ptolemy next considers a remark by Hipparchus, only to dismiss the concerns of his predecessor:

> Hipparchus himself [...] suspected that there is an irregularity in the length of the year, calculating on the basis of certain lunar eclipses, and declares that he finds that the variation in the length of the year, with respect to its mean value, is no more than ¾ of a day. This would be sufficiently great to take some account of, if it were indeed so; but it can be seen to be false from the very considerations which he adduces to support it [...] For on one occasion he found that the star Spica was 6½° from the autumnal equinox and on another occasion it was only 5¼° from it. Thence he concludes that, since it is impossible for a fixed star, Spica, to move so much in such a short time, it is plausible to suppose that the sun, which Hipparchus uses to determine the position of the fixed stars, does not have a constant period of revolution.[45]

Ptolemy then goes on to show that Hipparchus has argued in a circle: "surely it is perverse to use calculations based on the above foundations to impugn the very foundations on which they were based". Such is Ptolemy's

---

43 For an example of a list ascribed to Hipparchus, see PTOLEMY, *Almagest*, IX.3; TOOMER, *Almagest* (cit. note 1), p. 421.

44 PTOLEMY, *Almagest*, III.1, VII.2; TOOMER, *Almagest* (cit. note 1), pp. 132-134, 139, 327-329. Hipparchus was the first astronomer to be aware of precession: see NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), pp. 292-293.

45 PTOLEMY, *Almagest*, III.1; TOOMER, *Almagest* (cit. note 1), p. 135.

view of Hipparchus as a theoretician in this matter. Of particular interest is Ptolemy's description of Hipparchus's procedure for finding the longitude of the Moon at the middle of an eclipse. The method depends on finding the position of the Sun at that time and simply adding 180° to it; hence, our attention should focus on Hipparchus's way to find the position of the Sun. According to Ptolemy, Hipparchus started with the time of the vernal equinox in the year in which the eclipse took place and then found the solar position at the time of eclipse-middle.[46] No mention is made of finding a mean solar position and its anomaly, as is needed for Ptolemy's own procedure. No other relevant information is given, so how could Hipparchus get a true solar position? At this point we may have recourse to a plausible conjecture – suggesting a procedure Hipparchus may have used. But first let us consider Ptolemy's method as described later in Book 3 of the *Almagest*. To determine the true position of the Sun at a given time, one begins by adding the mean solar position at epoch (here the beginning of the reign of Nabonassar of Babylon, 747 B.C.) to the mean motion in the time since epoch, taking advantage of the tables of solar mean motion in *Almagest* iii.2, yielding the direction to the mean Sun.[47] In Fig. 3 point O is the observer, and we are looking down on the plane of the ecliptic: the solid lines represent directions, and the dashed lines are based on the model. The mean Sun lies in the direction $O\bar{S}$ and the vernal equinox in the direction OV. The model depends on the eccentricity, $e = OD$ (where the radius of the circle is taken to be 60), and the direction to the apogee OA. The true Sun, at S, then, lies on the circle whose center is D on a line parallel to the direction $O\bar{S}$. By trigonometry, one can compute $q = \text{angle } DSO = \text{angle } \bar{S}OS$. Finally, the longitude of the Sun is angle VOS, that is the algebraic sum of the mean longitude, angle $VO\bar{S}$, and the correction, q. Notice that the date of the vernal equinox plays no role in this procedure; rather, its direction is merely used as a guide-post for measuring longitude.

For Hipparchus let us consider another method. We begin by fixing the time of vernal equinox for a given year, using the period of the length of the year. In this way, the errors in the procedure in the course of a year cannot accumulate. We divide the year into zodiacal months: each zodiacal sign is exactly 30° and we determine the time the Sun spends in each of them.[48] In the scheme described by Geminus, the 365 days in a year are

---

46 *Ibidem*.

47 For an example of Ptolemy's method, see NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), p. 60. For chronology Ptolemy generally uses Egyptian years of 365 days each (with no intercalations), and the epoch for counting the years in Era Nabonassar is Thoth 1, Nabonassar 1, corresponding to Feb. 26, 747 B.C. Cf. TOOMER, *Almagest* (cit. note 1), p. 9.

