Wireless Communications and Cellular Network Fundamentals

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Telcom 2700 Slides 4

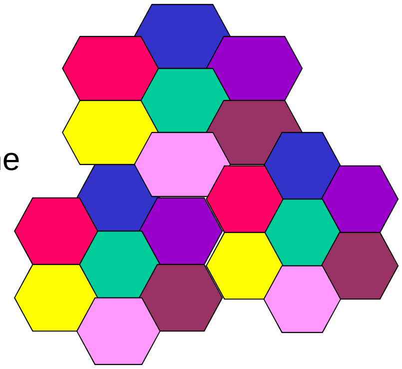
Cellular Concept

Proposed by Bell Labs 1971
Geographic Service divided into smaller "cells"

Neighboring cells do not use same set of frequencies to prevent interference

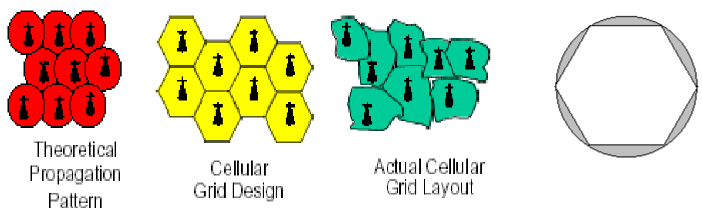
Often approximate coverage area of a cell by a idealized hexagon

Increase system capacity by frequency reuse.



Cellular Networks

- Propagation models represent cell as a circular area
- Approximate cell coverage with a hexagon - allows easier analysis
- Frequency assignment of *F* MHz for the system
- The multiple access techniques translates *F* to *T* traffic channels
- Cluster of cells *K* = group of adjacent cells which use all of the systems frequency assignment



Cellular Concept

- Why not a large radio tower and large service area?
  - Number of simultaneous users would be very limited (to total number of traffic channels T)
  - Mobile handset would have greater power requirement
- Cellular concept - small cells with frequency reuse
  - Advantages
    - lower power handsets
    - Increases system capacity with frequency reuse
  - Drawbacks:
    - Cost of cells
    - Handoffs between cells must be supported
    - Need to track user to route incoming call/message

Cellular Concept (cont)

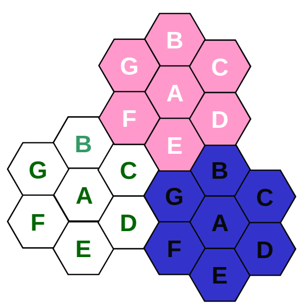
- Let T = total number of duplex channels

  K cells = size of cell cluster (typically 1, 4, 7,12, 21)

  N = T/K = number of channels per cell
- For a specific geographic area, if clusters are replicated M times, then total number of channels
  - system capacity = M *x* T
  - Choice of K determines distance between cells using the same frequencies – termed co-channel cells
  - K depends on how much interference can be tolerated by mobile stations and path loss

Cell Design - Reuse Pattern

- Example: cell cluster size K = 7, frequency reuse factor = 1/7, assume T = 490 total channels, N = T/K = 70 channels per cell



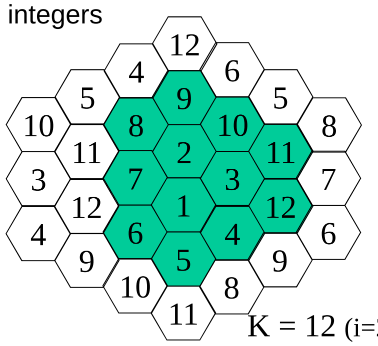
Assume T = 490 total channels, K = 7, N = 70 channels/cell

Clusters are replicated M=3 times

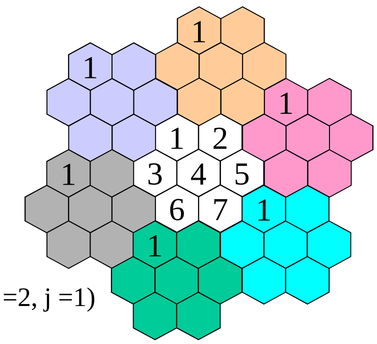
System capacity = 3x490 = 1470 total channels

Cluster Size

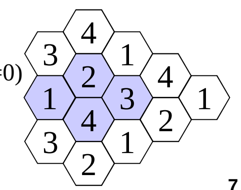
From geometry of grid of hexagons only certain values of K are possible if replicating cluster with out gaps
$K = i^2 + ij + j^2$ where i and j are non-negative integers



K = 12 (i=2, j=2)



