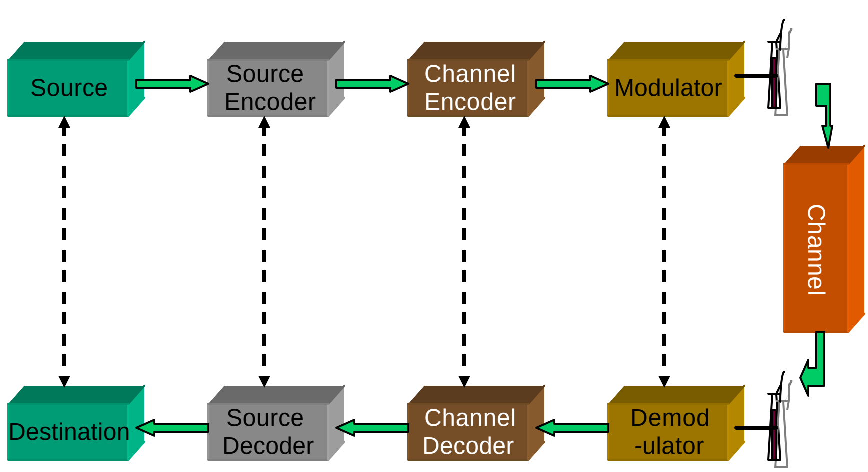
Digital Modulation

David Tipper
Associate Professor
Department of Information Science and Telecommunications
University of Pittsburgh

http://www.tele.pitt.edu/tipper.html

Typical Communication System



About Channel Capacity

- Channel Capacity (C)
  - the maximum rate at which data can be transmitted over a given communication path, or channel, under given conditions
- Data rate (bps)
  - rate at which data can be communicated , impairments, such as noise, limit data rate that can be achieved
- Bandwidth (B)
  - the bandwidth of the transmitted signal as constrained by the transmitter and the nature of the transmission medium (Hertz)
- Noise (N)
  - impairments on the communications path
- Error rate - rate at which errors occur (BER)
  - Error = transmit 1 and receive 0; transmit 0 and receive 1

Reasons for Choosing Encoding Techniques

- Digital data, digital signal
  - Equipment less complex and expensive than digital-to-analog modulation equipment
- Analog data, digital signal
  - Permits use of modern digital transmission and switching equipment
- Digital data, analog signal
  - Some transmission media will only propagate analog signals
  - E.g., unguided media (air)
- Analog data, analog signal
  - Analog data in electrical form can be transmitted easily and cheaply
  - E.g., AM Radio

Signal Encoding Criteria

- What determines how successful a receiver will be in interpreting an incoming signal?
  - Signal-to-noise ratio (SNR)
  - Data rate
  - Bandwidth (B)
  - Inter-related quantities
    - Increase in SNR decreases bit error rate
    - Increase in data rate increases bit error rate
    - Increase in bandwidth allows an increase in data rate
- Shannon Bound for AWGN non fading channel

Concepts Related to Channel Capacity

- Shannon Bound for AWGN non fading channel

$$C = B\log_2(1 + S/N)$$

- Nyquist Bandwidth
  - For binary signals (two voltage levels)
    - $C = 2B$
  - With multilevel signaling (M-ary signalling)
    - $C = 2B \log_2 M$
    - M = number of discrete signal or voltage levels
    - N= number of bits
    - $M = 2^N$

Example of Nyquist and Shannon Formulations

- Spectrum of a channel between 3 MHz and 4 MHz ; SNR_{dB} = 24 dB

$$B = 4\text{ MHz} - 3\text{ MHz} = 1\text{ MHz}$$
$$\text{SNR}_{\text{dB}} = 24\text{ dB} = 10\log_{10}(\text{SNR})$$
$$\text{SNR} = 251$$

- Using Shannon's formula

$$C = 10^6 \times \log_2(1+251) \approx 10^6 \times 8 = 8\text{Mbps}$$

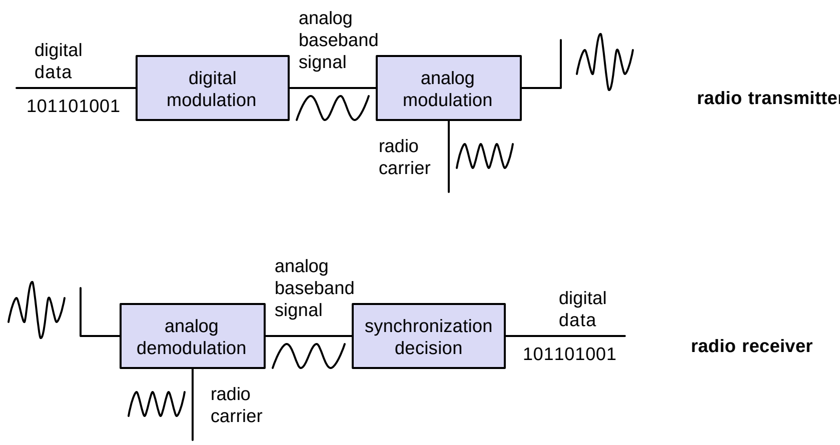
- How many signaling levels are required?

