Optimizing E-tailer Profits and Customer Savings: Pricing Multi-Stage Customized Online Bundles

ABSTRACT

Online retailing provides an opportunity for new pricing options that are not feasible in traditional retail settings. This paper proposes an interactive, dynamic pricing strategy from the perspective of customized bundling to derive savings for customers while maximizing profits for e-tailers. Given product costs, posted prices, shipping fees and customers' reservation prices, we propose a nonlinear mixed-integer programming model to increase e-tailers' profits by sequentially pricing customized bundles. The model is flexible in terms of the number and the variety of products customers may choose to incorporate during the various stages of their online shopping process. Our computational study suggests that the proposed model not only attracts more customers to purchase the discounted bundle, but also noticeably increases profits for e-tailers. The ODBP model is robust under various bundle sizes and scenarios. It performs the best when confronted with divergent views about product values, lower budgets, and higher cost ratios.

Key words: online retailing, customized bundle, multi-stage dynamic pricing, nonlinear mixed-integer programming, customer budget, reservation price

1. Introduction

The past decade has witnessed an astonishing growth of Internet retailing. Online shopping has become a daily phenomenon for some consumers, with estimated US Internet sales growing to $172.9 billion in 2010, and expected to reach $250 billion in 2014 (Schonfeld 2010). This upward trend is projected to continue over the years to come, and undoubtedly this rapid growth will attract further competition. For instance, when interested in a laptop computer, a customer can find one at every top Internet retailers (hereafter "e-tailers"). To remain competitive, e-tailers need to continue looking for ways to incentivize existing customers and to attract new ones.

Customer preferences and product prices have been documented as two main factors influencing purchasing decisions (Keeney 1999). In practice, e-tailers often motivate customers by recommending interesting products and presenting savings opportunities. For example, online recommendation systems (ORS) regularly use customers' browsing history to recommend products (Ansari *et al*. 2000), and retailers often promote merchandise through bundling at lower price. However, the current ORS tend to focus on customers' prior online behavior, with little emphasis on e-tailer profitability or customer savings. Also, the bundles promoted are usually established offline in advance by e-tailers. Customers do not have the freedom of selecting the content or the total number of items in a bundle. The potential for interactively pricing products in real-time is not made available to customers who may be looking for better deals.

Dana (2008) and Elmaghraby and Keskinocak (2003) point out that the extant literature has not fully incorporated pricing strategies into the Internet environment. The detachment of customers' online behavior from retailers' pricing strategies can lead to limitations. Without incorporating customer preferences and savings concerns, systems designed to maximize e-tailers' profits fail to capitalize on customers' interest and to convert online browsers into buyers;

whereas focusing on customer savings without linking them to the e-tailer's profit may improve sales volume, but not necessarily firm's profit. In the traditional cross-selling (*e.g*., an appliance and extended warranty), customers achieve savings only if they buy the specific package. Such fixed offerings can now be seen as overly rigid; customers receive no savings when un-tied products are purchased. To date, e-tailers have implemented a few relatively unsophisticated pricing strategies to encourage customer purchases. For example, Amazon.com's "everyday low price" regularly offers "buy A and get B at an additional 5% off" where the discounted price only link with certain products. Customers gain no savings when other products are chosen.

In this paper we propose an **o**nline **d**ynamic **b**undle **p**ricing (ODBP) model for e-tailers to exploit the real-time information available from tracking customers' decision making processes online. We offer e-tailers an interactive pricing scheme and provide a customized price to enhance consumers' savings and maximize e-tailers' profits in a manner not available to bricks-and-mortar retailers. Customers' online shopping behavior is a multi-stage process, where they sequentially add products to a shopping cart and often buy multiple products in one transaction (Häubl and Trifts 2000). After placing a few products in the shopping cart they may choose to remove certain items. Each "add" or "delete" event would update the shopping cart and advance the customer's shopping process to the next "stage". Once the shopping cart is updated, the ORS will generate a new recommendation list to further interest the shopper. For each recommended item, our ODBP model will determine the bundle price by combining the new product with the products already in the cart. Since the bundle price will inherently be cheaper than the sum of the individually posted price, which is defined as the standalone selling price shown on the e-tailer's website, customers are enticed to buy more products in one transaction.

Based on the data collected from Amazon.com and through numerical study, we found the

proposed approach is a "win-win" strategy as the ODBP model provides more profits for e-tailers and extra savings for customers. The contributions of the proposed model are threefold:

1. The model considers customer preferences, customer savings, and e-tailer profits. Compared with existing methods which explore the three aspects independently (Bodapati 2008, Geng et al. 2005, Ghosh and Balachander 2007), the ODBP model attracts more customers due to its emphasis on customers' savings. Also since the optimization model focuses on profit maximization and balances between discounts and profits, the price cut will not come at the expense of e-tailer's earnings due to higher sales volume.

2. Traditional pricing models regard customers' purchasing behavior as a buy-or-not-buy one-stage decision process. Our model more realistically allows customers to explore as many times as they please, in terms of the number and the variety of products. The ODBP model allows e-tailers to instantaneously provide attractive discount for any selected product mix. Incorporating real-time pricing capability significantly enhances the information available and provides better customer service.

3. The optimization model and heuristics developed in this paper advance the viability of real-time online pricing. It achieves near optimal solutions in a negligible time that satisfies the online interactive environment, while helping customers to make better decisions.

The rest of the paper is organized as follows. In §2, we review the literature, and identify the unique characteristics of our model; §3 proposes a nonlinear mixed-integer programming model and solution methodology to interactively solve the online adaptive bundle pricing problem. A computational study is conducted in §4 to understand the benefits of the proposed ODBP model.

In §5, we conduct sensitivity analyses and examine the robustness of the model. A summary, conclusions, and future research are given in §6.

2. Literature Review

Customers are key to firm survival; as a result several methods have been proposed to attract new patrons and to maintain old ones. Lu and Moorthy (2007) studied the application conditions of coupons and rebates, while Subramaniam and Venkatesh (2009) employed auction-based models. Among the available tactics, ORS and pricing strategies are two effective strategies that are gaining popularity. In this section, we review the theoretical and practical models of both.

2.1 Online Recommendation Systems (ORS)

An ORS is a decision aid that analyzes customers' prior online behavior and suggests products to meet the needs of a particular customer (Ansari *et al*. 2000). Most ORS gather data to extract information and to understand customer preferences, and then recommend the products most likely to be purchased by the customer based on her preferences as expressed through online behavior (Huang *et al*. 2007). To improve customer acceptance of an ORS, some researchers have focused on ORS design issues to enhance customers' shopping experience (Cooke *et al*. 2002). Others have examined the impact of recommendation systems on customer's buying decisions (Fitzsimons and Lehmann 2004; Fleder and Hosanagar 2009).

A practical online shopping aid should consider both customer need and e-tailer want (Bohte *et al*. 2004). To date, most systems are based on product prices and e-tailers' promotion strategies rather than customers' preferences or savings (Garfinkel *et al*. 2008; Chen *et al*. 2008); none of the ORS or shopbots (price comparison services) has been developed with such integration. Thus the existing systems are less successful in translating recommended products into sales than

the market potential (Wang *et al*. 2007, Wu *et al*. 2008). These findings motivate us to integrate all these aspects in one model so as to address the concerns of both e-tailers and customers.

2.2 Price Differentiation and Bundle Pricing

Price differentiation has been adopted in a number of industries as it is an important strategy that aids customer retention and creates a competitive advantage (Sahay 2008). All else being equal, economists favor price discrimination (differential pricing) since it is generally welfare-enhancing (Varian 1985). There are three degrees of price discrimination. The 1st degree relies on consumer identification. Since it appears discordant with current views on privacy, it is the least likely to be adopted. The 2nd degree discrimination is based on different product versions or quantities and is a more justifiable and acceptable method. Using observable group characteristics, the 3rd degree discrimination separates the market into segments, e.g. business vs. leisure classes, to maximize a seller's profit. It is also a more common market practice.

E-tailers may change prices, either across customers or across products, by dynamically updating the posted prices or by offering auctions or quantity discounts (Kannan and Praveen 2001). For example, Jain and Kannan (2002) study the pricing strategies for information goods online. Khouja and Park (2007) propose that customers with different attitudes towards piracy may be charged differently for digital goods such as music/video. In our model the price difference is due to different product variety and quantity chosen, an example of 2nd degree price discrimination.