48 See BOWEN and GOLDSTEIN, "Meton of Athens and Astronomy in the Late Fifth Century

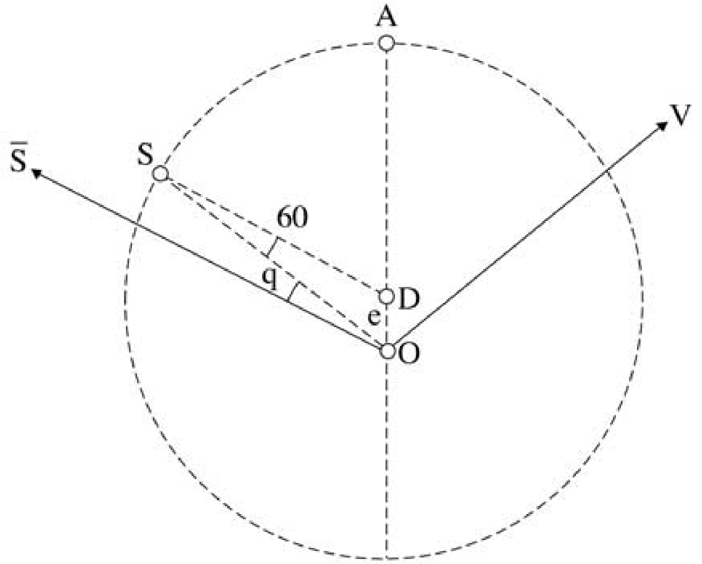

Fig. 3. Ptolemy's Solar Model

distributed in the 12 zodiacal months such that the minimum is 29 days and the maximum is 32 days [see Fig. 4]. Suppose we wish to find the position of the Sun 73 days after the vernal equinox in the given year. At equinox the position of the Sun is 0°; it takes 31 days to traverse Aries (from 0° to 30°) and another 32 days to traverse Taurus (from 30° to 60°). If we subtract 63 days from 73 days, we are left with 10 days of travel in Gemini. By linear interpolation, we can say that

$$x/30 = 10\text{d}/32\text{d}, \text{or } x = 300/32 \approx 9^{1}\!/_{3}°.$$

Hence, 73 days after the vernal equinox, the solar position was $69^{1}\!/_{3}°$. What makes this reconstruction plausible (but not certain) is that information on the time spent by the Sun in each zodiacal sign is given, for example, in Geminus's *Introduction to Astronomy*, and similar data are given in the *Ars Eudoxi*, mentioned above, that is, in one extant text composed shortly after Hipparchus and in another composed shortly before his time.

B.C.", in *A Scientific Humanist: Studies in Memory of Abraham Sachs*, edited by Erle Leichty, Maria deJ. Ellis, Pamela Gerardi (Philadelphia: University Museum, 1988), pp. 39-81, 58-63. For a different reconstruction, see JONES, "Hipparchus's Computations of Solar Longitudes", *Journal for the History of Astronomy*, 1991, *22*: 102-125.

Note that mean motion was not invoked in this procedure and, to repeat, Ptolemy never ascribes to Hipparchus a mean position of any celestial body or a table for finding the correction to a mean position.

By comparing his own observations with those taken centuries earlier, Ptolemy decided that the length of the tropical year is $365\frac{1}{4}$d less $\frac{1}{300}$d, or 365;14,48d in sexagesimal notation.[49] With this period, Ptolemy constructs a table of solar mean motion in days, months, years, and collections of 18 years. These mean motions do not depend on any model – they only depend on the period. And, although the period and the mean motions are mathematically equivalent, there is a conceptual shift that cannot be taken for granted, for it is possible to apply the periods without appealing to mean motions, as we have seen in the case of my reconstruction of Hipparchus's procedure for finding a solar position.[50] Indeed, similar applica-