$$C = 2B\log_2 M$$
$$8\times10^6 = 2\times(10^6)\times\log_2 M$$
$$4 = \log_2 M$$
$$M = 16$$

Digital Transmission

- Why Digital ?
  - Increase System Capacity
    - compression, more efficient modulation
  - Error control coding, equalizers,etc. possible to combat noise and interference => lower power needed
  - Reduce cost and simplify designs
  - Improve Security (encryption possible)
- Digital Modulation
  - Analog signal carrying digital data

Digital Modulation and demodulation



Modulation Review

- Modulation
  - Converting digital or analog information to a waveform suitable for transmission over a given medium
  - Involves varying some parameter of a carrier wave (sinusoidal waveform) at a given frequency as a function of the message signal
  - General sinusoid
    - $A \cos (2pf_C t + j)$

Amplitude — Frequency — Phase

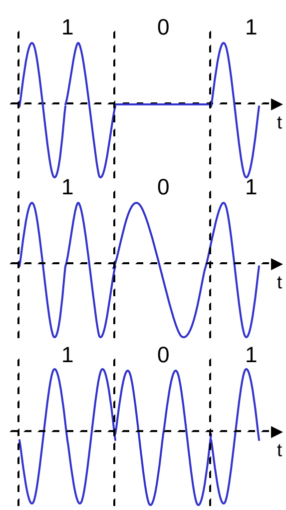
  - If the information is digital changing parameters is called "keying" (e.g. ASK, PSK, FSK)

Modulation

- Motivation
  - Smaller antennas (e.g., λ /4 typical antenna size)
    - λ = wavelength = c/f , where c = speed of light, f= frequency.
    - 3000Hz baseband signal => 15 mile antenna, 900 MHz => 8 cm
  - Frequency Division Multiplexing – provides separation of signals
  - medium characteristics
  - Interference rejection
  - Simplifying circuitry
- Modulation
  - shifts center frequency of baseband signal up to the radio carrier
- Basic schemes
  - Amplitude Modulation (AM) — Amplitude Shift Keying (ASK)
  - Frequency Modulation (FM) — Frequency Shift Keying (FSK)
  - Phase Modulation (PM) — Phase Shift Keying (PSK)

Digital modulation

- Amplitude Shift Keying (ASK):
  - change amplitude with each symbol – frequency constant
  - low bandwidth requirements
  - very susceptible to interference
- Frequency Shift Keying (FSK):
  - change frequency with each symbol
  - needs larger bandwidth
- Phase Shift Keying (PSK):
  - Change phase with each symbol
  - More complex
  - robust against interference



Basic Encoding Techniques

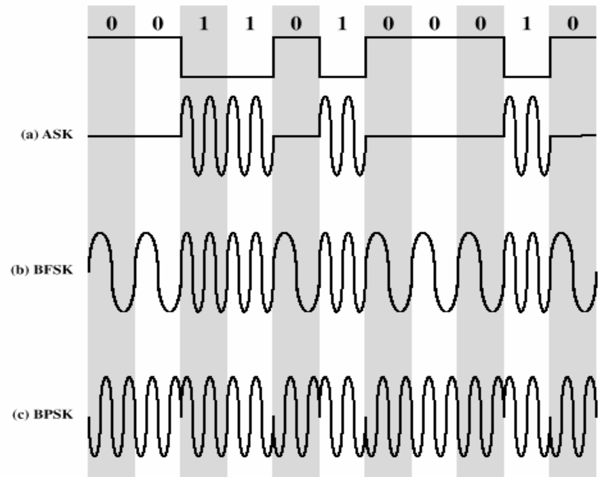


Figure 6.2 Modulation of Analog Signals for Digital Data

Amplitude-Shift Keying

- One binary digit represented by presence of carrier, at constant amplitude
- Other binary digit represented by absence of carrier

$$s(t) = \begin{cases} A\cos(2pf_c t) & \text{binary } 1 \\ 0 & \text{binary } 0 \end{cases}$$

  - where the carrier signal is $A\cos(2pf_c t)$
- Very Susceptible to noise
- Used to transmit digital data over optical fiber

Binary Frequency-Shift Keying (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency

$$s(t) = \begin{cases} A\cos(2pf_1 t) & \text{binary } 1 \\ A\cos(2pf_2 t) & \text{binary } 0 \end{cases}$$

  - where f_1 and f_2 are offset from carrier frequency f_c by equal but opposite amounts
  - $B = 2([f_2 - f_1]/2 + f_b)$
    - Where f_b = input bit rate

