Applied Elementary Number Theory (Math 1020)
University of Pittsburgh, Fall Semester 2020

Welcome to Number Theory! Please read this syllabus carefully. Let me know if you have any questions at all!

The course webpage on Canvas is where the course will really happen, whether we are meeting online or in person.

https://canvas.pitt.edu/courses/54149

1 Who I am

My name is Carl Wang-Erickson. You're welcome to call me "Carl," or "Dr. Carl," or "Professor Carl." Or "Dr. Wang-Erickson" or "Professor Wang-Erickson," but that is a mouthful. Let me know if you prefer me to call you by a name other than what I see in your official enrollment in the course (there will be a survey where you can tell me about that).

I am originally from Milwaukee, Wisconsin, USA. I finished my PhD studying number theory at Harvard University in 2013. Since then, I worked at Brandeis University and then Imperial College London, followed by Pitt starting in 2019. Because I am new to Pitt, I genuinely invite you to tell me more about Pitt and how things work here.

While in high school, I became interested in number theory, and started to learn about a few of the ideas that we will learn in this course. That's a major reason that I wanted to study math more, and after trying it out in college, I decided to try to get a PhD and do research. The topics that we will discuss in this course are fundamental to my research today, and I really enjoy them.

I am interested in promoting research and independent learning by undergraduates, especially taking the topics of this course as a starting point, and I welcome questions about how this could work for you.

Keep in mind that while the number theory we study is "elementary," that word is not meant to be pejorative. It does not mean that it's "simple" or "routine" either. It just means that we're studying the numbers we have known for some time: 1, 2, 3, 4, 5, and so forth. We will discover that they have much more room for exploration and discovery in them that one might first suspect.

How to contact me

Emailing me at carl.wang-erickson@pitt.edu is the best way to start. In particular, you are welcome to set up a meeting with me. If you would like to find out more about me, you might be interested in my webpage at https://www.pitt.edu/~caw203/.

Course TA, Lingyu Liu

I am pleased to have Lingyu Liu working with us as a TA in this course. He will be able to introduce himself to you in due course, along with his office hour times. He will work with me to assess coursework and hold office hours.

Emailing Lingyu Liu at lil133@pitt.edu is the best way to contact him.

2   Where and When: Flex@Pitt course delivery

This course will be delivered according to the Flex@Pitt guidelines (link). In summary, this means that we will be able to consistently hold course sessions and office hours via Zoom to allow for students who are not in the physical course location, whether or not the instructor is physically present there.

The location of the course sessions and office hours will depend upon Pitt's Operational Posture (link), which varies among three options, Guarded, Elevated, and High risk. I will also communicate with you to confirm whether there are in-person office hours and course sessions.

When

The **Course time** does not depend on the Operational Posture: Monday, Wednesday, and Friday, 4:30pm-5:20pm.

Similarly, the **Office hour time** is (tentatively) Monday and Wednesday, 2:00pm-3:00pm, regardless of Operational Posture. Office hours will be finalized after a survey of your schedules.

Location in Elevated or High risk Operational Postures

Zoom course sessions will occur in our Zoom meeting room *986 0889 4592*. For the password for the Zoom room, initially access it through the Zoom tab in Canvas.

Zoom office hours will occur in Zoom meeting room *934 9714 2096*. For the password for the Zoom room, initially access it through Canvas.

Location in Guarded risk Operational Posture

In-person course sessions are planned to be held at Chevron Science Center, Room 154. They will also be broadcasted to the same course sessions Zoom room as above. I would of course confirm any start of in-person instruction to you via the course Canvas page.

In-person office hours may potentially be held in my office, Thackeray Hall 421, and broadcasted to the office hours Zoom room. The default option even under Guarded risk will be to continue doing office hours on Zoom only; I will make a clear announcement on Canvas about a transition to in-person office hours.

3   What we are doing

Our goal is to understand the natural numbers 0, 1, 2, 3, 4, and so forth, usually considered within the *integers* $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$. Here are the BIG THEMES (or "learning objectives," if you like) that we will use to organize everything we do in this course.