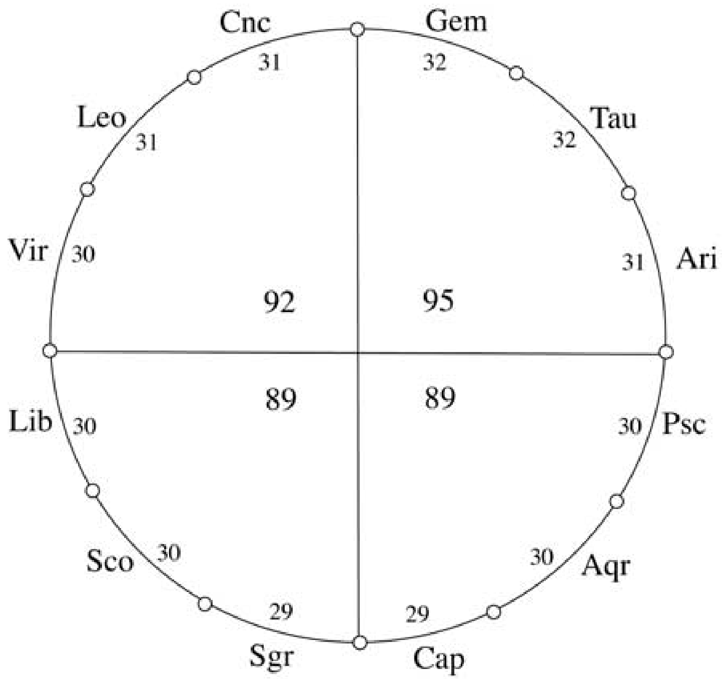

Fig. 4. Geminus's Solar Scheme

[49] Ptolemy, *Almagest*, III.1; Toomer, *Almagest* (cit. note 1), p. 140.

[50] For an example of periodicity in Babylonian mathematical astronomy without mean po-

tions of periods without mean motions have been found in Greek astronomical papyri where using the period keeps any errors from accumulating.[51] The evidence for Hipparchus using such a procedure was uncovered by Gerald Toomer and reported in a footnote to his translation of the *Almagest*. Ptolemy cites a lunar observational report by Hipparchus, which includes the following remark in the name of Hipparchus: "The progress of the Moon was that of day 241", meaning that the progress of the Moon on the day of the observation was its true daily motion on the 241st day of a 248-day period of lunar velocity (that is, 9 returns of lunar velocity correspond to about 248 days).[52] Based on tables preserved in Greek papyri, Alexander Jones has argued that the underlying scheme was a linear zigzag in the style of Babylonian astronomy, that is, an arithmetic scheme with fixed increments until reaching the maximum value and then fixed decrements until reaching the minimum, and so on. The scheme is reset at the beginning of each period of 248 days.[53]

The next chapter of Book 3 is devoted to proving by a geometrical argument that the eccentric and epicyclic models are equivalent. Then comes the determination of the parameters for the solar model, appealing to the eccentric model in the proof. For data Ptolemy has the length of the seasons, based on observations of solstices and equinoxes.[54] Next he applies a geometrical argument (with trigonometry) which leads to the result that the solar eccentricity, e, is $2\frac{1}{2}$ (where the radius of the circle is 60), and that the apsidal line, ODA, is $65\frac{1}{2}°$ from the vernal equinox [see Fig. 3]. From this model Ptolemy demonstrates that the greatest correction, q, for solar anomaly is 2;23°, and he then proceeds to construct a table for solar anomaly at intervals of 3° (occasionally 6°) from 6° to 360° [see Table 2].[55]

---

sitions, see AABOE, "On Period Relations in Babylonian Astronomy", *Centaurus*, 1965, *10*: 213-231.

51 See, e.g., JONES, "A Greek Saturn Table", *Centaurus*, 1984, *27*: 311-317.

52 PTOLEMY, *Almagest*, V.3; TOOMER, *Almagest* (cit. note 1), p. 224; cf. GOLDSTEIN and BOWEN, "Pliny and Hipparchus's 600-Year Cycle", *Journal for the History of Astronomy*, 1995, *26*: 155-158.