Phase-Shift Keying (PSK)

- Two-level PSK (BPSK)
  - Uses two phases to represent binary digits

$$s(t) = \begin{cases} A\cos(2pf_c t) & \text{binary } 1 \\ A\cos(2pf_c t + p) & \text{binary } 0 \end{cases}$$

$$= \begin{cases} A\cos(2pf_c t) & \text{binary } 1 \\ -A\cos(2pf_c t) & \text{binary } 0 \end{cases}$$

$B = f_b$

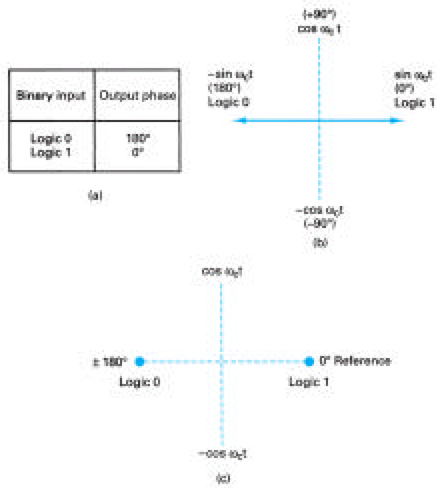
Selection of Encoding/Modulation Schemes

- Performance in an AWGN channel
  - How does the bit error rate vary with the energy per bit available in the system when white noise present
- Performance in fading multipath channels
  - Same as above, but add multipath and fading
- Bandwidth requirement for a given data rate
  - Also termed spectrum efficiency or bandwidth efficiency
  - How many bits/sec can you squeeze in one Hz of bandwidth for a given error rate
- Cost
  - The modulation scheme needs to be cost efficient
  - Circuitry should be simple to implement and inexpensive (e.g. detection, amplifiers)

Signal Constellation

- Given any modulation scheme, it is possible to obtain its signal constellation.
  - Represent each possible signal as a vector in a Euclidean space spanned by an orthonormal basis.
- If we know the signal constellation, we can estimate the performance in terms of the probability of symbol error or probability of bit error given the noise parameters.
- Probability of error depends on the minimum distance between the constellation points.

BPSK Signal Costellation



Tomasi
Electronic Communications Systems, 5e

Symbol Detection

- The receiver implementation can affect the performance.
  - Coherent detection
    - receiver will exploit the exact knowledge of the phase of the carrier to detect the signal better.
  - Non-coherent detection
    - involves making some approximations to the phase information that results in a loss in performance. However, it simplifies the circuitry.
- In symbol detection – decode incoming signal as closest symbol in the signal constellation space

Example of BPSK

A binary 1 is represented by:

$$s_1(t) = \sqrt{\tfrac{2E_b}{T}}\cos(2pf_c t) \quad , \quad 0 \le t \le T,\ f_c = \tfrac{n}{T}$$

A binary 0 is represented by:

$$s_2(t) = -\sqrt{\tfrac{2E_b}{T}}\cos(2pf_c t) \quad , \quad 0 \le t \le T$$

We can write $s_1(t) = \sqrt{E_b}\,\Psi(t)$

$$s_2(t) = -\sqrt{E_b}\,\Psi(t)$$

where $\Psi(\mathrm{t}) = \sqrt{\tfrac{2}{\mathrm{T}}}\cos(2pf_c t), \quad 0 \le \mathrm{t} \le \mathrm{T}$

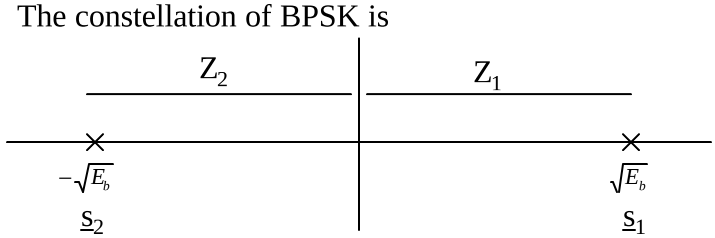
What is the energy of Ψ(t)?

$$E_\Psi = \int_0^T \Psi^2(t)dt = \tfrac{2}{T}\int_0^T \cos^2(2pf_c t)dt$$

$$= \tfrac{2}{T}\int_0^T \tfrac{1}{2}[1 + \cos(4pf_c t)]dt = 1$$

Note that the energy in one bit of BPSK is E_b.

The constellation of BPSK is



Detection

When do errors occur in BPSK?

- ***S_1*** is transmitted and the received point falls in ***Z_1*** region.
- ***S_2*** is transmitted and the received point falls in ***Z_2*** region.

