August 30, 2007

Chem. 1410

Problem Set 1, due Sept. 10, 2007

Do the following problems from Engel; these are *not* to be handed in for grading; solutions will be distributed via .pdf.

Chapter 1: P1.2, P1.7, P1.12, P1.17, P1.19

The following two problems are to be handing in for grading:

(1) **Density of states of electromagnetic modes in a blackbody cavity**: In class we asserted that at absolute temperature T the density of electromagnetic states ("modes") inside a blackbody is

$$D_\lambda(\lambda) = \frac{8\pi}{\lambda^4} \qquad [1]$$

(Note: the units of $D_\lambda(\lambda)$ are "# modes/[unit volume][unit wavelength]".) This means that in a small wavelength interval $[\lambda - \Delta\lambda/2, \lambda - \Delta\lambda/2]$, the number of electromagnetic modes per unit volume is $D_\lambda(\lambda)\Delta\lambda$. Show that Eq. [1] is equivalent to the following density of states in frequency space:

$$D_\nu(\nu) = 8\pi\nu^2 / c^3 \qquad [2]$$

where c is the speed of light. (Note: the units of $D_\upsilon(\upsilon)$ are "# modes/[unit volume][unit frequency]".)

The following remarks may be of use:

i) $\nu = c/\lambda$.

ii) $D_\lambda(\lambda)\Delta\lambda = D_\nu(\upsilon)\Delta\nu$, where $\Delta\lambda$ is a small wavelength interval and $\Delta\nu$ is the corresponding small frequency interval. [Why?]

<u>Note</u>: Eq. [2] is employed directly in Engel Eq. 1.1.

2) **Average thermal energy in a harmonic oscillator**. According to the principles of statistical physics, the relative probability to find a system in state j characterized by energy E_j is $\exp(-E_j / k_B T)$, where T is the absolute temperature and k_B is Boltzmann's constant.

a) In the case where the system is an electromagnetic cavity mode frequency ν, the allowed states j=0,1,2,… correspond to energy $E_j = jh\nu$ (i.e,. 0,1,2,… photons in the mode). Thus, the average thermal energy of a cavity mode of frequency ν at absolute temperature T is given by:

$$< E(\upsilon) >= h\nu \sum_{j=0}^{\infty} je^{-jh\nu / k_B T} / \sum_{j=0}^{\infty} e^{-jh\nu / k_B T} \ .$$

Why?

b) To evaluate $< E >$ explicitly, write $< E(\nu) >= h\nu N(\upsilon) / D(\upsilon)$, with

$$N(\nu) = \sum_{j=0}^{\infty} je^{-jh\nu / k_B T} \ \ ; \ \ D(\nu) = \sum_{j=0}^{\infty} e^{-jh\nu / k_B T}$$

i) Show that:

$$D(\upsilon) = [1 - e^{-h\nu / k_B T}]^{-1}$$

[Hint: D is a simple geometric series.]

ii) Show that

$$N(\nu) = \frac{-k_B T}{h} \partial D(\upsilon) / \partial \upsilon$$

iii) Combine the results of i) and ii) to obtain Eq. 1.5 of Engel.