53 See JONES, "The Development and Transmission of the 248-Day Schemes for Lunar Motion in Ancient Astronomy", *Archive for History of Exact Sciences*, 1983, *29*: 1-35, pp. 22 and 24.

54 It is possible that the observations were only made to confirm Babylonian data: for the argument that the length of time used by Ptolemy from the vernal equinox to the summer solstice ($94\frac{1}{2}$d) can be derived from a Babylonian scheme, see BOWEN and GOLDSTEIN, "Meton" (cit. note 48), pp. 68-69. Cf. VIGGO M. PETERSEN and OLAF SCHMIDT, "The Determination of the Longitude and Apogee of the Orbit of the Sun According to Hipparchus and Ptolemy", *Centaurus*, 1967, *12*: 73-96.

55 PTOLEMY, *Almagest*, III.6; TOOMER, *Almagest* (cit. note 1), p. 167.

Table 2 - *Ptolemy's Table for the Solar Anomaly*

Common Numbers		Correction
6	354	0;14
12	348	0;28
18	342	0;42
24	336	0;56
30	330	1; 9
...		
90	270	2;23
93	267	2;23
96	264	2;23
99	261	2;22
102	258	2;21
...		
171	189	0;24
174	186	0;16
177	183	0; 8
180	180	0; 0

Book 3 ends with a chapter on a new topic called, in modern astronomy, *the equation of time*.[56] Ptolemy seems to have been the first to pay attention in computing lunar positions that the time from one noon to the next varies throughout the year.[57] That is, after computing mean motions for a given number of days after the epoch, the result still has to be corrected for the time between mean noon (assuming the days are equal) and true noon (when the Sun crosses the meridian). As Ptolemy notes, this inequality of the days can amount to about half an hour which is significant for lunar motion. Once again, we see that Ptolemy's concern for mean motion has led him to recognize a characteristic feature of time as it affects astronomy.

Since the 18th century, Ptolemy has been roundly criticized for failing to meet the standards of 18th-century astronomy (which is hardly surprising), and for "errors" in his observations and his theories.[58] Much of this

56 PTOLEMY, *Almagest*, III.9; TOOMER, *Almagest* (cit. note 1), pp. 169-172; cf. NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), pp. 61-62.

57 There are two components to the equation of time and Ptolemy recognized both of them. GEMINUS (*Intro. Astr.* 6.1-4) was aware of one component qualitatively: see BOWEN and ROBERT B. TODD, *Cleomedes' Lectures on Astronomy* (Berkeley and Los Angeles: University of California Press, 2004), pp. 54-55.

58 For Le Monnier's comments, see JOHN KEILL, *Institutions astronomiques*, translated, enlarged, and preceded by an essay on the history of modern astronomy by PIERRE CHARLES LE MONNIER (Paris: Guerin & Guerin, 1746), pp. XIX-XX; translated in BRITTON, *Models and Preci-*

criticism (frequently directed against Ptolemy as an observer) misses an important point: Ptolemy did not appeal to authority and everything he presented is, in principle, reproducible. In all cases he stated the observations he used, both his own and those of his predecessors, and he described the instruments and the mathematical methods on which he depended. His confidence in his own observations was perhaps too great, but he hardly insisted that anyone take his data without checking them for himself. By modern standards, the observations ascribed to Hipparchus in the *Almagest* are often more accurate than those of Ptolemy, but this tells us little about the scientific methods he applied.[59] Ptolemy is particularly criticized for an error of a day in the time of an equinox that he observed. This error affected a fundamental parameter in the entire system and was propagated throughout it, leading some to "accuse" Ptolemy of making several errors.[60] So why did Ptolemy have confidence in his solar theory that seems to lie on a shaky foundation? As Ptolemy noted, the solar theory plays a major role in determining the parameters of the lunar theory, and his lunar theory is well confirmed by data over 8 centuries. How can this be? It turns out that for the lunar theory Ptolemy uses the differences between solar positions at the times of lunar eclipses; the error in the initial point drops out in these differences and so the error in the solar theory had no effect on the lunar theory.[61]