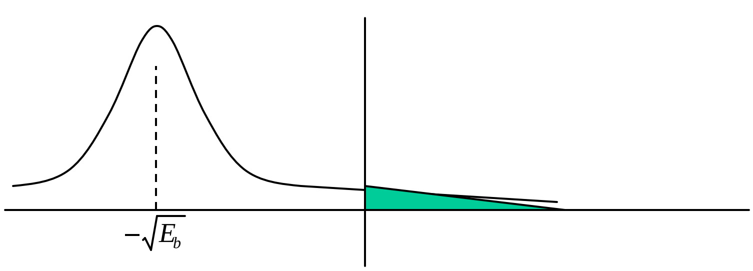
Why will a point fall elsewhere?

- The received signal point is $\underline{s_i} + \underline{n} = \underline{r}$, i =1,2
- $\underline{n}$ is the noise vector that is normally distributed with mean zero and variance $N_0/2$. This can shift the transmitted signal to some other value.

Bit Error Rate

- What is the probability that the signal point $\underline{r}$ falls in ***Z_1*** given $\underline{s_2}$(t) was transmitted? (Conditional probability)
- $\underline{r} = \underline{s_2} + \underline{n}$ is a normally distributed random variable with mean -ÖE_b and variance $N_0/2$.

$$P_e = \int_0^{\infty} \frac{1}{\sqrt{\frac{N_0}{2}}\sqrt{2p}} \exp - \left[\frac{\left(x+\sqrt{E_b}\right)^2}{N_0} \right] dx$$



Normal Distribution Review

$$Let \quad Z = \frac{\left(x+\sqrt{E_b}\right)}{\sqrt{\frac{N_0}{2}}}, \quad dZ = \frac{dx}{\sqrt{\frac{N_0}{2}}}$$

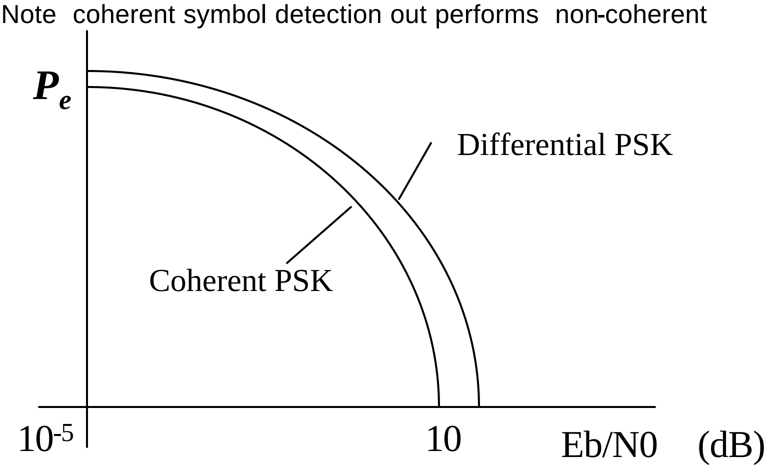
$$When \;\; x = 0, \quad Z = \sqrt{\frac{2E_b}{N_0}}$$

$$P_e = \int_{\sqrt{\frac{2E_b}{N_0}}}^{\infty} \frac{1}{\sqrt{2p}} \exp\left(\frac{-Z^2}{2}\right) dz = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$= Q(g_b)$$

PSK Error Performance

Note coherent symbol detection out performs non-coherent



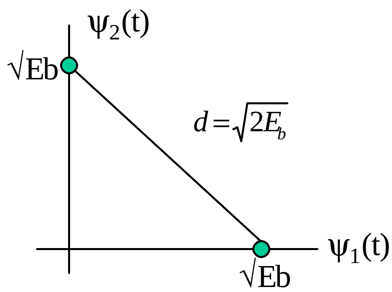
Remarks

a) We use Eb/N0 as a figure of merit because it provides a good comparison of the "power efficiency" or "energy efficiency" of a modulation scheme. Sometimes called SNR per bit.

b) We will not derive the bit error rate performance of different modulation schemes but we will only use the results.

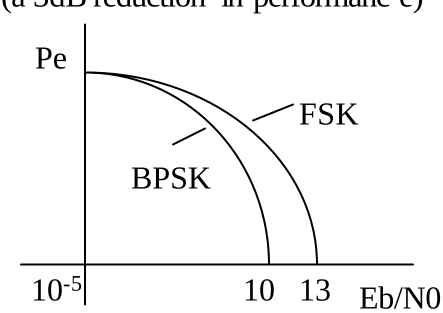
Some Remarks (cont.)

c) The constellation of orthogonal FSK looks like this



$$P_e(FSK) = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

(a 3dB reduction in performanc e)