1. Distinguish and study the *additive* and *multiplicative* aspects of the integers

2. Apply *modular arithmetic* to enhance our understanding of the integers

3. Outline how modular arithmetic is used in *cryptography*

4. *Construct explanations and arguments* about the integers, and

5. *Analyze and constructively criticize* your fellow students' explanations and arguments

Within the curriculum of the math major, the point of this course is to apply what you have learned in Math 430 (intro. to algebra) to study numbers, and build upon your skills in argumentation (proofs) and explanation (exposition) that you have and will develop in many courses. We will also touch on how the study of number theory influenced the development of algebra.

4   How we will do it

Here are the major tools and resources that we will engage in order to accomplish this.

4.1   Textbook

The textbook will be a useful resource to learn about the content of the course. Sometimes we will follow it closely, and sometimes we will use it as a resource that complements the content in lectures.

Elementary Number Theory, 6th Edition, by Kenneth H. Rosen.

It has been conveyed to me that the e-book version will be available through Pitt's *RedShelf Inclusive Access* program for the price of $40, which is also viewable on our course's Canvas page. The book's official website may be found at this link.

4.2   Written assignments

There will be weekly written assignments that you can submit through Canvas. Assignments completed using LaTeX, which is the standard typesetting package in mathematics and many other fields, will receive a 5% bonus. LaTeX is free and can be downloaded to your computer. Alternatively, you can use a free online version such as Overleaf.[1]

The emphasis in these assignments will be to either

- argue and explain your proof clearly, or

- argue and explain your proof as far as you can, and then identify what remains that you do not understand or cannot figure out.

Accordingly, I do not expect the assignments to have as many problems as typical problem sets in math courses, since the focus is on convincing your audience or explaining what is in the way of solving the problem, and there is a revision process (see below). In fact, an assignment may consist of just one "problem."

Initial drafts of written assignments are due at the beginning of class on Wednesdays. They should leave space for comments – I will supply a LaTeX template with large margins.

Late initial drafts of assignments will not be accepted, unless you seek approval of an extension from me *in advance*. Any assignment not turned in by the deadline receives a grade of zero, but the lowest initial assignment grade is dropped and the second lowest initial assignment grade is weighted at 50% compared to the rest of them.

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[1]To access a Pitt-sponsored Overleaf account, go to https://www.overleaf.com/edu/pitt.

4.2.1   Collaboration policy

You may collaborate with fellow students on any stage or your own written assignments, provided that you

- write up your explanations independently, and

- list the names of those that you collaborated with on your written assignment.

4.2.2   Sourcing policy

You may use materials from the textbook and course sessions without limitation and without citing them, unless *upcoming content* from the course somehow obviates the assigned work – I expect you to use your own reasoning to avoid doing that.

You may also seek out and use inspiration from all sources, including the internet, provided that you follow standard professional and academic practice of $\boxed{\text{citing your sources}}$, and genuinely understand what you are writing down.

To assist you in following these policies and getting the most out of this course, I will also purposely design the assigned work so that it encourages your own personal voice and cannot be "looked up" to any appreciable extent.

4.2.3   Citing your sources

I will provide an introduction to the standard practice of citing your sources in mathematics. It is not so different than other fields, so my point is to make it as easy as possible for you.

4.2.4   Why these policies?

Assiduously following these collaboration and sourcing policies are very important relative to your

- respect for your fellow students and what they are trying to achieve in this course,

- respect for your instructor(s), and

- most importantly, your respect for yourself and your own personal and professional development.

Please respect yourself by believing that you can be successful in this course by getting any help you need to reach your goal, rather than short circuiting your potential for growth by violating these policies.

I will be taking both of these requirements seriously, and welcome discussion about them. In particular, I promise to grade *much more highly* a discussion that identifies what you do not understand about how to solve a problem, as opposed to stumbling your way around implementing a solution that you found online but do not understand.