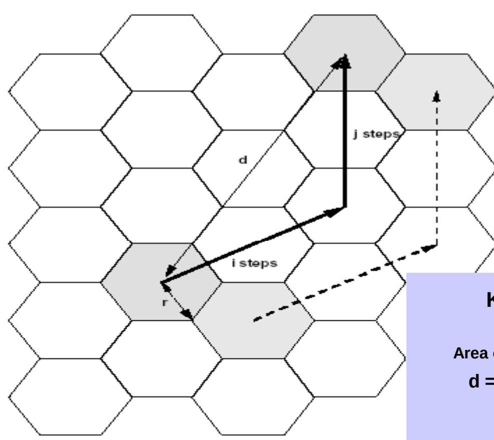
K = 7 (i =2, j =1)



K = 4 (i =2, j=0)

Cellular Concepts

- To find co-channel neighbors of a cell, move i cells along any chain of hexagons, turn 60 degrees counterclockwise, and move j cells (example: i=2, j=2, K=12)



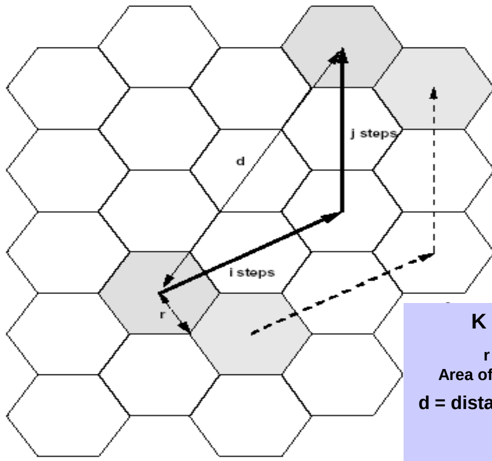
$K = i^2 + ij + j^2$

r = cell radius
Area of hexagon = 2.61 r^2

d = distance to co-channel cell

Cellular Concepts

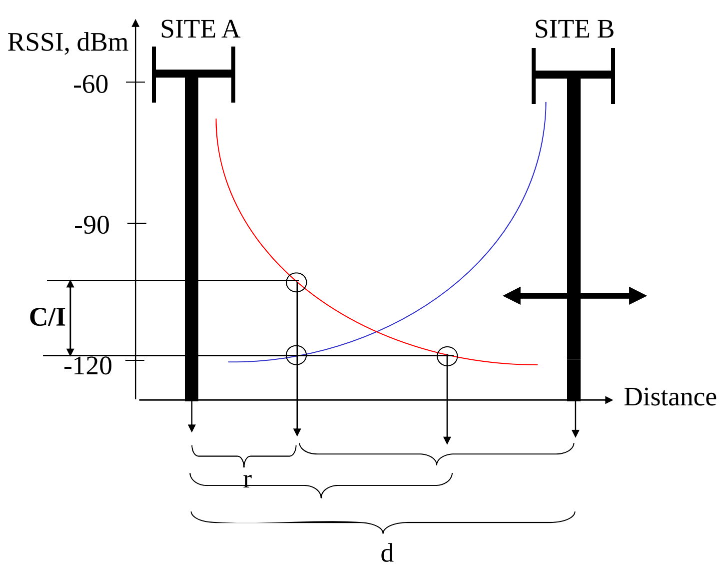
- From hexagonal geometry $d = r\sqrt{3K}$
- The quantity d/r is called the co-channel reuse ratio $d/r = \sqrt{3K}$



$K = i^2 + ij + j^2$

r = cell radius
Area of hexagon = 2.61 r^2
d = distance to co-channel cell

Frequency Reuse



Frequency Reuse

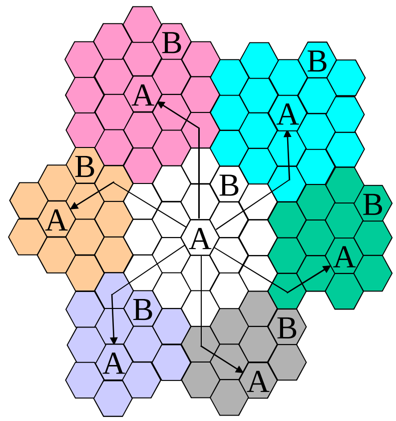
Relate cluster size to carrier to co-channel interference ratio *C/I* at the edge of a cell

propagation model of the form

$$P_r = P_t L d^{-\alpha}$$

L = constant depending on frequency,
d = distance in meters,
α = path loss coefficient,
Then at edge of a cell in center of network the C/I is given by

$$\frac{C}{I} = \frac{P_t L r^{-\alpha}}{\sum_{j=i}^{6} P_t L d^{-\alpha}} = \frac{1}{6}\left(\frac{r}{d}\right)^{-\alpha}$$



