Cooling Constraints on Planet Formation via Gravitational Instabilities

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Abstract

In the conventional model of planet formation, small solid particles in a protoplanetary disk come together to form ever larger particles. Eventually, these objects become large enough that they are able to gravitationally accrete material. However, there is an alternative scenario where planets can form through direct fragmentation of the protoplanetary disk via gravitational instabilities (Kuiper 1951; Cameron 1978; Boss 1998, 2000). For this second scenario to occur, the disk must be unstable to self-gravitation; a situation quantified by the Toomre Q parameter (Toomre 1964). In addition, for an instability to collapse, it must be able to cool within a few orbital periods (Rice et al. 2003). We use realistic dust opacities and Monte Carlo radiative transfer to calibrate our cooling time calculations. We are thus able to bracket disk surface densities that allow gravitational instability as a function of orbital distance. This allows us to place limits on which regions of a protoplanetary disk allow planet formation via gravitation instability. Additionally, we explore the effects that different stellar and disk parameters have on our surface density limits. We find that for a typical T Tauri star, gravitational instabilities are allowed for distances greater than ~40AU. Variations in the disk temperature structure and opacity can modify this value by a few to tens of AU. FU Ori outbursts potentially allow an "island" of instabilities at distances of a few AU.

Physical Model

- Star + flared circumstellar disk
- Power-law surface density and scale height
- Temperature structure from Whitney et al. (2003)
- Rosseland and Planck mean dust opacities from Semenov et al. (2003)

Cooling Time

Analytic

We construct an analytic expression for the cooling time of a small temperature perturbation (i.e. ΔT << T) within a circumstellar disk.

Assumptions:
- Plane-parallel
  - 1+1D
- Externally illuminated
- Gray atmosphere
  - Eddington approx.
- Optically thick and thin limits

A temperature perturbation corresponds to an increase in internal energy, ΔE, as well as an increase in luminosity, ΔL. The cooling time is given by

$$t_{cool} = \frac{\Delta E}{\Delta L}.$$

In the optically thick and thin limits, the above evaluates to

$$t_{cool}^{thick} = \frac{c_s^2}{\gamma - 1} \frac{1}{\sigma T^4} \frac{1}{\chi_R} \left(\frac{3 z'(1-z')\tau^2}{32} + \frac{\tau}{4} \right)$$

and

$$t_{cool}^{thin} = \frac{c_s^2}{\gamma - 1} \frac{1}{\sigma T^4} \frac{1}{\kappa_P} \frac{1}{16} \left(\frac{d \ln(\kappa_P)}{d \ln(T)} + 1 \right)^{-1}.$$

Where z' is τ'/τ, i.e. the optical depth of the perturbation divided by the total optical depth of the disk (z'=0.5 is the disk midplane).

To account for the transition from thick to thin, we use the approximation

$$t_{cool} \approx t_{cool}^{thick} + t_{cool}^{thin}.$$

Numerical Radiation Transfer

To test the accuracy of our analytic cooling time, we use a numerical radiative transfer code and compare the results.

Code Details
- Monte Carlo with wavelength-dependent dust opacity
  - Dust opacities from Wood et al. (2002)
- 1+1D

Comparisons are made to both gray and non-gray atmospheres to test the effect this assumption has on our analytic cooling time (fig. 1).

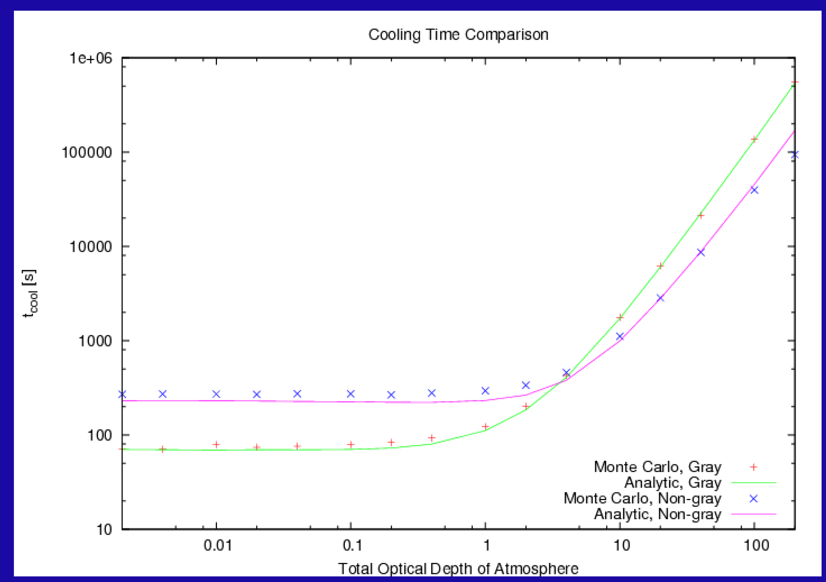


Fig. 1 – A comparison of our analytic cooling time to the results of a 1+1D Monte Carlo code. The perturbation is located at the disk midplane. Note that the non-gray case is only slightly less accurate than the gray case.