Naturally, these policies also apply to all of the work that you do for this course: the Ts & Qs, midterm exam, and final project, as well. They are particular instances of Pitt's academic integrity policies, which also apply in all cases.

4.3   Peer feedback and revision of written assignments

You will also be asked to give feedback on your fellow students' assignments. Constructive and sincere feedback is a great way to identify misunderstandings or misplaced emphases in proofs that you read. Roughly, the process will work like this.

1. Your assignment is turned in on Canvas on Wednesday by 4:15pm.

2. I (the instructor) do a quick check for initial completeness on Wednesday evening or by Thursday at noon at the latest, and assign an initial draft completeness score.
   - "Completeness" means that I do not check for correctness in detail, but only check that the solutions thorough exposition and a complete attempt at addressing the problem(s), either by completely solving the problem or identifying gaps in a partial solution.

3. I send your assignment to one of your classmates by Thursday at noon, using the Canvas peer review function.

4. Your classmate will read and provide praise and constructive criticism written on your assignment, and then turn it in on Canvas by Monday at 4:15pm. Your classmate's work is assessed by me as a "peer editing" score.

5. Most Fridays' class sessions will include an opportunity to discuss feedback with your classmate, as long as this can be done safely (if in person) and practically.

6. You have an opportunity to revise in light of received feedback, and submit this revision on Canvas by the following Thursday at 6:00pm. In particular, you might change an initial draft's explanation of what confused you about a problem into a complete solution in the final draft.

More detailed information about exactly *how* this process will happen will be provided in class and on Canvas. It is experimental, so I desire and welcome your feedback! I want to emphasize that I want to make the written assignments concise enough that this review and revision process provides an appropriate challenge and takes a reasonable amount of your time.

As in the initial draft, please let me know in advance if you cannot participate in peer editing for a week, whether or not it is "excusable" grade-wise. This is important so that everyone who submitted an assignment can get feedback: I will adjust who is assigned to peer edit based on absences.

Here are a few more notes on how peer review and revision for the final version will work.

- If you do not turn in the initial draft on time, you will not be assigned a peer review. The idea behind this practice is that it's good to have gone through an assignment yourself before reviewing someone else's attempt.

- Even if you do not turn in an initial draft and do not do the peer review, you are still welcome to turn in the problem set by the final version deadline. According to the grading formula (below), this will get you at most 50% credit for that assignment.

4.4 Thoughts and Questions

On a weekly basis, probably due each Monday by 3:00pm, I will ask you to post on Canvas a brief thought or question about upcoming topics in the course – I will give you a prompt for what to post about. These "Ts & Qs" will be viewable by all students and instructors under the Discussions tab on Canvas.

Each one will be graded out of two points, where "2" means that you did it thoughtfully, "1" means that you did it but not very thoughtfully, and "0" means that you did not do it by Monday at 3:00pm.

You (and I) are welcome and encouraged to respond to each other's Ts & Qs at any time; these further chats will not be graded or otherwise counted in assessments.

4.5   Midterm exam

There will be a midterm in class in the middle of October. Currently, I am aiming for this midterm to occur on Monday, October 12, but this is tentative. The final date for the midterm will be confirmed later in the semester.

The midterm will be cumulative up to that point. It will involve stating major definitions and theorems, applying them to solve problems in some cases, and justifying perspectives on the additive and multiplicative nature of the integers. More details will be released closer to the time of the midterm.

4.6   Final project

Your final project has two parts. It is a combination of

1. an expanded version of a single problem on a final written assignment, and
2. an optional Zoom final presentation and feedback component.