With an aim of selling two or more products jointly, bundling is an attractive marketing practice (Ghosh and Balachander 2007; Venkatesh and Mahajan 1993). Bundle pricing research determines whether products should be sold as pure component (only individual products), pure-bundling (only product bundle) or mixed-bundling (individual products and product bundle), and

how to price them. Other researchers have studied bundling subject to the constraints of customer demand, arrival process, supply information, and fixed-price bundles (Bakos and Brynjolfsson 2000, Basu and Vitharana 2009, and Hitt and Chen 2005). Our multi-stage online pricing system allows customers to interactively select the products of their choice and provides shoppers dynamic price menu in real-time.

2.3 Distinctive Features of our ODBP Model

The ODBP model proposed in this research contains a number of advances over prior work. First, the model emphasizes motivating customers, since emphasizing profits only without inspiring customers to participate is less likely to improve sales, *ceteris paribus*. Given the posted prices of the recommended products, ODBP determines bundle prices most likely to entice customers and generate profits. Second, traditional pricing models regard customers' purchasing behavior as a one-stage process, where they decide to buy or not to buy a product or a bundle as a single decision. However, given that online shopping is a multi-stage process (Häubl and Trifts 2000), how to attract customers at each interactive stage becomes very important.

Our model allows shoppers to realistically have flexibility and choice in terms of when and what to put in their shopping cart. The model incorporates customers' view and seeks savings for customers, thereby enhancing customer satisfaction with the likely concomitant increase in e-tailers' sales. Compared with cross-selling (Netessine *et al*. 2006), our bundles are formed freely by online shoppers, and guarantee to derive savings every time a new item is added to the cart.

3. The Online Dynamic Bundle Pricing Model

3.1 Problem Description and Model Assumptions

In this section we formulate the ODBP problem as a nonlinear mixed-integer programming model for e-tailers. Before detailing the model, we first discuss four model assumptions, (i)

following Wu *et al*. (2008), we assume the default reservation price for a bundle is the sum of the reservation prices of individual products in the bundle. For products with dependent reservation prices, e-tailers can employ the superadditive or subaddititive methods to increase or decrease the reservation prices for the bundle (Venkatesh and Kamakura 2003, Jedidi *et al*. 2003). (ii) Customers' purchasing decisions are governed by their consumer surplus, defined as the reservation price minus the price paid. Customers would prefer a product (bundle) that has the highest consumer surplus. (iii) Because the individual posted price is a function of market competition and demand popularity, we deem it the retailer's optimal price which generates the highest possible profit when sold alone. Methods for establishing optimal posted prices can be found in McCardle *et al*. (2007)[1]. (iv) Customers have budget limits that restrict their ability to pay. See Ulkumen *et al*. (2008) for methods to estimate customer budgets.

Two types of data are needed for our model. One is product information, *e.g.* product cost, posted price, and shipping rate. The other is customer information, *e.g.* purchasing history and reservation price. On the individual level the shopper's online interaction with the system, such as shopping cart contents, needs to be tracked. At the group level e-tailers have to survey or analyze past data to estimate reservation prices, budgets, and customers' shipping preferences. Sufficient transaction records are necessary to generate online recommendations to customers.

To estimate customers' reservation price, Wertenbroch and Skiera (2002), Jedidi *et al*. (2003), Wang *et al*. (2007), and Bitran and Ferrer (2007) have proposed several practical methods. For mature products which have known prices and demand records, e-tailers can use the posted price

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[1] When customers' reservation prices for product g_n follow the distribution $f(x)$ between $[r_l, r_u]$, e-tailers' profits can be determined by $Profit_n = (M * \int_{p_n}^{r_u} f(x)dx) * (p_n - c_n)$. Function $f(x)$ can be $U(u-b, u+b)$, $N(u, \sigma^2)$, or any other shape. The value of $M * \int_{p_n}^{r_u} f(x)dx$ corresponds to the actual demand of product g_n when market size is M and g_n is sold at price p_n. The optimal selling price p_n^* can be found at the point that maximizes profit: $profit^* = Max_{p_n}[(M * \int_{p_n}^{r_u} f(x)dx)] * (p_n - c_n)$.

to approximate the reservation price (see footnote 1). For new products, e-tailers could first estimate reservation prices through a market survey and later adjust them according to the customer's response to the price changes and market condition (Jedidi and Zhang 2002).

3.2 The Proposed Online Dynamic Bundle Pricing (ODBP) Model

Suppose an e-tailer has N products and M potential customers who might patronize the business (Wu *et al*. 2008; Venkatesh and Kamakura 2003). Each product has a posted price, and different customers have different reservation prices for each product. Suppose I products are already in the shopping cart, $G_S = \{g^S_1, \ldots, g^S_i, \ldots, g^S_I\}$ and the corresponding posted prices, costs, reservation prices, and shipping fees are $\{p^S_1, \ldots, p^S_i, \ldots, p^S_I\}$, $\{c^S_1, \ldots, c^S_i, \ldots, c^S_I\}$, $\{r^S_{m,1}, \ldots, r^S_{m,i}, \ldots, r^S_{m,I}\}$, $\{f^S_{m,1}, \ldots, f^S_{m,i}, \ldots, f^S_{m,I}\}$, $m = 1, 2, \ldots, M$, $i = 1, 2, \ldots, I$. The shipping fee $f^S_{m,i}$ is established based on the number of products to ship, customer's shipping option (e.g. express or ground shipping) and the e-tailer's shipping rate. Once the customer places products into the shopping cart and selects a shipping option, the shipping fee of the order can be determined. The bundle price of the shopping cart is p^S, and $p^S = p^S_1$ if there is only one product in the shopping cart. Based on the products in G_S, ORS recommends additional J products, $G_R = \{g^R_1, \ldots, g^R_j, \ldots, g^R_J\}$.

The corresponding posted prices, costs, reservation prices, and shipping fees are $\{p^R_1, \ldots, p^R_j, \ldots, p^R_J\}$, $\{c^R_1, \ldots, c^R_j, \ldots, c^R_J\}$, $\{r^R_{m,1}, \ldots, r^R_{m,j}, \ldots, r^R_{m,J}\}$, $\{f^R_{m,1}, \ldots, f^R_{m,j}, \ldots, f^R_{m,J}\}$, $j = 1,2, \ldots, J$. The budgets of the M potential customers are denoted as $\{b_1, \ldots, b_m, \ldots, b_M\}$. For each recommended product g^R_j in G_R, we combine it with the products already in the cart G_S to form a new bundle, $\{g^S_1, \ldots, g^S_i, \ldots, g^S_I, \boldsymbol{g^R_j}\}$, and determine the best bundle price that would win over a customer purchase while generating profit for the e-tailer. In all, J different new bundles are formed and J bundle prices are calculated by the ODBP in each shopping stage. Customers will

presumably pick one out of the J candidate new bundles and continue to the next shopping stage. We use X_m, a binary decision variable, to denote whether customer m will purchase the specific bundle under consideration. Another decision variable p is the bundle price for bundle $\{g^S_1,\ldots, g^S_i,\ldots, g^S_I, g^R_j\}$. X_m is dependent on the price p. The percentage of customers who actually made the purchase among all M shoppers is: $\frac{\sum_{m=1}^{M} X_m}{M}$. In addition, p_{-i} is the bundle price before product g^S_i is added to the bundle, i.e. p_{-i} is the price of bundle $\{g^S_1, \ldots, g^S_{i-1}, g^S_{i+1}, \ldots, g^S_I, g^R_j\}$.

The objective of the ODBP model is to maximize profit for the e-tailer when recommending product g^R_j to potential customers who have already picked G_S. Thus, the e-tailer's profits are the sum of profits obtained from all customers who would choose to buy g^R_j and Gs:

$$\max \sum_{m=1}^{M} (p - (\sum_{u=1}^{I} c^S_u + c^R_j)) X_m \qquad (1)$$

The profit obtained from each customer is the difference between the bundle price (p) and the total bundle cost, which includes the cost of all products already in the cart: $\sum_{u=1}^{I} c^S_u$, and the cost of the newly recommended product c^R_j. The binary variable, X_m, in the objective function (1) equals "1" if customer m chooses to buy the bundle $\{g^S_1,\ldots, g^S_i,\ldots, g^S_I, g^R_j\}$ at price p, and "0" otherwise. Therefore, among the M shoppers in the market, only those with decision variables X_m equal to "1" are the actual buyers.