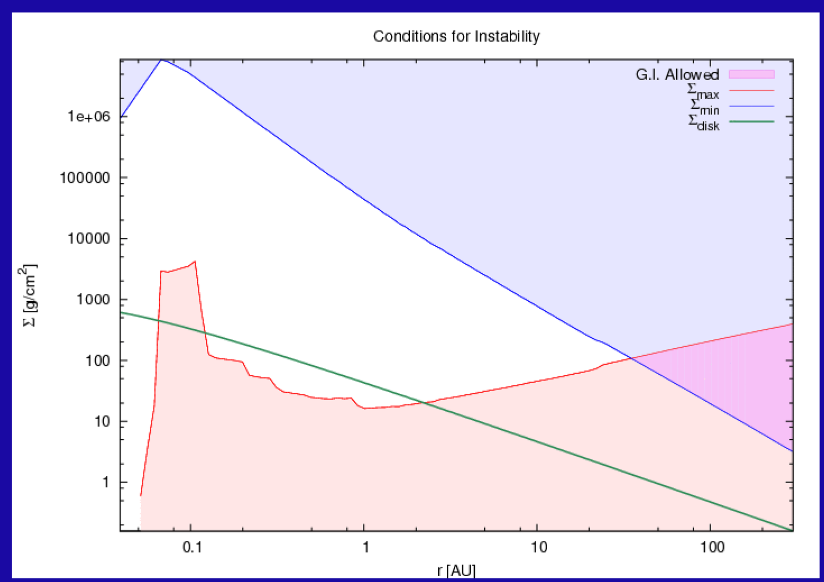
Conditions for Instability

The formation of gravitational instabilities requires that

$$Q \equiv \frac{\Omega c_s}{\pi G \Sigma} < Q_0 \quad \text{and} \quad t_{cool} < \frac{\xi}{\Omega},$$

Where $Q_0 \approx 1$ (Toomre 1964) and $\xi \approx 3$ (Rice et al. 2003).

These conditions place lower and upper limits, respectively, on unstable disk surface densities as a function of radius.



Parameters
- $M_* = 0.5\ M_\odot$
- $R_* = 2\ R_\odot$
- $T_{eff} = 4000$ K
- $\dot{M} = 7.5 \times 10^{-9}\ M_\odot$/yr
- $M_{disk} = 0.01\ M_\odot$
- $R_{disk} = 300$ AU

Fig. 2 – The results for a typical Class II T Tauri star. The blue line is a lower limit on surface density imposed by the Toomre Q parameter. The red line is an upper limit imposed by t_{cool}. The blue and red shaded regions are surface densities that meet one criteria, but not both. The violet shaded region is where gravitational instabilities are allowed. The green line shows the surface density that corresponds to the minimum-mass solar nebula. Gravitation instability would require a significant surface density enhancement at large radii, such as that found during early evolutionary times.

Modifications to Cooling Time

Clumping

An analytical treatment of clumpy media shows that opacity can be reduced by up to a factor of five at large optical depths (Hobson and Scheuer 1993). We estimate the effect of clumping by decreasing our Rosseland mean opacity by a factor of five.