K = 19

Frequency Reuse

Solving for *d/r* results in

$$\frac{d}{r} = \left(\frac{6C}{I}\right)^{1/\alpha}$$

Remember $d / r = \sqrt{3K}$, which results in

$$K = \frac{1}{3}\left(\frac{6C}{I}\right)^{2/\alpha}$$

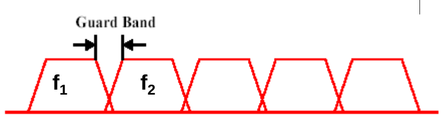
Example: Consider cellular system with a C/I requirement of C/I = 18 dB and a suburban propagation environment with α = 4 , determine the minimum cluster size.
18 dB ➔ 18 = 10log(x) ➔
1.8 = log(x) ➔ x = $10^{1.8}$ ➔
X = 63.0957,

K = 1/3 x (6 x 63.0957)$^{0.5}$ = 6.4857 ,

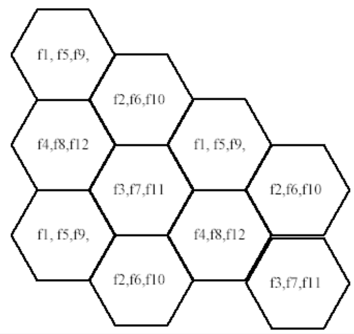
Since K must be an integer round up to nearest feasible cluster size => K = 7

Frequency Assignment

- Typical C/I values used in practice are 13-18 dB.
- Once the frequency reuse cluster size K determined frequencies must be assigned to cells
- Must maintain C/I pattern between clusters.
- Within a cluster – seek to minimize adjacent channel interference
- Adjacent channel interference is interference from frequency adjacent in the spectrum

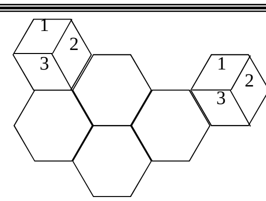


Example: You are operating a cellular network with 25KHz NMT traffic channels 1 through 12. Labeling the traffic channels as {f1, f2, f3, f4, f5, f6, f7, f8, f9, f10, f11, f12}
Place the traffic channels in the cells below such that a frequency reuse cluster size of 4 is used and adjacent channel interference is minimized



Sectoring

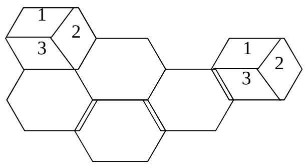
- Sectoring
  - used to improve the C/I ratio
  - make cluster size K smaller



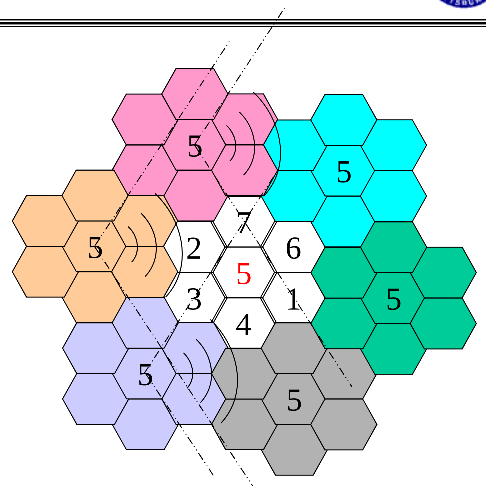
120 sectoring

- Use directional antennas rather than omni-directional
  - cell divided into 3 (120º sectoring) or 6 (60º sectoring) equally sized sectors
- Frequencies/traffic channels assigned to cells must partitioned into 3 or 6 disjoint sets
- Reduces the number of co-channel cells causing interference
- Disadvantages: need *intra-cell* handoff, increases complexity

Sectoring



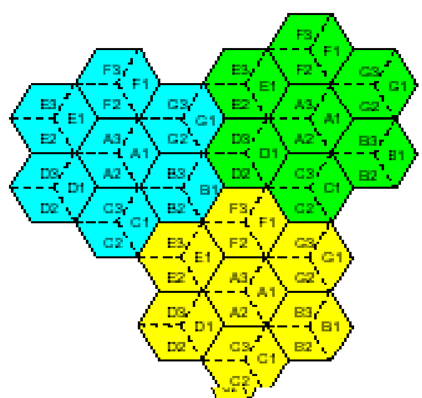
120 sectoring



120° sectoring reduces number of interferers from 6 to 2

Sectored Frequency Planning

- Example: Allocate frequencies for a GSM operator in U.S. PCS B-block who uses a 7 cell frequency reuse pattern with 3 sectors per cell
- Use a Frequency Chart – available from FCC web site
- Groups frequencies into 21 categories Cells A-G and sectors 1-3 in each cell