A serious difficulty faced by firms is that deep discounts offered to boost sales do not drive enough traffic volume to generate a profit. To avoid such a predicament, we carefully incorporate both product costs and customers' reservation prices into the proposed ODBP model by employing nine constraints. Constraints (2)-(5) determine whether customers would be interested in buying the bundle at price p. The reservation prices for all products customer m placed in the shopping cart is $\sum_{u=1}^{I} r^S_{m,u}$, while $\sum_{u=1}^{I} r^S_{m,u} + r^R_{m,j} - p - f_m$ is customer m's consumer surplus when

buying the recommended bundle $\{g^S{}_1, \ldots, g^S{}_i, \ldots, g^S{}_I, g^R{}_j\}$ at price p and paying shipping fee f_m. To attract customer m to buy the recommended $g^R{}_j$, the customer's consumer surplus derived from $G_S \cup \{g^R{}_j\}$ has to be greater than that generated by buying $g^R{}_j$ alone at the posted price. In other words, the consumer surplus of the new bundle should be no less than that of $g^R{}_j$:

$$[(\sum_{u=1}^{I} r^S_{m,u} + r^R_{m,j} - p - f_m) - (r^R_{m,j} - p^R_j - f^R_{m,j})]X_m \geq 0, \ m=1,\ldots,M \qquad (2)$$

Similarly, customer m's consumer surplus derived from the bundle has to be at least equal to that of buying $g^S{}_i$ individually. Only when the consumer surplus from the bundle is no less than that of purchasing the individual item at price $p^S{}_i$ would the customer keep $g^S{}_i$ in the bundle:

$$[(\sum_{u=1}^{I} r^S_{m,u} + r^R_{m,j} - p - f_m) - (r^S_{m,i} - p^S_i - f^S_{m,i})]X_m \geq 0, \ i=1,\ldots,I, \ m=1,\ldots,M \qquad (3)$$

In constraints (4), customer m would buy the bundle only if her reservation price for the bundle is no less than her expenses:

$$(\sum_{u=1}^{I} r^S_{m,u} + r^R_{m,j} - p - f_m)X_m \geq 0, \ m=1,2,\ldots,M \qquad (4)$$

Constraints (5) assure that customers' actual expenditure is no more than their budget; otherwise, customers cannot afford the products:

$$(p + f_m - b_m)X_m \leq 0, \ m=1,2,\ldots,M \qquad (5)$$

When adding product $g^R{}_j$ to the shopping cart constraint (6) ensures that the marginal bundle price $(p - p^s)$ for the newly recommended product, $g^R{}_j$, is no more than its posted price. It would be irrational to pay a higher marginal price for $g^R{}_j$ in the bundle than to buy the item separately:

$$p - p^S - p^R_j \leq 0 \qquad (6)$$

Besides adding product $g^R{}_j$ to the shopping cart there are additional I ways to form bundle $\{g^S{}_1, \ldots, g^S{}_i, \ldots, g^S{}_I, g^R{}_j\}$. That is, instead of having $g^R{}_j$ as the last item to be added into the cart, customers may actually add $g^S{}_i$ to the cart that already contains $\{g^S{}_1, \ldots, g^S{}_{i-1}, g^S{}_{i+1}, \ldots, g^S{}_I, g^R{}_j\}$. Recall p_{-i} be the bundle price of $\{g^S{}_1, \ldots, g^S{}_{i-1}, g^S{}_{i+1}, \ldots, g^S{}_I, g^R{}_j\}$. Constraints (7) assure that the

marginal bundle price $(p - p_{-i})$ is no more than its posted price p^S_i. The bundle prices for all I cases in equation (7) are the same, since the same contents $\{g^S_1 , \ldots, g^S_{i-1}, g^S_i, g^S_{i+1}, \ldots, g^S_I, g^R_j \}$ are in the bundle. The cart entering sequence of products i does not affect the bundle price:

$$p - p_{-i} - p^S_i \leq 0 , \ i = 1,\ldots, I \tag{7}$$

Constraint (8) guarantees that the bundle price is no less than the total cost of the products in the bundle. Otherwise, e-tailers will incur a loss when selling the bundle:

$$p \geq \sum_{u=1}^{I} c^S_u + c^R_j \tag{8}$$

Constraint (9) ascertains that the bundle price of the shopping cart is no more than the sum of the individual posted price of all products in the bundle:

$$p \leq \sum_{u=1}^{I} p^S_u + p^R_j \tag{9}$$

However, given constraints (6) and (7), constraint (9) becomes redundant, so we will remove it when solving the model. Constraints (10) define X_m as a binary variable, and it equals "1" if customer m purchases the bundle at price p, "0" otherwise:

$$X_m = 0 \text{ or } 1, \ m = 1,\ldots, M \tag{10}$$

Given constraints (2)-(10), the nonlinear mixed-integer ODBP model aims to maximize (1). The optimal bundle price, p, derived by the ODBP is based on the reservation prices of all M potential customers, rather than by a single customer's valuation of the product. This is because in each shopping stage the J bundles and their ODBP derived prices serve as an online price menu; all shoppers are quoted the same price as long as the same bundle contents are selected.

However, if an e-tailer chooses to differentiate prices across customers based on their profile, he can estimate the reservation price distribution for the customer segment and personalize the prices by entering the respective reservation price distributions directly into the ODBP model.

According to the customer's online browsing and purchasing behavior, the e-tailer can also re-match the customer to the appropriate customer segment in real-time. This way, when the online information tracked suggests that the customer may be willing to pay more (or less) than others for that item, the customer's reservation price distributions can be revised accordingly, and ODBP can thus set the bundle price in accordance with customer's online behavior and willingness to pay. In short, when adopting the 1st degree price discrimination or when engaging in target recommendations, the ODBP implementation is the same as before except for the segment match and real-time update of the reservation price distributions. Market survey data and online information tracked are key inputs to dynamically determine the reservation price distribution and thus bundle price for potential customers.

3.3 Illustration of the Multi-Stage Online Shopping Process

Figure 1 illustrates shopper's decision making process and its relationship with the proposed ODBP. Suppose the e-tailer sells 8 products (A-H), with posted prices (p^R_j) of $9.00, $11.99, $16.47, $13.72, $7.53, $6.59, $14.75, and $15.63 respectively. The shopper's reservation price for each product is shown in the 2nd row of each sub-table. For ease of illustration, we assume the shopper chooses ground shipping and the shipping fee is $3.00+$0.99×#products. The Marginal_price_UBL = standalone posted price + marginal shipping fee = p^R_j +$.99; while Marginal_Price_BL = change in bundle price from last stage + marginal shipping fee=$(p - p^s)$+$.99.

After the shopper logs on the ORS recommends products {A, B, C} based on the customer profile and shopping history. To maximize consumer surplus $ (16-11.99-3.99 shipping fees) the shopper selects B. At this point she may check out or continue. If she chooses to continue, the ORS recommends {A, C, D} and the ODBP instantly determines the optimal bundle price for {B, A}, {B, C}, and {B, D}, whose corresponding marginal bundle prices are $8.69, $17.18, and

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$14.51. Note that without the bundle discount the shopper will not buy product A, due to the negative consumer surplus $(9.50-9.99). However, since the marginal price of the bundle ($8.69) is smaller than her reservation price ($9.50) for product A, she is better motivated to buy the bundle. By applying the maximum consumer surplus rule she will choose A among the recommended {A, C, D}; and again in stage 3, choose F owing to its maximum surplus.

After adding products B, A, and F to the shopping cart the customer, now in shopping stage 4, may decide to remove A. Once again, the ODBP is applied to determine the bundle price for {B, F}. Thereafter, the ORS may recommend products {C, E, H}[2] in stage 5.

3.4 Solution Methodology

Three relationships are defined between products: substitutes, complements, and independence (Venkatesh and Kamakura, 2003). For example, to an *HP Laptop*, an *Acer Laptop* is a substitute (Relation 1), a *MP3 Player* is an independent product (Relation 2), and a *Notebook Case* is a complement (Relation 3). Customers generally buy only one of the substitutes at a time, not both. Thus, the following two cases should be solved differently when implementing ODBP.