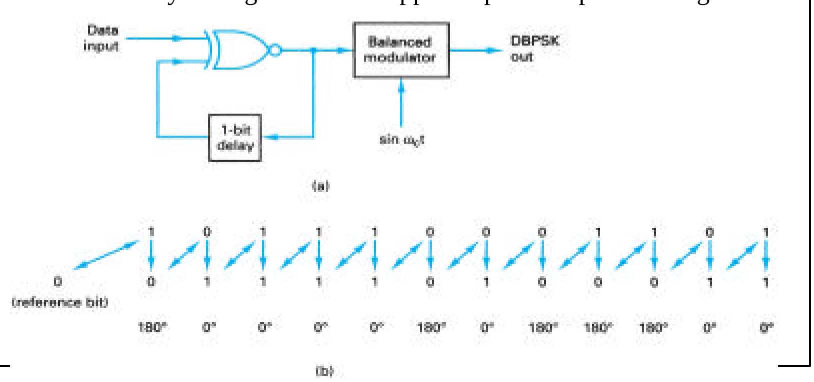


Performance

- Bandwidth of modulated signal (B_T)
  - ASK, PSK $B_T=(1+r)R$
  - FSK $B_T=2DF+(1+r)R$
    - R = bit rate
    - 0 < r < 1; related to how signal is filtered
    - D$F = f_2-f_c=f_c-f_1$

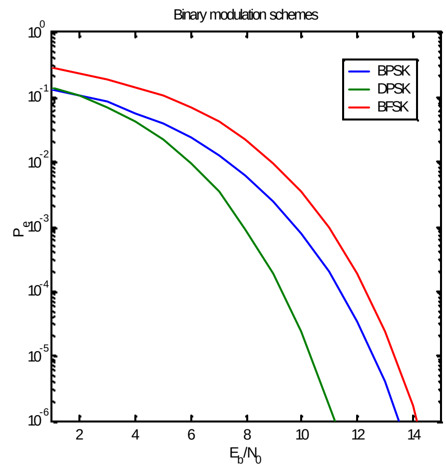
D Phase-Shift Keying (PSK)

- Differential PSK (DPSK)
  - Phase shift with reference to previous bit
    - Binary 0 – signal burst of same phase as previous signal burst
    - Binary 1 – signal burst of opposite phase to previous signal burst



Performance in AWGN channels

- Similar to BPSK analysis have P_e for FSK, and DPSK



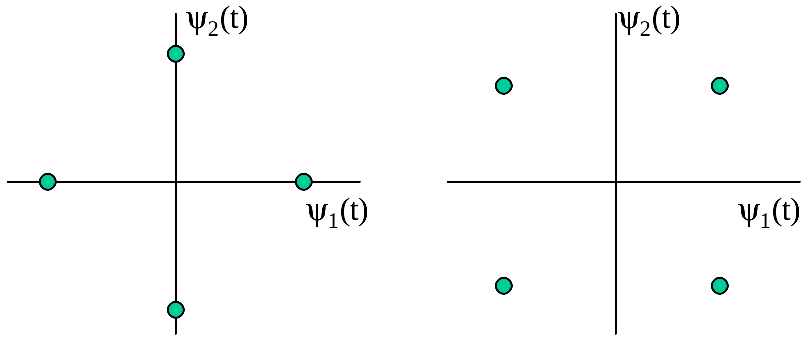
M-ary Signaling/Modulation

- What is *M*-ary signaling?
  - The transmitter considers ‘k’ bits at a times. It produces one of *M* signals where $M = 2^k$.

  Example: QPSK ($k = 2$)

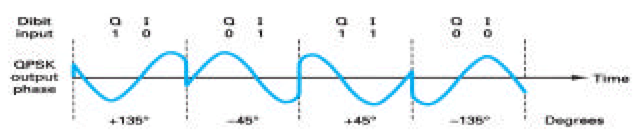
Input:	Signal :
00	$\sqrt{\frac{2E}{T}}\cos\left(2pf_c t\right), \quad 0 \le t \le T$
01	$\sqrt{\frac{2E}{T}}\cos\left(2pf_c t + \frac{p}{2}\right), 0 \le t \le T$
11	$\sqrt{\frac{2E}{T}}\cos\left(2pf_c t + p\right), 0 \le t \le T$
10	$\sqrt{\frac{2E}{T}}\cos\left(2pf_c t + \frac{3p}{2}\right), 0 \le t \le T$

QPSK Constellations



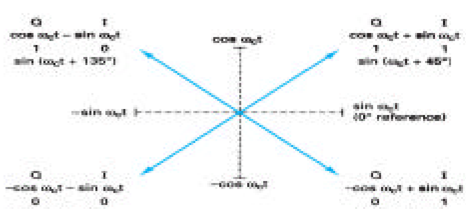
Rotated by $\pi/4$

$\pi/4$ QPSK

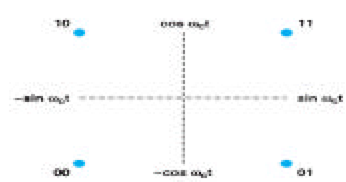


Binary input Q	Binary input I	QPSK output phase
0	0	−135°
0	1	−45°
1	0	+135°
1	1	+45°