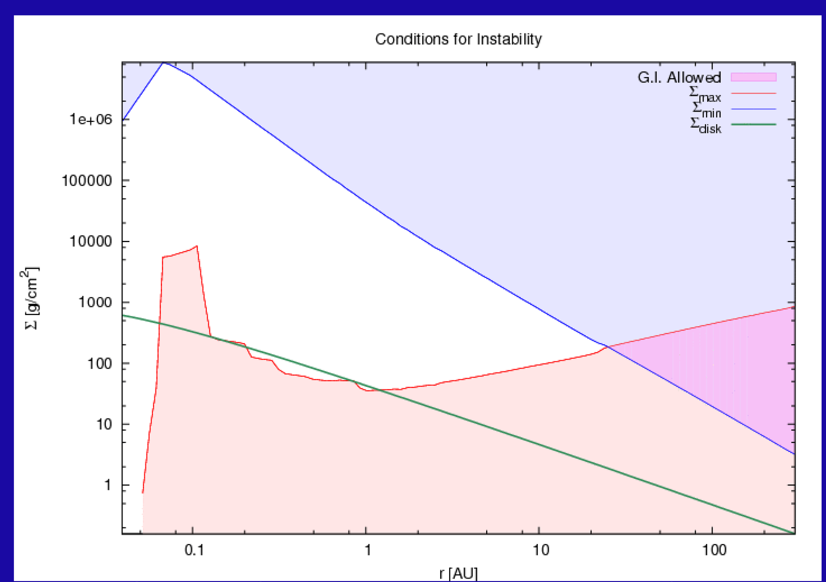


Fig. 3 – Same as fig. 2, but with the opacity modified to simulated clumping. Note that clumping serves to decrease the cooling time, thus potentially allowing gravitational instabilities at slightly smaller radii.

Perturbations at Different Depths

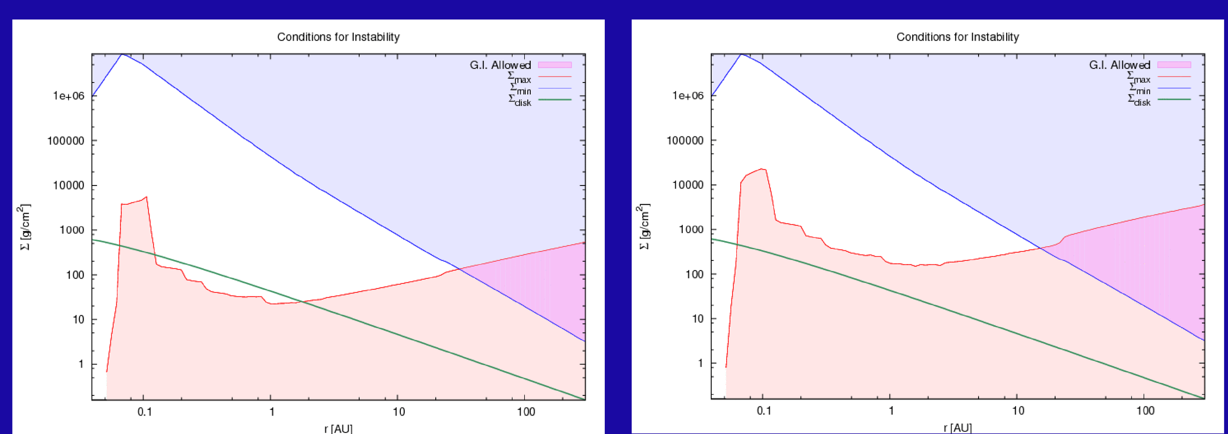
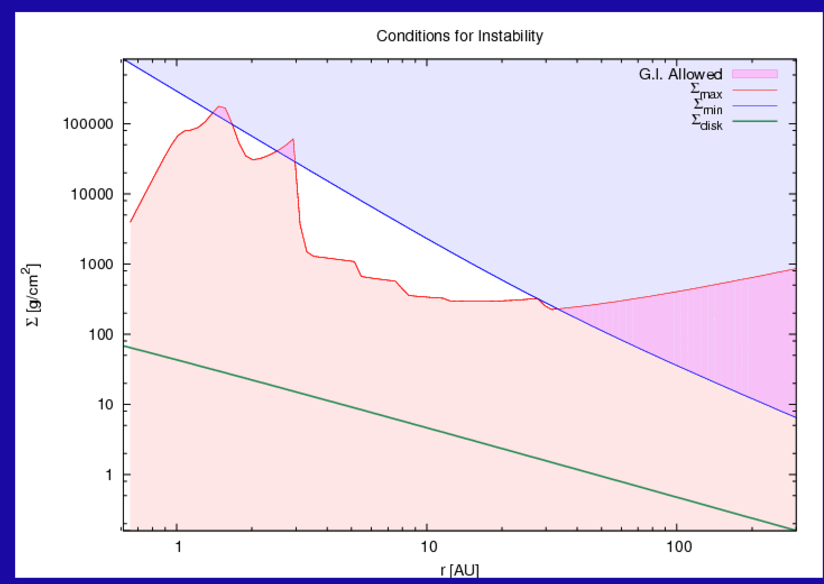


Fig. 4 – Same as fig. 2, but with the perturbation located at one scale height (left) and three scale heights (right) above the midplane. Like clumping, moving the perturbation closer to the surface acts to reduce the cooling time. Note that the potentially unstable region continues to grow as the perturbation approaches the surface.