For relation 1, given that product g^R_j is a substitute for g^S_I, the ODBP should calculate the bundle price of $\{g^R_j\}\cup\{g^S_1, \ldots, g^S_i, \ldots, g^S_{I-1}\}$. For Relations 2 and 3 the recommended product is independent or complementary to the products in the cart. The proposed model can be employed directly to determine the price of the bundle, $\{g^R_j\}\cup\{g^S_1, \ldots, g^S_i, \ldots, g^S_{I-1}, g^S_I\}$. In addition, if customers choose a product not in the recommendation list, the ODBP can also compute the bundle price by combining the selected product with those in the shopping cart.

Note that constraints (7) inherently demand solving the model repeatedly *I* times. If a customer's transaction contains many products, the recurring application of the nonlinear mixed-

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[2] Different products in the cart portray different customer characteristics and thus a unique recommendation list. After removing product *A* from the cart, bundle {*B*, *F*} may communicate new customer attributes and extract a different recommendation list.

integer programming algorithm would require excessive computation time to reach optimality, which is computationally infeasible for a commercial online environment. Since conventional optimization techniques, such as relaxation and decomposition methods (Nowak 2005), are relatively slow and cannot solve our problem in real-time, we propose a quasi-optimal method to achieve a near-optimal solution, as outlined in Figure 2 and described next.

3.4.1. The Quasi-Optimal Method

Although solving small-sized non-linear mixed integer programming problems with optimization software is possible, a drawback is that they are generic tools and do not consider the special structure of the problem in question. Consequently, they require a relatively long solution time. In the online shopping environment waiting time of more than a few seconds is unacceptable and may cause customers to renege. Therefore, rapid response time is essential.

To promptly determine the optimal bundle price p for $\{g^S_1 , \ldots, g^S_i, \ldots, g^S_I, g^R_j\}$, we need to establish the prices of p_{-i}, for $i = 1,2,\ldots,I$. This requires repetitive execution of the procedures in Figure 2, with $\sum_{v=2}^{I-1}{}_I C_v +1$ nested loops to arrive at the solution. Clearly, this would be a time-consuming procedure. To make it practical for an online real-time application, we propose a computationally efficient heuristic method in Figure 3 to establish the price for p_{-i}, which is then used to replace p_{-i} in step 2 of Figure 2.

3.4.2. A Heuristic Algorithm to Expedite the Quasi-Optimal Method

The rationale for the need of a heuristic in Figure 3 is best illustrated with an example. Suppose there are three products: g_1, g_2, g_3 with prices of \$10, \$100 and \$200 respectively. The bundle $\{g_1, g_2, g_3\}$ may be formed in three ways: (i) After the cart has already contained g_2 and g_3, g_1 is added, i.e. $\{g_2, g_3\}\cup\{g_1\}$; (ii) $\{g_1, g_3\}\cup\{g_2\}$; and finally (iii) $\{g_1, g_2\}\cup\{g_3\}$. Suppose the existing bundle price for $\{g_2, g_3\}$ in scenario (i) is \$285. To attract customers the ODBP

model has to increase customer savings when g_1 is added. Therefore $\{g_2, g_3\} \cup \{g_1\}$ should be $<$ **$295** (=$285+$10). Similarly, if $\{g_1, g_3\}$ is $200 and $\{g_1, g_2\}$ $104, the corresponding upper bounds for scenarios (ii) and (iii) would be **$300** (=$200+$100), and **$304** (=$104+$200) respectively. In the end, the value $295 (=Min ${295, 300, 304}) would be the effective upper bound for the bundle price of $\{g_1, g_2, g_3\}$, regardless of which scenario has formed the bundle.

The sequence independence prerequisite is thus imperative, since it would be unacceptable to charge the same bundle, like the above $\{g_1, g_2, g_3\}$, differently just because the product's cart entering sequence differs. Similarly, when removing products from a cart, the price of the cart needs to stay rational. For example, in Figure 1 after removing A from {B, A, F}, the price of the reversely updated cart {B, F} should equal that of {F, B}, regardless of the sequence the items entered the cart. These prerequisites are taken for granted from the shopper's perspective, but they are computationally complex and require extra attention in designing the solution approach.

In Figure 3, we define the upper bound for p_{-i} as $p_{-v} + p_v$, while its lower bound is the sum of the costs of all products in the cart, $\sum_{u=1}^{i-1} c_u^S + \sum_{u=i+1}^{I} c_u^S + c_j^R$. In the heuristic, the nested loop will only be executed $(I-1)$ times to determine p_{-i} and $I \times (I-1)+1$ times to determine p if I is larger than 2. By taking advantage of the problem structure, we develop the heuristic in Figure 3 to provide the p_{-i} for step 2 of Figure 2. The heuristic-based solution approach is effective (accurate) and efficient (with small execution time) when solving the ODBP (see numerical study in §4.2).

4. Numerical Study of the ODBP Model

In this section we study the effectiveness of the proposed ODBP model from the perspectives of both e-tailer profit improvement and customer savings, and compare the ODBP with the existing pricing method. We also investigate the computational efficiency of the proposed

heuristic method so as to understand its suitability for online real-time implementation.

4.1 The Data Sets and Experiment Procedure

4.1.1 Posted Price and Product Costs

Data of the top 100 ranked books from Amazon.com were collected in April 2010, including

book titles and posted prices. Although Amazon.com knows the exact costs of their products, due

to confidentiality we are not able to obtain this cost information. We therefore follow the

literature (Sampson, 2007) and assume book costs are uniformly distributed at U(.60, .80) of the

posted price. Impacts of potential cost variation on e-tailer profits are examined in §5.5.

4.1.2 Reservation Prices

Reservation prices have been widely used in the literature to develop customized pricing

(Chen and Iyer 2002), auction (Yao and Mela 2008), etc. Following Wu *et al.* (2008) and Bitran

and Mondschein (1997), we assume customers' reservation prices are uniformly distributed $U(r_l,$

$r_u)$. The prices posted online by Amazon.com are assumed to give optimal profit since they are

the results of market competition. For uniformly distributed reservation prices, McCardle *et al.*

(2007) show that the optimal selling price, p_n, equals the average of the upper bound of

reservation price and the product's cost, i.e. $p_n^* = \frac{r_u + c_n}{2}$. The upper bound of the reservation

price thus can be derived by $r_u = 2 \times p_n - c_n$. As to the lower bound of reservation prices, we let

$r_l = (1 - \beta) \times p_n$, with β being the range index which signifies the degree of heterogeneity in

customers' valuation of the product. Profits are generated when a customer's reservation price is

greater than p_n. Equally any reservation price lower than p_n will not generate revenue for the e-

tailer. A larger β gives a smaller r_l, which corresponds to a wider dispersion of reservation prices.

Taking the book *The Outliers* as an example, Figure 4 shows three different uniform

reservation price distributions, with β ranging from 0.15 to 0.20 and 0.25 and the posted price p_n

being \$13.72. By assuming that p_n is Amazon's optimal price, we found $r_u = 2 \times p_n - c_n = \$16.60$,

and the lower bounds computed by $r_l = (1 - \beta) \times p_n$ are \$11.66, \$10.98, and \$10.29 respectively.

The probabilities corresponding to areas a, b, and c are 0.58, 0.51, and 0.46, indicating the

chance of selling the product at $p_n = \$13.72$ under different customer valuations. The small

probability in c shows that at a high β, fewer customers are willing to buy the product at p_n. The

values of r_l and r_u under other probability distributions can be derived similarly (see footnote #1).

4.1.3 Shipping Charge

In line with Amazon.com we assume three shipping options: next-day, 2-day, and ground

shipping. Shipping charges are determined by:

Shipping fee per order = "Per Shipment" charge + # items per order × "Per Item" fee.

The "Per Shipment" charge by Amazon.com for the ground, 2-day, and next-day shipping are

\$3.00, \$9.99 and \$12.99, while the "Per Item" fees are \$0.99, \$1.99 and \$4.99, respectively. To

be consistent with our survey results that most customers choose ground shipping when shopping

online, and only a few opt for express shipping, the percentage of customers who require ground,

2-day and next day shipping is estimated at 70%, 20%, and 10% respectively.