Ptolemy can hardly be faulted for the veneration he was accorded after his death – for many he became an "authority". Indeed, some medieval scholars who criticized Ptolemy's theories felt obliged to apologize for this practice. For example, Levi ben Gerson (d. 1344), one of the most innovative astronomers in the Middle Ages, had this to say:

---

sion (cit. note 7), p. XIX. See also JOSEPH JÉRÔME DE LA LANDE, "Mémoire sur les équations séculaires", *Histoire de l'Académie Royale des Sciences: Année 1757* (Paris: Imprimerie royale, 1762), pp. 411-470: 420-421; translated in CURTIS WILSON, essay review of NEWTON's, *The Origin of Ptolemy's Astronomical Parameters* (Baltimore: Johns Hopkins University Applied Physics Laboratory, 1982), in *Journal for the History of Astronomy*, 1984, *15*: 37-47, pp. 37-38.

59 For a balanced assessment of the errors in different types of observations reported by Ptolemy, see BRITTON, *Models and Precision* (cit. note 7), pp. 71-77, 122-124. Britton (*ibid.*, p. 118) shows that for a certain class of observations (elongations between the Sun, the Moon, planets, or fixed stars) those made by Ptolemy are slightly more accurate than those made by Hipparchus.

60 For the relation of the error in the position of the Sun and its effect on Ptolemy's star positions and his value for precession, see *ibid.*, *Models and Precision* (cit. note 7), p. 78, n. 22.

61 On Ptolemy's method for finding the eccentricity and apsidal line for the Moon, see NEUGEBAUER, *Ancient Mathematical Astronomy* (cit. note 3), pp. 73-79.

> When deciding to dissent from the teachings of the ancients, one should do so with extreme care and scrutiny, deviating from these teachings as little as possible. This is appropriate because the ancients were lovers of truth and endeavored to approach it as closely as possible even when their principles prevented them from reaching it entirely. Therefore, Ptolemy devised all these strategems and postulated many strange features in order that the planetary models he proposed be arranged so as to make the computation of the planetary paths according to his principles come as close as possible to the truth. Therefore, we first tried to solve some of the difficulties raised against him by our predecessors with respect to his postulates concerning eccentric spheres and epicycles, seeking to find observational evidence to establish his hypotheses.[62]

Levi continues by saying that only after the failure of these attempts to save Ptolemy did he turn to a new set of principles. There is no comparable apology in Ptolemy – he did not share this deferential attitude towards the "ancients". And yet Levi did all the "right" things (that is, he met the standards for astronomical research set by Ptolemy): for example, he gave preference to his own observations, which were numerous by ancient and medieval standards, over those of his predecessors. Among his many contributions to astronomy, he invented new observational instruments which he described in detail; he introduced new geometrical models for planetary motion and explicitly derived the parameters for them from his own observations; and then he constructed tables that represent these new models, for ease of computation.[63]

IV. Conclusion

To sum up, Ptolemy made astronomy the paradigm of a quantitative science by invoking mathematical theorems, computational methods, historical data, observational data, descriptions of instruments, geometrical models (including equant models for the planets), and reducing models to numerical tables. At a fundamental level, however, his most important innovation – that reflected his theological commitments – was the intro-

---

62 Levi ben Gerson, *Astronomy*, chapter 46, translated in Goldstein, "A New Set of Fourteenth-Century Planetary Observations", *Proceedings of the American Philosophical Society*, 1988, *132*: 371-399, p. 385.

63 See Goldstein, *The Astronomical Tables of Levi ben Gerson* (New Haven: Connecticut Academy of Arts and Sciences, 1974); Id., *The Astronomy of Levi ben Gerson (1288-1344)* (New York and Berlin: Springer-Verlag, 1985); and Id., "The Physical Astronomy of Levi ben Gerson", *Perspectives on Science*, 1997, *5*: 1-30.

duction of quantitative procedures for finding planetary positions based on mean motions and anomalies.

Acknowledgments

I am most grateful to Alan C. Bowen and Giora Hon for reading a draft of this paper and offering valuable comments on it.