Frequency Chart . 612-685 represent B-block frequencies for GSM

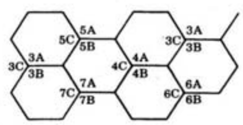
A1	B1	C1	D1	E1	F1	G1	A2	B2	C2	D2	E2	F2	G2	A3	B3	C3	D3	E3	F3	G3
612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632
633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653
654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674
675	676	677	678	679	680	681	682	683	684	685										

Sectored Frequency Planning

- Example: Allocate frequencies for a 1G Analog AMPS operator in cellular B-block who uses a 7 cell frequency reuse pattern with 3 sectors per cell
- Use a Frequency Chart – available from FCC web site
  - Groups frequencies into 21 categories Cells 1-7 and sectors A-B in each cell

Block B

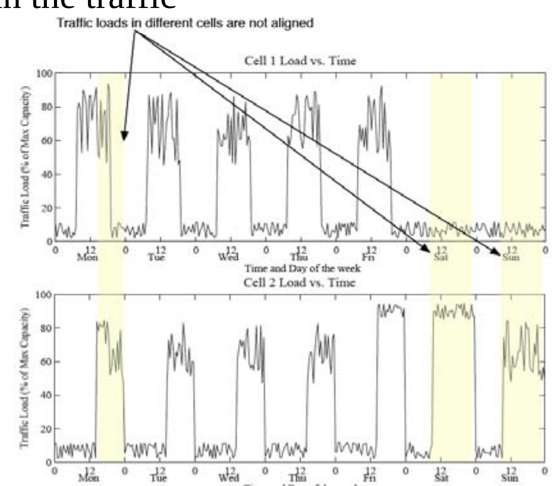
1A	2A	3A	4A	5A	6A	7A	1B	2B	3B	4B	5B	6B	7B	1C	2C	3C	4C	5C	6C	7C
334	**335**	**336**	**337**	**338**	**339**	**340**	**341**	**342**	**343**	**344**	**345**	**346**	**347**	**348**	**349**	**350**	**351**	**352**	**353**	**354**
355	356	357	358	359	360	361														375
376	377	378	379	380	381	382														396
397	398	399	400	401	402	403														417
418	419	420	421	422	423	424														438
439	440	441	442	443	444	445														459
460	461	462	463	464	465	466														480
481	482	483	484	485	486	487														501
502	503	504	505	506	507	508														522
523	524	525	526	527	528	529														543
544	545	546	547	548	549	550														564
565	566	567	568	569	570	571														585
586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606
607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627
628	628	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648
649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	717	718	719
720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740
741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761
762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782
783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799				



*Boldface numbers indicate 21 control channels for Block A and Block B respectively.

Cellular Network Traffic

- Large time of day variations in the traffic
- Spatial variations in the traffic



Traffic Engineering

- Given or N = T/K traffic channels per cell – what is grade of service (GoS) or how many users can be supported for a specific GoS
- Required grade of service?
  - Usually 1-2% blocking probability during busy hour
  - Busy hour may be
    1. busy hour at busiest cell
    2. system busy hour
    3. system average over all hours

- Basic analysis called *Traffic Engineering or Trunking*
  - same as circuit switched telephony
  - use Erlang B and Erlang C Models

Traffic Engineering

- Estimate traffic distribution?
  - Traffic intensity is measured in Erlangs (mathematician AK Erlang)
  - One Erlang = completely occupied channel,
  - Example: a radio channel occupied for 30 min. per hour carries 0.5 Erlangs
- Traffic intensity per user A_u

  A_u = average call request rate x average holding time = λ x t_h
- Total traffic intensity = traffic intensity per user x number of users = A_u x n_u
- Example 100 subscribers in a cell

  20 make 1 call/hour for 6 min => 20 x 1 x 6/60 = 2E

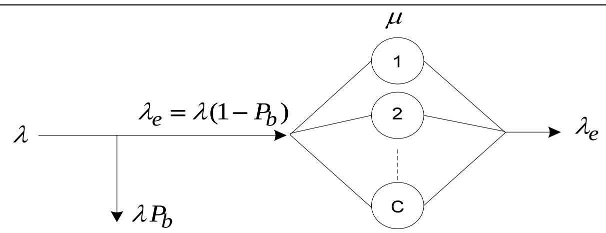
  20 make 3 calls/hour for ½ min => 20 x 3x .5/60 = .5E