4.2 Improvement of E-tailer Profit and Customer Savings

Chen *et al*. (2008) suggest that, in general, customers order no more than eight items in one

transaction. We thus assume the number of products in a bundle follows a uniform distribution

$U(1, 8)$. The recommendation method used is the item-to-item collaborative filtering technology

of Amazon.com (Linden 2003). Similar to that in Figure 1, each customer starts with a list of

recommendations and selects the product that gives her the highest consumer surplus. From that,

the ORS generate another recommendation list, the ODBP determines the corresponding bundle

prices, and the customer picks one to add to the shopping cart. The process continues until she

checks out. Customers may pick items not recommended and can leave the process any time.

To examine the benefit of adopting ODBP, we let M, the number of shoppers, equal 500. Note that M could assume any number. However, a larger M takes a longer time to replicate. Table 1 shows the simulation results, where column (1) displays the e-tailer's total profits when buyers pay at Amazon's prices and column (2) gives the total profits when buyers pay the ODBP bundle prices. The positive values of % profit improvement in Column (3) indicates the ODBP outperforms its counterpart in profit generation, while column (6) represents average customer dollar savings. In addition, we found that profits and savings increase with the bundle size. For example, the % profit improvement increases from 121% (=(860.69-389.41)/389.41) to 386.4% (=(6545.49-1345.65)/1345.65) when bundle size rises from two to eight. Similarly, average savings per customer increases from $3.13 to $12.78 (see column (6)), while the number of new buyers increases from approximately 3.6 to 8.4 times (see column (7)).

Because customers' purchasing decisions are very much influenced by product price and shipping cost, offering an additional savings opportunity for larger bundles is an effective promotion strategy. Rational customers are better motivated to buy larger orders under the ODBP discounts. Thus, although the seller's unit profit decreases, overall profits in ODBP increase due to the greater sales volume. The sizeable increase in profit provides a convincing argument as to the attractiveness of the ODBP strategy and gives e-tailers a clear incentive to adopt the model.

To ensure rapid response time of the ODBP so as to meet the online requirement, we apply the heuristic-based method by replacing Figure 3 for p_{-i} in Figure 2. Table 2 shows that when the bundle is small, both the quasi-optimal method (Figure 2) and heuristics-based method require essentially the same computation time. However, when the size of bundle increases, the computational efficiency of the heuristic-based method is much more pronounced.

The efficiency improvement due to the heuristic-based method is up to 99.8% for the 8-product bundles. While improving the execution efficiency markedly, the heuristic-based method continues to reach the same solution quality as that of the quasi-optimal method, as evidenced by the same bundle prices shown on the rightmost two columns. Thus, the more efficient heuristic-based method is used to solve the ODBP model and for sensitivity analysis next.

5. Sensitivity Analysis

In this section we investigate the robustness of the proposed strategy by examining how uncertainties in input parameters affect the performance of the ODBP model. To understand how sensitive system performances are to possible changes in our assumptions, we first study the impacts of changes in the range and shape of customer reservation price distribution on both the e-tailer and the customers. Subsequently, we examine the performance of ODBP when customers employ different purchase decision rules, and when their budgets vary. Next, we analyze how changes in the cost ratios of products impact the e-tailer's profitability. Finally, a multiple factor sensitivity analysis is conducted to study the overall model robustness.

5.1 Changes in the Range of Reservation Prices

We use the range index, β, to measure the heterogeneity of customers' valuations of a product. A large β indicates there are diverse views regarding the value of the merchandise. Table 3 gives the simulation results with regard to e-tailer profits and to customer savings when β varies from 0.10 to 0.25. We found that a larger β corresponds to a higher % profit improvement. When the range is big, many customers in the market have low valuations of the product (see Figure 4 for example); correspondingly, fewer customers are willing to pay for the product at the posted price. This implies that the price discount from a large bundle has a better chance to attract previously uninterested customers in the market. Therefore, the % profit improvement under a higher β and

larger bundle size is more evident.

Customer savings have shown the same trend. For the 8-product bundles customers' average savings is \$14.72 at β=0.25, while it is \$7.96 when β=0.1. Again, this is because a larger β implies that many shoppers have lower valuations of the product. In order to attract more purchases, the ODBP must increase the discounts offered to customers when product valuation is diverse. Likewise, a higher discount (savings) is needed to lure customers for more purchase.

5.2 Changes in the Distributions of Reservation Price

Four cases are examined to understand the impacts of customers' reservation price distributions. Recall in §4.1.2 we assume that the posted price is optimal when reservation prices are uniformly distributed, from which we determine r_u. To make a fair comparison when reservation prices follow other distributions, we derive the optimal posted prices based on McCardle *et al*. (2007), i.e. $profit^* = \underset{p_n}{Max}[(M * \int_{p_n}^{r_u} f(x)dx)] * (p_n - c_n)$.

Cases 1 (*Normal*) and 2 (*Uniform*) in Table 4 examine the benefits of ODBP when reservation prices are normally and uniformly-distributed, respectively. For the normal distribution, the reservation prices follows $N(u,\sigma^2)$, where u is $(r_u + r_l)/2$, and σ is the standard deviation with $\sigma = (r_u - r_l)/4$. Case 3 (0.3N&0.7U) and Case 4 (0.7N&0.3U) consider the cases where customers' valuations follow different distributions. In Case 3 (Case 4), we randomly select 30% (70%) of the products and assume that customers' reservation prices for those products follow normal distributions, and the other 70% (30%) of the products follow uniform distributions.

Table 4 shows that, regardless of distribution type, the ODBP strategy universally outperforms the unbundling strategy, as evidenced by all the positive values. Under the same bundle size the improvements are comparable among all cases. However, the performance improves with the bundle sizes, *e.g.* for the 8-product bundle in Case 4, the % profit improvement using ODBP is

312.1% and customer savings is $10, much higher than those of the 2-product bundle.

5.3 Changes in Purchase Decision Rules

To examine whether the proposed ODBP remains valid when a customer's purchase decision rule changes, we vary the rule from maximizing consumer surplus to maximizing price savings, defined as sum of all posted prices in the bundle minus the ODBP bundle price. We found in Table 5 that although customers receive slightly better savings when applying the "max price savings" rule due to its direct focus on price, e-tailers can nonetheless obtain comparable % profit improvements when customers employ the surplus rule. This is because the customer's price savings (sum of posted price – bundle price) do not automatically counteract the e-tailer's profit margin (bundle price – bundle cost) and therefore applying the max price savings rule does not necessarily cause more profit decline for e-tailers than applying the surplus rule. Overall, purchase decision rules do not significantly affect e-tailer's profit or customer savings.

5.4 Changes in Budget Level

Suppose customers' budgets follow a normal distribution $N(u_b, \sigma^2)$, with $\sigma^2 = 0.2 \times u_b$. We vary u_b from $80 to $160, and in Figure 5 show the effects of such changes on the e-tailer. Limited by budgets, customers cannot afford certain products even though their reservation prices are higher than the offered prices. Therefore, relative to no budget limitation, the e-tailer's overall profits will naturally fall. For small bundles the % profit improvements at low budget are comparable to that of unlimited budgets. This is because even under a low budget, customers who are interested in the few products can mostly afford and would buy them. Therefore, unlimited budgets do not stimulate many more buyers for small bundles and do not generate extra profits.

However, Figure 5 shows that when the bundle size is large, the % profit improvement is more significant under the lower budget situation, *e.g.* the profit improvement for the 8-product

bundle is 801.0% when u_b is 100, but it is only 537.9% when u_b is 140. For the larger bundle size the ODBP has more room to offer savings opportunities, which, in turn, bring more sales, and make more profits. The simulation results indicate that our model performs significantly better when the budget is tight. This implies that the ODBP is more valuable in improving e-tailers' profit when customers' purchasing power decreases, as in an economic downturn. E-tailers should provide greater discounts during recessions when consumer budgets are low.

5.5 Changes in the Cost Ratio of Products

To understand the impact of product cost changes on ODBP performance, we vary the e-tailer's cost ratio (unit cost/posted price) from 60% to 80%, in increment of 5%. Because the e-tailer has limited room to earn profit when the cost ratio is high, the dollar profit decreases at the rise of the cost ratio (Figure 6a). However, the % improvement by the ODBP grows as the cost ratio increases (Figure 6b). Since at high cost ratio, e-tailers often charge a high price and such a decision would deter potential customers. Using the ODBP bundle discount, the e-tailer would attract more customers, boost sales volume, offset the high costs, and continue to make profits.