(a)

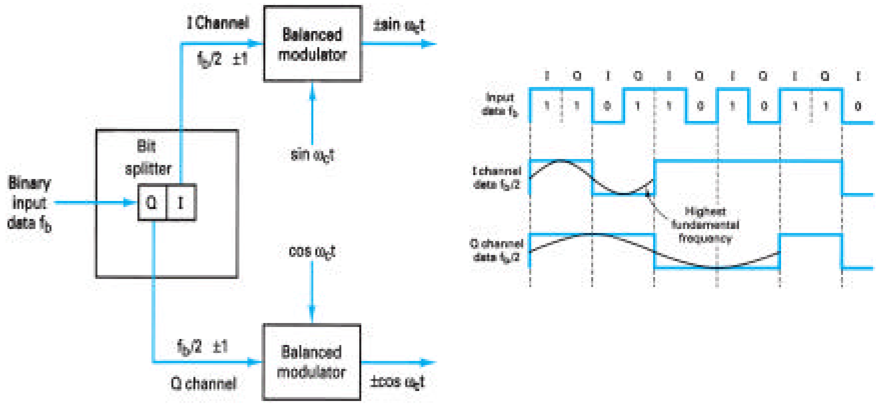


(b)



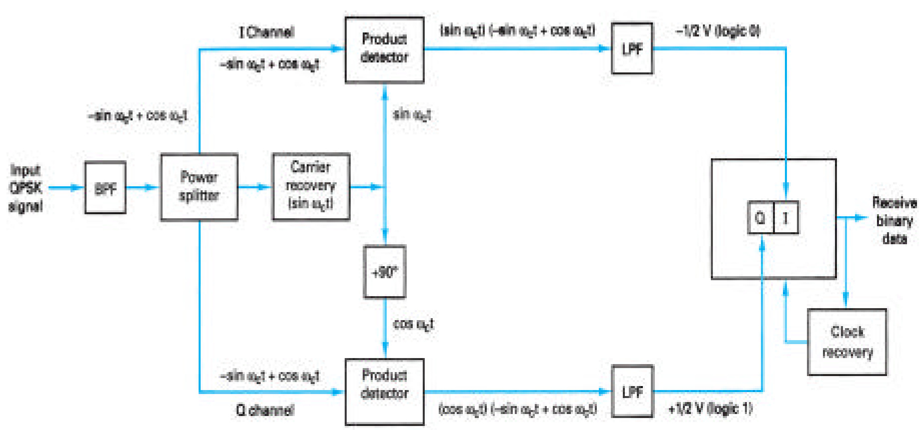
(c)

π/4 –QPSK Modulation



Can use simple AM balanced modulator

π/4 QPSK Coherent Demodulator

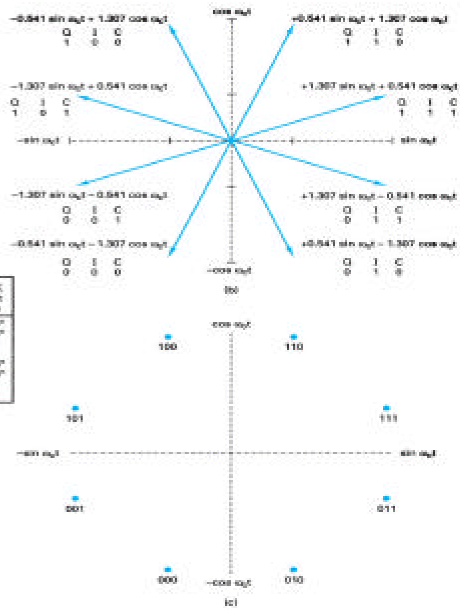


8 –PSK

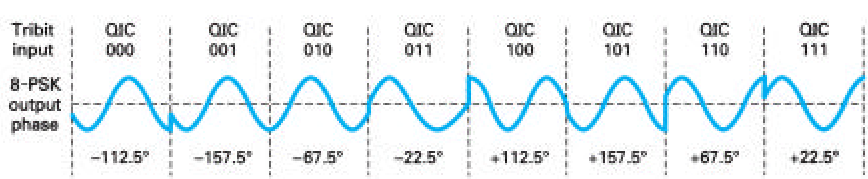
Increasing the number of levels increases the data rate – but Increases the symbol error rate as the symbols are closer together in the constellation space

Binary input			8-PSK output phase
Q	I	C	
0	0	0	−112.5°
0	0	1	−157.5°
0	1	0	−67.5°
0	1	1	−22.5°
1	0	0	+112.5°
1	0	1	+157.5°
1	1	0	+67.5°
1	1	1	+22.5°