  60 make 1 call/hour for 1 min => 60 x 1 x 1/60 = 1E

100 users produce 3.5 E load or 35mE per user

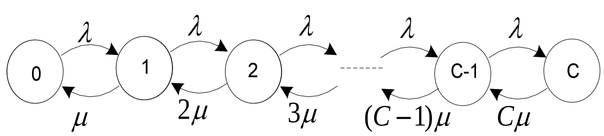
Erlang B Model M/M/C/C queue

- The system has a finite capacity of size ***C***, customers arriving when all servers busy are *dropped* ➔ stable system
- *Blocked calls cleared model (BCC)*
- Assumptions
  - ***C*** identical servers process customers in parallel.
  - Customers arrive according to a Poisson process with mean rate λ
  - Customer service times exponentially distributed with mean rate $1/\mu$
  - Offered load in Erlangs is $a = \lambda/\mu$



M/M/C/C

- Let π_i denote the steady state probability of *i* customers in the system, then the state transition diagram for n(t) is given by



flow out state *j* = flow in state *j*

$$\lambda\pi_0 = \mu\pi_1 \qquad j = 0$$

$$(\lambda + j\mu)\pi_j = \lambda\pi_{j-1} + (j+1)\mu\pi_{j+1} \qquad 1 \le j < C$$

$$(C\mu)\pi_c = \lambda\pi_{c-1} \qquad j = C$$

Normalization condition $\sum_{j=0}^{\infty} \pi_j = 1$

M/M/C/C

Probability of a customer being blocked *B(c,a)*

$$B(c,a) = \frac{\dfrac{a^c}{c!}}{\displaystyle\sum_{n=0}^{c} \frac{a^n}{n!}}$$

B(c,a) ⇐ Erlang's *B* formula, Erlang's blocking formula
Erlang B formula can be computed from the recursive formula

$$B(c,a) = \frac{a \cdot B(c-1,a)}{c + a \cdot B(c-1,a)}$$

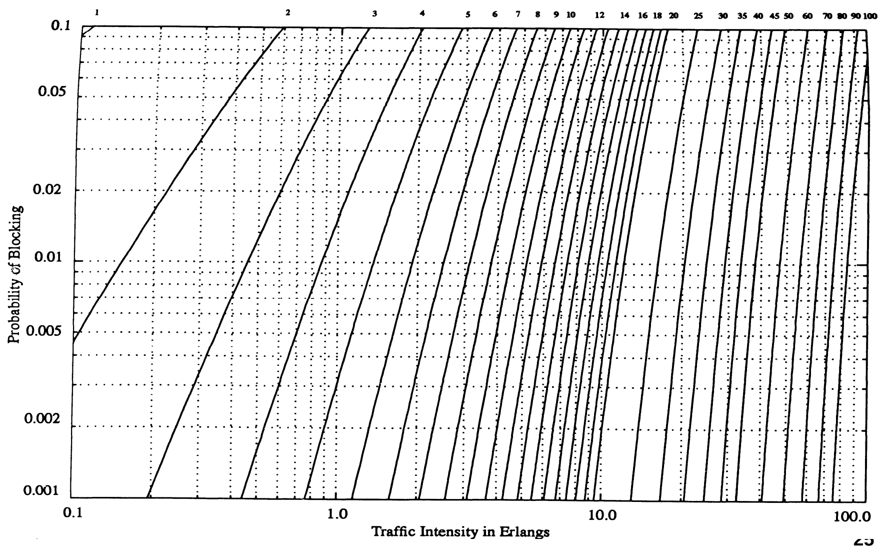
Usually determined from table or charts – many software programs
Example for 100 users with a traffic load of 3.5 E – how many channels are need in a cell to support 2% call blocking ?
From Erlang B table with 2% call blocking need 8 channels

Traffic Engineering Erlang B table

Appendix 1.1
Blocked-Calls-Cleared
(Erlang *B*)