From the above experiments we found that the ODBP is most effective when customers have a diverse view about the value of the product (high β), when the customer budget is low, and when the cost ratio is high. It is indifferent to the distribution of reservation prices and to the customer's purchase decision rules. To understand the ODBP's overall quality in withstanding uncertainties in different environment, we conduct a multi-factor sensitivity analysis next.

5.6 Multi-Factor Robustness Study

The ODBP model is deemed robust if it is capable of coping with much uncertainty in its operating environment. Using a $3^4 \times 2$ sensitivity analysis, we conduct 162 experiments for each bundle size to concurrently test five factors: (i) reservation price distribution, (ii) reservation

price range, (iii) product cost ratio, (iv) customer budget, and (v) purchase decision rule (see Table 6). The quantitative factor levels assumed in §4 are varied by ±15% of their average values. The corresponding factor levels for the experiment are: [Uniform, Normal, Mixed=0.5U&0.5N], β=[0.17, 0.20, 0.23], cost ratio=[0.595, 0.70, 0.805], budget=[76.5, 90, 103.5]; and purchase decision rule at two levels: [max consumer surplus, max price savings]. Each experiment (scenario) corresponds to a unique combination of these factors. For example, in Table 6 the first experiment uses uniformly distributed reservation price, with β=0.23, cost ratio=0.805, customer budget=$76.5, and the max price savings decision rule. The profit improvements are shown in the three rightmost columns, whose complete values are graphically displayed in Figure 7.

Figure 7 shows that the highest profit is at $2,882 under the 4-product bundle case, while the smallest profit improvement is $222 under the 2-product bundle case. For ease of illustration and clarity, we only show the results of bundle size of 2-4. Similar patterns are found for the bundle size of 5-8 products, i.e. the profit improvements are all positive and again increase with bundle size. The ODBP model is robust under various bundle sizes and scenarios. As far as % profit improvement is concerned, we found that when confronted with divergent views about product values, lower budgets, and higher cost ratios, the ODBP performs significantly better, while the customers' purchase decision rule is inconsequential, as is the reservation price distribution. These results are consistent with those found in §5.1–5.5.

6. Summary and Conclusions

Attractive and profitable pricing is essential for business survival and success. This paper provides a new approach to promote online customer spending by offering interactive bundling and pricing, contingent on the products chosen by shoppers at various browsing stages. Product bundling is a widely used tactic for differential pricing. But, because the number of prices

increases exponentially with the number of products, pricing all possible bundle combinations and displaying them offline is a practical impossibility. As a result, traditional pricing strategies pre-specify discount rates, bundle sizes and bundle contents in promotions. Such approaches severely limit the choices to customers and their application online would squander the valuable information available from the interactive shopping environment.

The method we proposed starts with an insight into consumer motivation, and ends in a stream of profit enhancement. It includes product selection flexibility in terms of bundle size and product variety, coupled with a dynamic pricing model that integrates customers' preferences, customers' savings, and e-tailers' profits. Furthermore, the incorporation of customers' multi-stage purchasing behavior in the decision process and the development of the heuristics, afford the ODBP model the capability to provide real-time online pricing information that appeals to customers regardless of the mixture of the products they choose.

The proposed model ensures that the price presented online is independent of the sequence of products entering into the shopping cart ─ customers see the same price for the same bundle. Any product combination is allowed in the shopping cart, and an extra discount is guaranteed when additional products are selected. The price of the customized bundle can be prompted instantaneously online. To sensibly implement the proposed model we design a heuristic-based solution procedure which is capable of arriving at a near-optimal bundle price with negligible computation time. The numerical studies show that the proposed model is a win-win strategy. It offers monetary savings for customers, enhances product differentiation with numerous discount scenarios, helps firms gain competitive advantage, and ultimately enhances e-tailers' profits.

In terms of future research one possibility is that in actual applications, e-tailers may adjust the selling price according to their inventory level. They could provide a bigger discount when the

inventory level of a product is high and a smaller discount when the inventory level is low. Incorporating a product's inventory level to the dynamic bundle pricing strategy could be a future extension to our model. Of course, such an extension would only be of value for sellers of tangible goods. Sellers of digital goods, e.g. information goods such as software, videos, news reports, stock prices, etc., would not have inventories subject to this type of constraint. Another possible extension is to link the ODBP model with other marketing strategies, such as coupon offering. For sellers to make the optimal coupon offering decision endogenously, it is necessary to have information about the market response of the deal-prone segment, the number of customers who are loyal to the brand, the profit margin and the cost of managing coupons. Integrating coupon offering decision with the ODBP sequential bundle pricing model through understanding the market structure would be an important and interesting research topic.

In recent years there has been a burst of activities in online retailing. As is typical in the adoption of information technology the initial applications are inclined to model the virtual world implementations directly after their real world analogs. As e-commerce progresses we expect e-tailers to actively take advantage of the unique characteristics of the online environment and, in particular, the opportunity to dynamically customize and price goods in ways that benefit both sellers and buyers.

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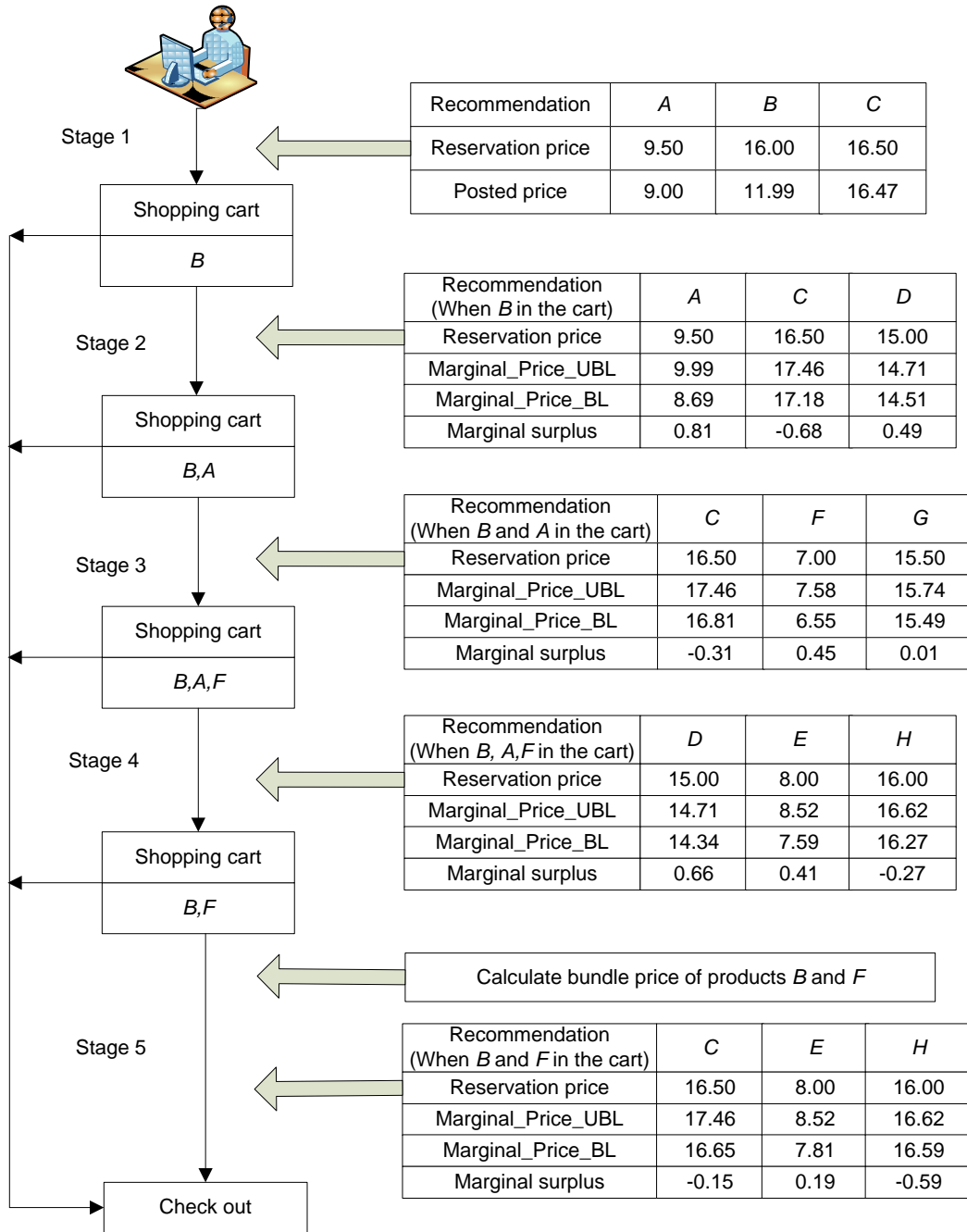Yao, S. and C.F. Mela. 2008. Online auction demand. *Marketing Sci.* **27**(5) 861-885.