(a)



8-PSK Output



M-ary Error Performance

- MPSK, as M increases
  - the bandwidth remains constant,
  - the minimum distance between signals reduces => increase in symbol error rate
- MFSK, as M increases
  - the bandwidth increases
  - the performance improves but the minimum distance between signals remains the same

Performance

- Bandwidth of modulated signal (B_T)
  - MPSK $$B_T = \left(\frac{1+r}{L}\right)R = \left(\frac{1+r}{\log_2 M}\right)R$$
  - MFSK $$B_T = \left(\frac{(1+r)M}{\log_2 M}\right)R$$
    - L = number of bits encoded per signal element
    - M = number of different signal elements

Quadrature Amplitude Modulation

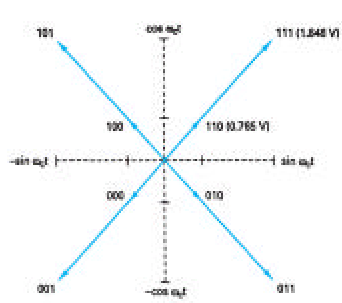
- QAM is a combination of ASK and PSK
  - Two different signals sent simultaneously on the same carrier frequency –
  - Change phase and amplitude as function of input data
  - Simple case 8 QAM (two amplitudes – 4 phases)

$$s(t) = d_1(t)\cos 2pf_c t + d_2(t)\sin 2pf_c t$$

8 - QAM

Binary input			8-QAM output	
Q	I	C	Amplitude	Phase
0	0	0	0.765 V	−135°
0	0	1	1.848 V	−135°
0	1	0	0.765 V	−45°
0	1	1	1.848 V	−45°
1	0	0	0.765 V	+135°
1	0	1	1.848 V	+135°
1	1	0	0.765 V	+45°
1	1	1	1.848 V	+45°

(a)



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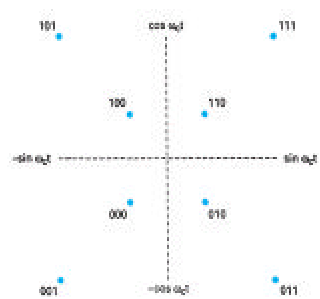
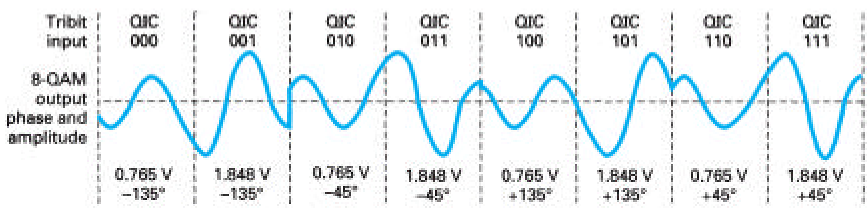


(c)

8-QAM Output



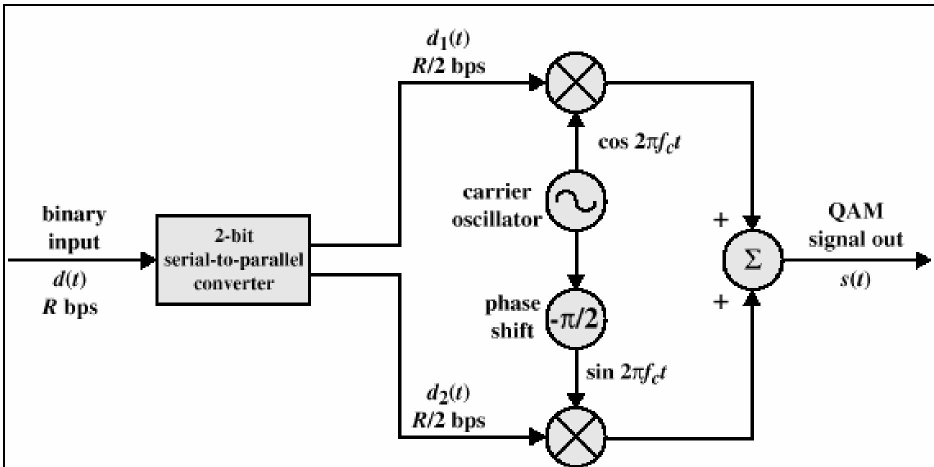
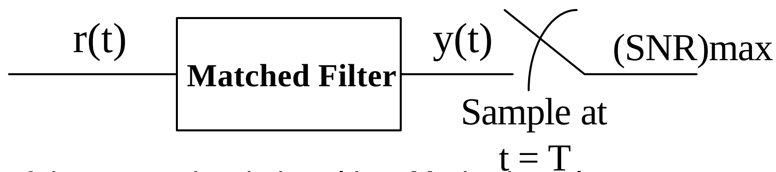