	A, erlangs												
	B												
N	1.0%	1.2%	1.5%	2%	3%	5%	7%	10%	15%	20%	30%	40%	50%
1	.0101	.0121	.0152	.0204	.0309	.0526	.0753	.111	.176	.250	.429	.667	1.00
2	.153	.168	.190	.223	.282	.381	.470	.595	.796	1.00	1.45	2.00	2.73
3	.455	.489	.535	.602	.715	.899	1.06	1.27	1.60	1.93	2.63	3.48	4.59
4	.869	.922	.992	1.09	1.26	1.52	1.75	2.05	2.50	2.95	[illegible]	5.02	6.50
5	1.36	1.43	1.52	1.66	1.88	2.22	2.50	2.88	3.45	4.01	5.19	6.60	8.44
6	1.91	2.00	2.11	2.28	2.54	2.96	3.30	3.76	4.44	5.11	6.51	8.19	10.4
7	2.50	2.60	2.74	2.94	3.25	3.74	4.14	4.67	5.46	6.23	7.86	9.80	12.4
8	3.13	3.25	3.40	3.63	3.99	4.54	5.00	5.60	6.50	7.37	9.21	11.4	14.3
9	3.78	3.92	4.09	4.34	4.75	5.37	5.88	6.55	7.55	8.52	10.6	13.0	16.3
10	4.46	4.61	4.81	5.08	5.53	6.22	6.78	7.51	8.62	9.68	12.0	14.7	18.3
11	5.16	5.32	5.54	5.84	6.33	7.08	7.69	8.49	9.69	10.9	13.3	16.3	20.3
12	5.88	6.05	6.29	6.61	7.14	7.95	8.61	9.47	10.8	12.0	14.7	18.0	22.2
13	6.61	6.80	7.05	7.40	7.97	8.83	9.54	10.5	11.9	13.2	16.1	19.6	24.2
14	7.35	7.56	7.82	8.20	8.80	9.73	10.5	11.5	13.0	14.4	17.5	21.2	26.2
15	8.11	8.33	8.61	9.01	9.65	10.6	11.4	12.5	14.1	15.6	18.9	22.9	28.2
16	8.88	9.11	9.41	9.83	10.5	11.5	12.4	13.5	15.2	16.8	20.3	24.5	30.2
17	9.65	9.89	10.2	10.7	11.4	12.5	13.4	14.5	16.3	18.0	21.7	26.2	32.2
18	10.4	10.7	11.0	11.5	12.2	13.4	14.3	15.5	17.4	19.2	23.1	27.8	34.2
19	11.2	11.5	11.8	12.3	13.1	14.3	15.3	16.6	18.5	20.4	24.5	29.5	36.2
20	12.0	12.3	12.7	13.2	14.0	15.2	16.3	17.6	19.6	21.6	25.9	31.2	38.2

Traffic Engineering



M/M/C/C

Other performance metrics can be related to Erlang B formula *B(c,a)*
The carried load

$$\lambda_e = \lambda \cdot (1 - B(c,a))$$

$\Leftarrow$ Effective throughput of the system

Mean server utilization $$\rho_e = \frac{a}{c} \cdot (1 - B(c,a))$$

Mean number in the system $$L = \frac{a}{\mu} \cdot (1 - B(c,a))$$

Average delay in the system $$W = \frac{1}{\mu}$$

Traffic Engineering Example

- Consider a single analog cell tower with 56 traffic channels, when all channels are busy calls are blocked. Calls arrive according to a Poisson process at a rate of 1 call per active user an hour. During the busy hour 3/4 the users are active. The call holding time is exponentially distributed with a mean of 120 seconds.
- (a) What is the maximum load the cell can support while providing 2% call blocking?

  From the Erlang B table with c= 56 channels and 2% call blocking the maximum load = 45.9 Erlangs
- (b) What is the maximum number of users supported by the cell during the busy hour?

  Load per active user = 1 call x 120 sec/call x 1/3600 sec = 33.3 mErlangs

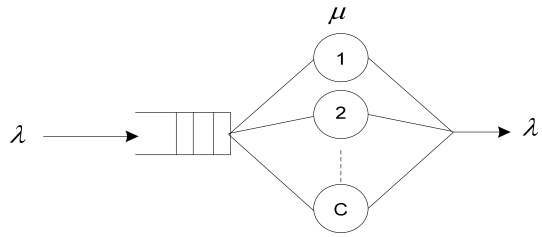
  Number active users = 45.9/(0.0333) = 1377

  Total number users = 4/3 number active users = 1836
- Determine the utilization of the cell tower ρ