Stage 1

Recommendation	*A*	*B*	*C*
Reservation price	9.50	16.00	16.50
Posted price	9.00	11.99	16.47

Shopping cart
B

Stage 2

Recommendation (When *B* in the cart)	*A*	*C*	*D*
Reservation price	9.50	16.50	15.00
Marginal_Price_UBL	9.99	17.46	14.71
Marginal_Price_BL	8.69	17.18	14.51
Marginal surplus	0.81	-0.68	0.49

Shopping cart
B,A

Stage 3

Recommendation (When *B* and *A* in the cart)	*C*	*F*	*G*
Reservation price	16.50	7.00	15.50
Marginal_Price_UBL	17.46	7.58	15.74
Marginal_Price_BL	16.81	6.55	15.49
Marginal surplus	-0.31	0.45	0.01

Shopping cart
B,A,F

Stage 4

Recommendation (When *B, A,F* in the cart)	*D*	*E*	*H*
Reservation price	15.00	8.00	16.00
Marginal_Price_UBL	14.71	8.52	16.62
Marginal_Price_BL	14.34	7.59	16.27
Marginal surplus	0.66	0.41	-0.27

Shopping cart
B,F

Calculate bundle price of products *B* and *F*

Stage 5

Recommendation (When *B* and *F* in the cart)	*C*	*E*	*H*
Reservation price	16.50	8.00	16.00
Marginal_Price_UBL	17.46	8.52	16.62
Marginal_Price_BL	16.65	7.81	16.59
Marginal surplus	-0.15	0.19	-0.59

Check out

Figure 1. Illustration of the Role of the ODBP in the Shopper's Decision Making Process

Note that Marginal_Price_UBL = standalone posted price + marginal shipping fee= p^R_j +$.99
Marginal_Price_BL = change in bundle price from the last stage + marginal shipping fee=$(p - p^s)$+$.99
Marginal surplus=Reservation price of individual product - Marginal_Price_BL

1 Calculate the lower bound *lowBound* of p according to constraint (8).

$$lowBound = \sum_{i=1}^{I} c_i^S + c_j^R.$$

2 Calculate the upper bound *upBound* of p according to constraints (6), and (7).

  For $i = 1$ to I

    Calculate the bundle price p_{-i} of the products $\{ g^S_1, \ldots, g^S_{i\text{-}1}, g^S_{i+1}, \ldots, g^S_I, g^R_j \}$.

  End For

$$upBound = min\{ \{ p_{-i} + p_i^S, \ i = 1, \ldots, I \} \cup \{ p^S + p_j^R \} \}.$$

3 Search the optimal price with the following fixed step length method

  Set the step length *stepLen* in the search of optimal price to a constant integer.
  Initialize the alternative price point: *altPrice = upBound*.
  Initialize the maximum profit and intermediate variable of profit: *maxProfit = altProfit = 0*.
    While *altPrice >= lowBound*, Do
      Count the number *NC* of customers whose budget and reservation prices for the bundle are
        both larger than the sum of *altPrice* and shipping fee f_m.
      Calculate the intermediate variable of maximum profit: *altProfit = NC×(altPrice – cost)*.
        Real number *cost* is the cost of the bundle.

4   If *altProfit* is larger than *maxProfit*
        p = *altPrice*; *maxProfit = altProfit*.
      End If
      altPrice = altPrice – (upBound – lowBound)/stepLen
    End While

Figure 2. The Quasi-optimal Method for the Proposed ODBP Model

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The heuristic method to calculate the bundle price p_{-i}:

Calculate the lower bound of p_{-i}:

$$lowBoundP_{-i} = \sum_{u=1}^{i-1} c_u^S + \sum_{u=i+1}^{I} c_u^S + c_j^R.$$

Calculate the upper bound of p_{-i} as follows:
  Find the product g^S_v from $\{ g^S_1, \ldots, g^S_{i\text{-}1}, g^S_{i+1}, \ldots, g^S_I, g^R_j \}$ that has the lowest price.
    Calculate the upper bound of p_{-i} as follows:

$$upBoundP_{-i} = p_{-v} + p_v.$$

  where p_{-v} is the optimal price for the product bundle:
    $\{ g^S_1, \ldots, g^S_{i\text{-}1}, g^S_{i+1}, \ldots, g^S_I, g^R_j \} - \{ g^S_v \}$,
  which is also calculated by the heuristic method.
Search the optimal price of p_{-i} in [$lowBoundP_{-i}$, $upBoundP_{-i}$] with the fixed step length method.

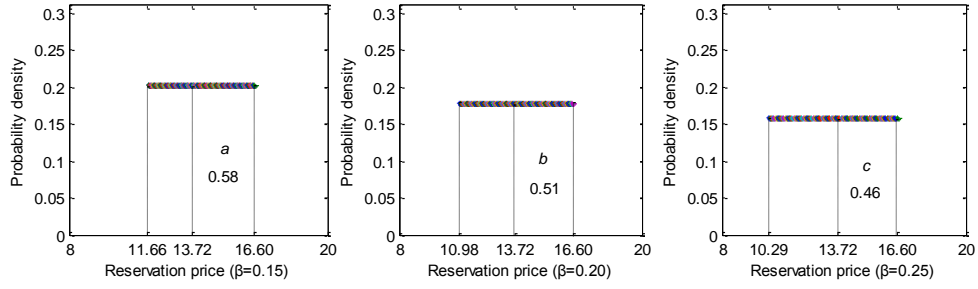*Figure 3*. The Heuristic Method to Calculate p_{-i} in step 2 of Figure 2

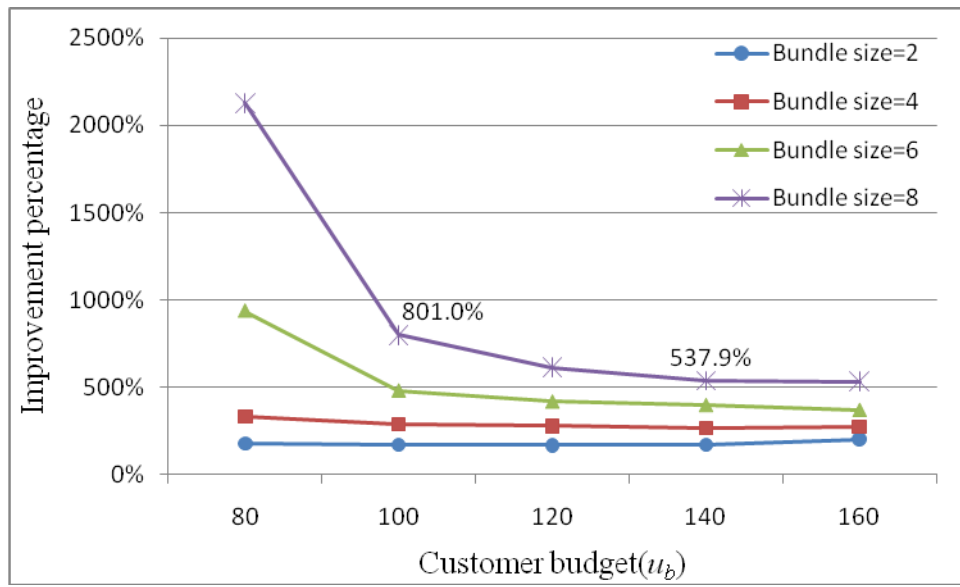*Figure 4.* The Reservation Price $U(r_u, r_l)$ under Uniform Probability Distribution