Instability in FU Ori Outbursts

FU Ori outbursts are interesting to consider, because their greatly enhanced accretion rate increases both the disk temperature and surface density, thus loosening the constraints on gravitational instability.



Parameters
- $M_* = 0.5\ M_\odot$
- $R_* = 2\ R_\odot$
- $T_{eff} = 4000$ K
- $\dot{M} = 1.0 \times 10^{-4}\ M_\odot$/yr
- $M_{disk} = 0.01\ M_\odot$
- $R_{disk} = 300$ AU

Fig. 5 – Same as fig. 2, but for an FU Ori outburst. This circumstellar disk is much hotter than that of a T Tauri star, thus the cooling time is decreased. In addition, the high temperatures cause the opacity to be lowered, further decreasing the cooling time. The result is two new potentially unstable regions at 1.5 and 3 AU. The green line shows the T Tauri surface density for comparison. Note that during an FU Ori outburst, the surface density is likely to be greatly enhanced at small radii due to the increased accretion rate.

Temperature Requirements for Instability

Disk instability requires two conditions:
- $\Sigma_{max} > \Sigma_{min}$
- $\Sigma_{min} < \Sigma < \Sigma_{max}$

The second condition selects a range of disk masses. The first condition depends on temperature, primarily via the opacity, as well as the distance from the star.

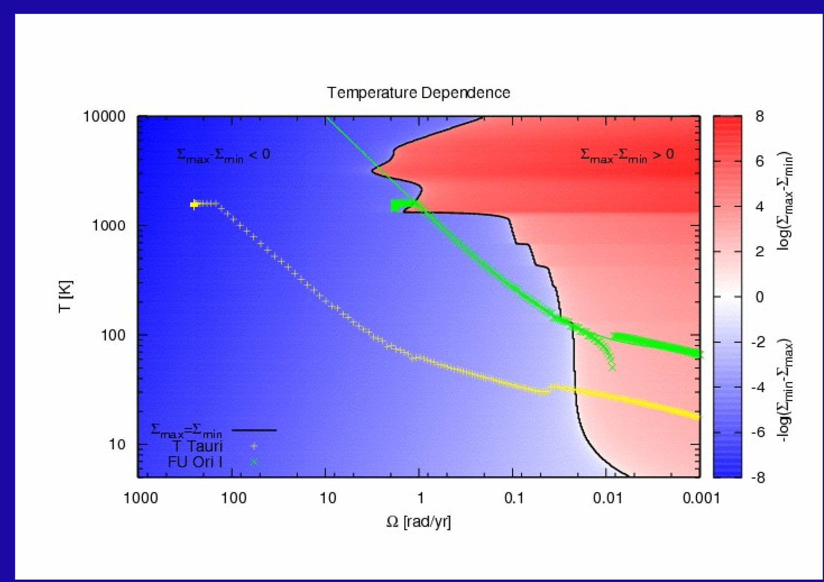


Fig. 6 – A plot of Σ_{max}-Σ_{min} with respect to temperature and distance. Distances are plotted in terms of orbital angular frequency rather than radius to remove the dependence on the stellar mass. The black line corresponds to $\Sigma_{max} = \Sigma_{min}$. Gravitational instabilities are allowed in the red region, and disallowed in the blue region. The temperature structures used for the T Tauri star (yellow) and the FU Ori outburst (green) are plotted for reference. In the case of the FU Ori outburst, a double-power-law fit was used to estimate the disk temperature within the sublimation radius (a limitation imposed by the temperature code we used). Note that it seems impossible to form a gravitation instability at distances less than about 1 AU, regardless of disk temperature and density.

Summary

Gravitational instabilities may occur:
- In the outer regions of a circumstellar disk (greater than a few tens of AU)
- At distances of a few AU, for temperatures around a few 1000 K
- Never at distances less than a ~1 AU
- Only for relatively large surface densities
  - Either local enhancements of 2-3 orders of magnitude over the minimum-mass solar nebula
  - Or a global enhancement leading to disk masses of 1-10 $M_\odot$

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Acknowledgments

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