  $\rho = \alpha/c$ = 45.9/56 = 81.96%

Erlang C M/M/C Model

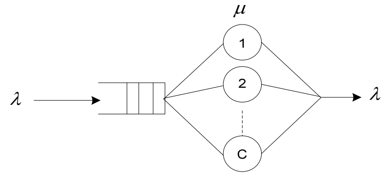
- C identical servers processes customers in parallel.
- Customers arrive according to a Poisson process with mean rate λ
- Customer service times exponentially distributed with mean rate $1/\mu$
- Infinite system capacity – all customers are eventually served – if servers are busy customers queue up
- *Blocked calls delayed model (BCD)*



M/M/C

The server utilization (ρ)

$$\rho = \frac{\lambda}{C\mu}$$



The traffic intensity (a) $\Leftarrow$ offered load (Erlangs)

$$a = \frac{\lambda}{\mu}$$

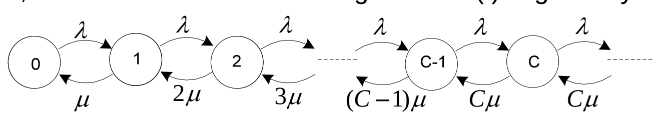
The stability requirement

$$\rho = \frac{a}{C} < 1 \qquad \Rightarrow \qquad a < C$$

With traffic intensity *a* Erlangs, *C* is the minimum number of servers for stability.

M/M/C

Let π_i denote the steady state probability of *i* customers in the system, then the state transition diagram for n(t) is given by



Flow Balance equations

$$\lambda\pi_0 = \mu\pi_1 \qquad j = 0$$

$$(\lambda + j\mu)\pi_j = \lambda\pi_{j-1} + (j+1)\mu\pi_{j+1} \qquad 1 \le j < C$$

$$(\lambda + C\mu)\pi_j = \lambda\pi_{j-1} + C\mu\pi_{j+1} \qquad j \ge C$$

Normalization condition $\sum_{j=0}^{\infty} \pi_j = 1$

M/M/C

Probability of a customer being delayed *C(c,a) is important metric*

$$C(c,a) = \sum_{j=c}^{\infty} \pi_j = \frac{\dfrac{a^c}{(c-1)!(c-a)}}{\displaystyle\sum_{n=0}^{c-1} \frac{a^n}{n!} + \frac{a^c}{(c-1)!(c-a)}}$$

C(c,a) ⇐ Erlang's *C* formula, Erlang's delay formula

In the telephone system, *C(c,a)* represents a blocked call delayed (BCD)
Widely used to determine call center staffing
Difficult to compute due to factorials - several software packages built around it (see links on web site)
Tables and plots available (table on class web page)