In your final written assignment, you will be asked to

- Introduce a question in number theory in a way that engages an audience of your peers
- Introduce a theorem that addresses the question
- Prove something related to the theorem (you may prove the whole theorem, but this does not have to be the case)
- Discuss the implications or applications of the theorem or its proof, including and also beyond addressing the original question
- Use proper style: paragraphs, section headings, bibliography/works cited
- Aim to do this in no more than 10 pages of LaTeX (not counting bibliography and pictures/figures), but note that assignments around at least 5 pages long may be excellent

Then your assignment will be assessed by both a fellow student and the instructor (but only the instructor's assessment will be numerical and count toward your grade, and the instructor will assess it before your fellow student does to avoid bias). Then, you can submit a revised final version of your essay.

I may assign the same question to all students, or I may give students partial freedom of choice of a topic. This will be clarified in due course. Likewise, I will produce a very clear schedule and rubric for how the final project will be assessed, and share it with you well in advance.

This course will not have a final exam, but the assigned final exam period will be used to allow for a presentation on your paper to the class via Zoom. Your final project grade can potentially be improved by presenting on your paper and by showing up to watch others' presentations and providing written feedback. If you choose to not present or not watch others' presentations, your final project grade will not be affected.

More specifically, the grade for your final project will be calculated as the maximum of the two numbers

$$W \qquad \text{and} \qquad \frac{3}{5}W + \frac{2}{5}P,$$

where W is your final written assignment score, and P is your final presentation and feedback score (all out of 100). The rationale behind this grading formula is that written assignments have been practiced all term, while presentations have not been. Therefore, it is sensible to give a bonus for doing well at something that you have not previously practiced, but you should not be penalized for doing poorly at something that you have not practiced before.

4.7 Communication!

4.7.1 Between me and you-singular

Communication between me and you is key for promoting your success in this course. The standard ways I communicate *to* you are

- the lecture components of course sessions

- two-stage feedback on your written assignments and final project, and grading/feedback on midterm exam and Ts & Qs

- other interactions, such as office hours, appointments you request, email, etc.

The standard ways you communicate with me are reciprocal to the above: showing up to course sessions, submitting all of the required grades, and showing up to office hours.

In addition, I need to and want to hear from you if you are finding making the deadlines difficult. I know that it can be hard to reach out to me to ask for extensions, but I really encourage you to start that conversation in advance of falling behind. While I will try to reach out to you if I notice you falling behind or otherwise uncommonly struggling in the course, I ask you to agree to take primary responsibility for your success. Conditions in all of our lives may be difficult from time to time, and Covid-19 only makes things worse.

To promote communication between me and you, I will create an (optional) opportunity for you to talk with me "in person" toward the beginning of the course. There and on an initial survey, I am really interested in knowing your response to "what do you want to get from this course?"

4.7.2 Between me and you-plural

Also, on a whole-class level, I want to hear from you (or, in Pittsburgh, yinz) about the course is going, and I am looking forward to hearing your voices during course sessions and outside them. Maybe we will be going "too slow" or "too fast", or I didn't appreciate something about Pitt or its undergraduate experience that is important for me to know – I would like to know your opinion about it in real time, not only once the course is over via OMET. I will create some opportunities for formal feedback as we go along, and I also want to hear from you about these topics "in person" or over email, whatever you feel most comfortable with. I also encourage you to discuss what you think of the course with each other, one benefit being that you can give even better feedback to me.

4.7.3 Among you-plural

Our peers can often be our best teachers. I started practicing this lesson only after graduating from college, which was my own loss. While it might possibly feel unusual to do as much peer discussion of math as is required in this course, it is, in fact, an important part of being a mathematician, and I want to treat you like mathematical adults. That is why I am

- putting emphasis on *explanation* on top of mere *correctness* in your written assignments, and

- including a peer review component.

I think that there is a common image of mathematics being a solitary, locked-in-your-room-by-yourself endeavor, but that's not actually the way it usually goes (when it goes well, anyway). I'd be happy to talk with you more about how peer interaction is going well (or not) in this course, and how we can work together to improve our exercises of our ability to communicate about mathematics with each other.