Figure 5. The Impact of Customers' Budgets on the E-tailer's Profits
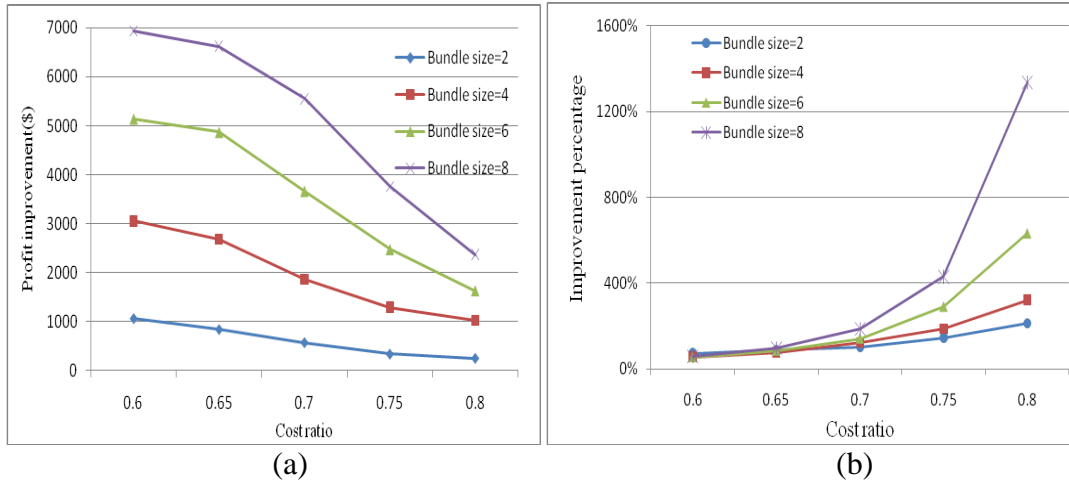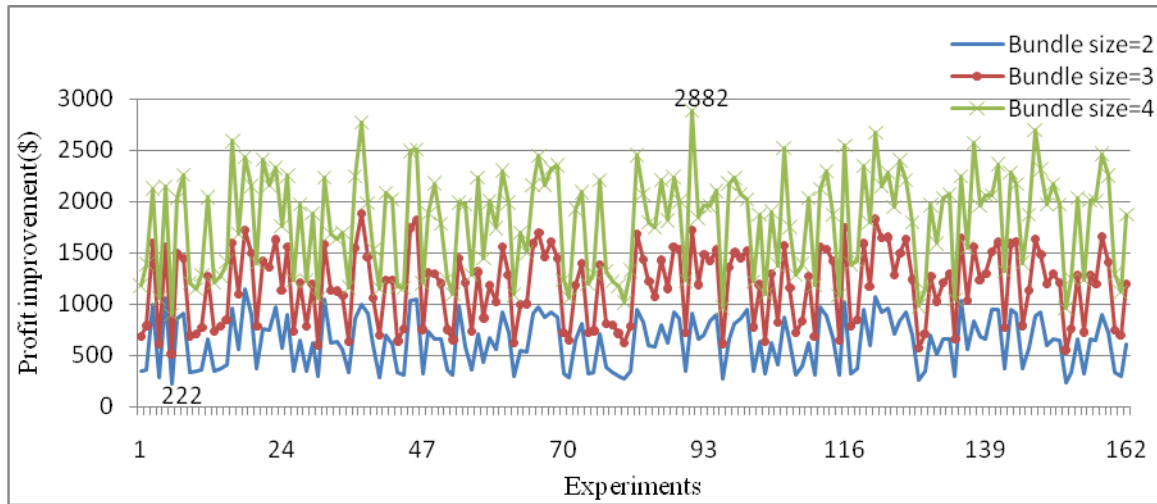
Figure 6. The Impact of E-tailer's Cost Ratio

Figure 7. Profit Improvements under Multiple-factor Sensitivity Analysis

Table 1. E-tailer's Profits and Customers' Savings By the Proposed Strategy

# of products	E-tailer's profits			Average customer savings in $			(7) % of New Buyers
	(1) Unbundling profit	(2) Bundling Profit	(3)= ((1)-(2))/((1) Profit improvement	(4) Unbundling price	(5) Bundling price	(6)= (4)-(5) ODBP Savings	
2	389.41	860.69	121.0%	30.29	27.16	3.13	358.3%
3	678.43	1698.98	150.4%	46.53	41.72	4.81	406.4%
4	1033.27	2702.22	161.5%	61.44	55.18	6.26	452.6%
5	1177.21	3737.69	217.5%	76.33	68.32	8.01	515.1%
6	1396.55	4729.40	238.6%	90.60	81.09	9.51	554.2%
7	1256.96	5624.85	347.5%	105.85	94.54	11.31	770.3%
8	1345.65	6545.49	386.4%	120.33	107.54	12.78	844.3%

Table 2. Comparison of the Heuristic Method and Quasi-optimal Method

# of products	Execution time (sec.)		Efficiency improvement	Average Optimal price in $	
	Quasi-optimal method	Heuristic-based method		Quasi-optimal method	Heuristic-based method
2	0.000	0.000	0.0%	27.16	27.16
3	0.000	0.000	0%	41.72	41.72
4	0.004	0.001	75%	55.19	55.18
5	0.017	0.003	82.3%	68.32	68.32
6	0.105	0.005	95.2%	81.09	81.09
7	0.729	0.008	98.9%	94.55	94.54
8	6.215	0.013	99.8%	107.54	107.54

Note: The Quasi-optimal method is given in Figure 2.
       The Heuristic-based method uses Figure 3 to replace p_{-i} in Figure 2.

Table 3. Impact of Changes in the Range of Reservation Prices

# of products \ β	E-tailer's profit improvement				Customers' savings			
	0.1	0.15	0.2	0.25	0.1	0.15	0.2	0.25
2	97.4%	118.1%	121.0%	156.3%	2.83	3.01	3.13	3.13
3	94.0%	127.2%	150.4%	175.0%	3.84	4.32	4.81	5.01
4	94.7%	123.3%	161.5%	215.6%	4.62	5.44	6.26	6.84
5	110.8%	154.2%	217.5%	308.6%	5.58	6.85	8.01	9.03
6	105.4%	166.5%	238.6%	357.3%	6.16	8.00	9.51	10.63
7	130.1%	215.1%	347.5%	512.6%	7.28	9.44	11.31	12.99
8	128.1%	239.4%	386.4%	623.8%	7.96	10.50	12.78	14.72

Table 4. Impacts of the Distribution Type of Reservation Prices

# of products \ Distribution	E-tailer's profit improvement				Customers' savings			
	Case 1: Normal	Case 2: Uniform	Case 3: 0.3N&0.7U	Case 4: 0.7N&0.3U	Case 1: Normal	Case 1: Uniform	Case 1: 0.3N&0.7U	Case 1: 0.7N&0.3U
2	191.8%	121.0%	152.1%	159.10%	2.73	3.13	2.83	2.59
3	190.3%	150.4%	177.2%	177.70%	4.07	4.81	4.45	4.15
4	201.0%	161.5%	200.5%	206.70%	5.19	6.26	5.85	5.43
5	224.9%	217.5%	242.0%	228.70%	6.29	8.01	7.23	6.64
6	258.5%	238.6%	284.4%	255.00%	7.25	9.51	8.51	7.81
7	298.0%	347.5%	331.6%	283.40%	8.19	11.31	9.83	8.92
8	325.6%	386.4%	350.9%	312.10%	9.18	12.78	10.98	10.00

Table 5. Impacts of Purchase Decision Rule

Decision rule / # of products	Applying maximum consumer surplus rule		Applying maximum price savings rule	
	E-tailer's profit improvement	Customers' $ savings	E-tailer's profit improvement	Customers' savings
2	121.0%	3.13	111.2%	3.48
3	150.4%	4.81	140.6%	5.20
4	161.5%	6.26	155.4%	6.68
5	217.5%	8.01	212.3%	8.41
6	238.6%	9.51	247.1%	9.91
7	347.5%	11.31	341.1%	11.82
8	386.4%	12.78	395.5%	13.20

Table 6. A $3^4 \times 2 = 162$ Sensitivity Analysis and Results

ID	Level for sensitivity factors					Profit improvement		
	Reservation price distributions	Reservation Price range (β)	Cost ratio	Customer budget	Purchase decision rule	Bundle size =2	Bundle size =3	Bundle size =4
1	Uniform	0.23	0.805	76.5	Price Saving	346.25	680.44	1179.73
2	Uniform	0.17	0.805	76.5	Price Saving	396.03	832.05	1387.76
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
38	Mixed	0.20	0.595	90	Surplus	914.17	1454.0	1980.61
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
91	Uniform	0.23	0.595	76.5	Surplus	905.29	1716.2	2882.23
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
162	Normal	0.20	0.70	103.5	Surplus	638.26	1188.9	1872.89