Erlang C model

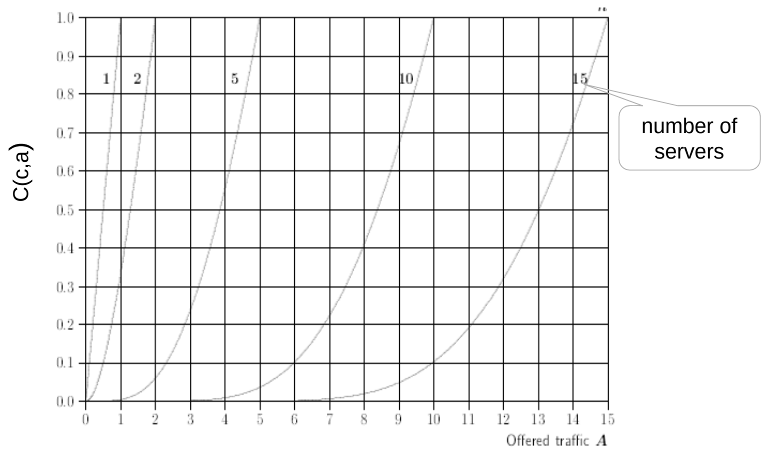
Tables and plots available

Erlang C Traffic Table

Maximum Offered Load Versus B and N
B is in %

N/B	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0100	.0200	.0500	.1000	.1500	.2000	.3000	.4000
2	.0142	.0319	.0452	.1025	.1465	.2103	.3422	.5000	.6278	.7403	.9390	1.117
3	.0860	.1490	.1894	.3339	.4291	.5545	.7876	1.040	1.231	1.393	1.667	1.903
4	.2310	.3533	.4257	.6641	.8100	.9939	1.319	1.653	1.899	2.102	2.440	2.725
5	.4428	.6289	.7342	1.065	1.259	1.497	1.905	2.313	2.607	2.847	3.241	3.569
6	.7110	.9616	1.099	1.519	1.758	2.047	2.532	3.007	3.344	3.617	4.062	4.428
7	1.026	1.341	1.510	2.014	2.297	2.633	3.188	3.725	4.103	4.406	4.897	5.298
8	1.382	1.758	1.958	2.543	2.866	3.246	3.869	4.463	4.878	5.210	5.744	6.178
9	1.771	2.208	2.436	3.100	3.460	3.883	4.569	5.218	5.668	6.027	6.600	7.065
10	2.189	2.685	2.942	3.679	4.077	4.540	5.285	5.986	6.469	6.853	7.465	7.959
11	2.634	3.186	3.470	4.279	4.712	5.213	6.015	6.765	7.280	7.688	8.336	8.857
12	3.100	3.708	4.018	4.896	5.363	5.901	6.758	7.554	8.099	8.530	9.212	9.761
13	3.587	4.248	4.584	5.529	6.028	6.602	7.511	8.352	8.926	9.379	10.09	10.67
14	4.092	4.805	5.166	6.175	6.705	7.313	8.273	9.158	9.760	10.23	10.98	11.58
15	4.614	5.377	5.762	6.833	7.394	8.035	9.044	9.970	10.60	11.09	11.87	12.49
16	5.150	5.962	6.371	7.502	8.093	8.766	9.822	10.79	11.44	11.96	12.77	13.41
17	5.699	6.560	6.991	8.182	8.801	9.505	10.61	11.61	12.29	12.83	13.66	14.33
18	6.261	7.169	7.622	8.871	9.518	10.25	11.40	12.44	13.15	13.70	14.56	15.25
19	6.835	7.788	8.263	9.568	10.24	11.01	12.20	13.28	14.01	14.58	15.47	16.18
20	7.419	8.417	8.914	10.27	10.97	11.77	13.00	14.12	14.87	15.45	16.37	17.10

Erlang C Plots



Erlang C Model

Other performance measures expressed in terms of C(c,a)

$$L_q = \left(\frac{a}{c-a}\right) \cdot C(c,a)$$

$$L = L_q + a$$

$$W_q = \frac{L_q}{\lambda} = \frac{\frac{1}{\mu} C(c,a)}{c-a}$$

$$W = W_q + \frac{1}{\mu}$$

Erlang C model

Distribution of the waiting time in the queue

$$P\{w_q \le t\} = 1 - C(c,a) \cdot e^{-c\mu(1-\rho)t}$$

The *p*th percentile of the time spent waiting in the queue t_p

$$t_p = \frac{-\ln\left(\frac{1-p}{C(c,a)}\right)}{c\mu(1-\rho)}$$

Note: $p > 1 - C(c,a)$

Traffic Engineering Example 2

- A telephone company has five operators to handle inquires for directory assistance. Inquires arrive according to a Poisson process with an average rate of λ = 4.5 calls/minute. The time to process an inquiry is exponentially distributed with a mean of $1/\mu$ = 1 minute/call. If an arriving call sees all operators busy it is placed on hold until an operator becomes free.

(a) What is the probability that a caller will have to wait on hold?

*The offered load in Erlangs is 4.5 calls/min * 1 min/call = 4.5 erlangs with 5 operators from the Erlang C graphs given in the class handout the probability a caller will be delayed = C(c,a) = .75 computing an exact value from the Erlang C formula one gets C(5,4.5) = .7625*

(b) What is the 95 percentile of the time on hold?

note $\rho = a/c$ = 4.5/5 = .9, p = .95

$$t_p = \frac{-\ln\left(\frac{1-p}{C(c,a)}\right)}{c\mu(1-\rho)}$$

yields t_p = 5.4491 minutes

Traffic Engineering Example 3

- A service provider receives unsuccessful call attempts to wireless subscribers at a rate of 5 call per minute in a given geographic service area. The unsuccessful calls are processed by voice mail and have an average mean holding time of 1 minute. When all voice mail servers are busy – customers are placed on hold until a server becomes free.
- Determine the minimum number of servers to keep the percentage of customers placed on hold $<$ or equal to 1%

  The offered load is a = 5 call per minute x 1 minute/call = 5

  From the Erlang C tables 13 servers are needed.
- Determine the .995% of the delay in access the voice servers
- With p = .995, C(c,a) = .01, c = 13, and μ = 1

$$t_p = \frac{-\ln\left(\frac{1-p}{C(c,a)}\right)}{c\mu(1-\rho)}$$

yields t_p = .0866 minute = 5.2 secs

Summary

- Cellular Concept
  - Small cells
  - Frequency Reuse
- Frequency Planning
  - Assignment of frequencies to cells
  - Maintain C/I requirements
- Traffic Engineering
  - Erlang B model
  - Erlang C model