5 Assessment

Your final numerical course grade, out of 100, will be calculated as

$$\text{Course grade } = \frac{1}{2}H + \frac{1}{4}F + \frac{3}{20}M + \frac{1}{10}TQ,$$

where H = Homework, F = Final project, M = Midterm, and TQ = Ts & Qs grades, where each of H, F, M, TQ has been scaled to an out-of-100 scale before this calculation.

The Homework grade will be calculated as follows.

$$\frac{1}{4}I + \frac{1}{4}E + \frac{1}{2}F,$$

where

- I is the average of all of your *initial completeness* scores (which is influenced by the 5% LaTeXbonus and the dropping low scores rule described above),

- E is the average of your *peer editing* scores (which is <u>not</u> influenced by the 5% LaTeXbonus, but *does* following the dropping scores rule),[2] and

- F is the average of all of your *final draft* scores (which is influenced by the 5% LaTeXbonus and the dropping low scores rule described above, independently of the initial scores)

and I, E, and F have been scaled to a 100-point scale before the calculation.

Your final letter grade for the course will be calculated on an *absolute* basis to the maximum extent that I can achieve. In other words, a student will get an A for "superior" work, no matter the performance of other students; and so forth, according to the official grade scale.[3] While it is undeniable and natural that your peers' level of achievement may affect your grade in some sense, I use the overall class achievement level as only one calibration factor for letter grades. I welcome more discussion in person on this topic, and would be happy to occasionally provide an estimate of your letter grade based on your current grade status once the midterm exam has passed.

6 Other policies and procedures

- **Audit option.** Please contact me to set up a discussion of an audit option, if this interests you.

- **Attendance.** Attendance is not "taken" nor graded, but I do expect you to attend course sessions. In particular, while I will aim to put all of the most important announcements for the course online, if you miss an announcement or explanation of how to do something that happens during a course session, it will be "on you" to find out about it.

- **Email and Canvas communication.** Students will be expected to be aware of updates about the course that are sent via email to their Pitt email account or posted on the course's Canvas page. To see how to make sure you get the right Canvas notifications, see <u>this link</u>.

- **Disability resources.** If you have a disability for which you are or may be requesting an accommodation, you are encouraged to contact both your instructor and the Office of Disability Resources and Services, 140 William Pitt Union, 412-648-7890, as early as possible in the term. Disability Resources and Services will

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[2]*Excused in advance* absences from the peer editing process will not be held agains your peer editing grade, just like the I and F scores.

[3]https://catalog.upp.pitt.edu/content.php?catoid=188& navoid=17780#grading-systems

verify your disability and determine reasonable accommodations for this course. More information may be found at `http://www.studentaffairs.pitt.edu/drs/`.

- **Academic integrity.** Cheating/plagiarism will not be tolerated. Students suspected of violating the University of Pittsburgh Policy on Academic Integrity, from the February 1974 Senate Committee on Tenure and Academic Freedom reported to the Senate Council, will be required to participate in the outlined procedural process as initiated by the instructor. A minimum sanction of a zero score for the quiz or exam will be imposed. (In particular, this includes following the collaboration and sourcing policy described above.)

- **Scheulding conflicts.** It your responsibility to take stock of schedule conflicts, for example, those resulting from athletic participation or religious observance. These should be discussed with the instructor in the first two weeks of the semester, and will be handled according to university guidelines.

- **Health and Safety.** In the midst of this pandemic, it is important that you abide by public health regulations and Pitt health standards and guidelines. While in class, at a minimum this means that you must wear a face covering and comply with physical distancing requirements; other requirements may be added by the University during the semester. These rules have been developed to protect the health and safety of all community members. Failure to comply with these requirements will result in you not being permitted to attend class in person and could result in a Student Conduct violation. For the most up-to-date information and guidance, please visit http://coronavirus.pitt.edu and pay attention to your Pitt email for announcements from the university.

7 Have a great semester!

Please get in touch with me to ask about any questions that arise from this syllabus! I am looking forward to working with you – all of us have a lot to learn this semester. In particular, just as I am expecting continuous improvement from you in your learning, I am expecting continuous improvement in my teaching.