Figure 6.10 QAM Modulator

Matched Filter

- In order to detect a signal at the receiver, a linear filter that is designed to provide the maximum output SNR in AWGN for a given symbol waveform is used. This filter is called a “matched filter” (section 3.2.2)



- If the transmitted signal is ***s(t),*** the impulse response of the matched filter can be shown to be

$$h(t) = \begin{cases} k \cdot s(T-t), & 0 \le t \le T \\ 0 & \text{, outside} \end{cases}$$

This assumes that ***s(t)*** exists only for a duration of T seconds. Let us look at the output for k = 1.

$$\begin{aligned} y(t) &= r(t) * h(t) \\ &= \int r(t)s(T-(t-t))dt \\ &= \int r(t)s(T-t+t)dt \\ &= \int r(t)s(t+T-t)dt \end{aligned}$$

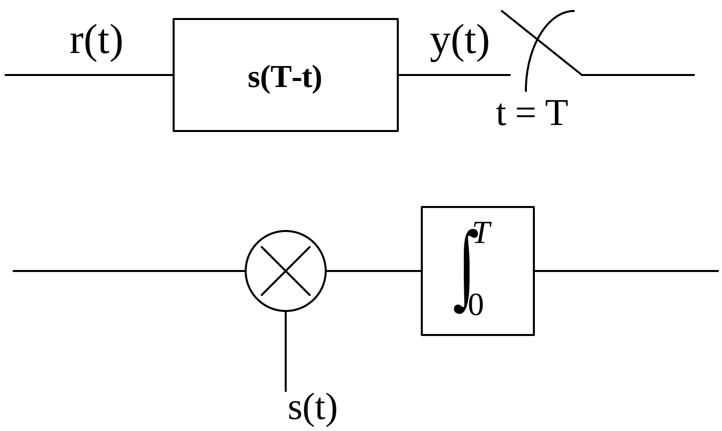
Compare with cross-correlation: $R_{rs}(t) = \int r(t)s(t-t)dt$

The output of the matched filter is the cross-correlation of the received signal and the time shifted transmitted signal.

At $t = T,\ y(T) = \int r(t)s(t)dt = R_{sr}(0)$

If $s(t) = r(t),\ y(t) = E_s$ or $-E_s$

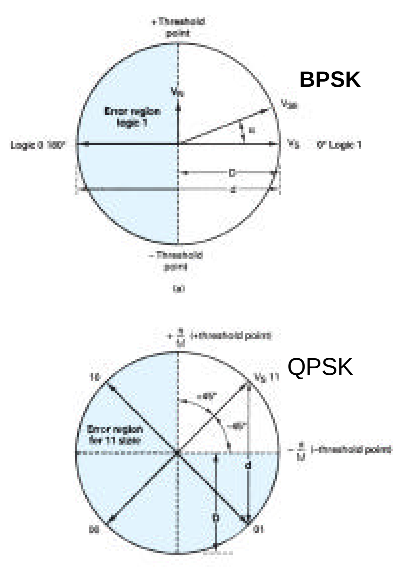
Correlation Implementation of Matched Filter



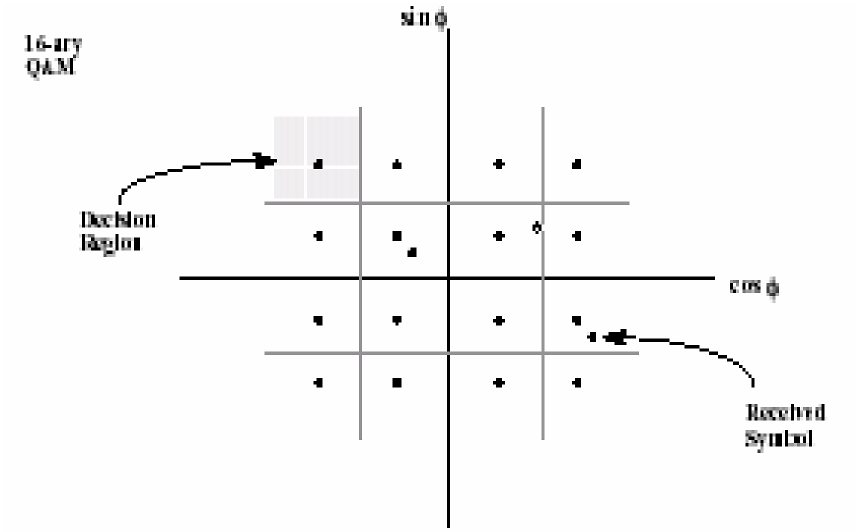
M-ary Error Performance

A received symbol is decoded into the closest the symbol in the signal constellation

As the number of symbols in the signal space increases the decoding region for each symbol decreases

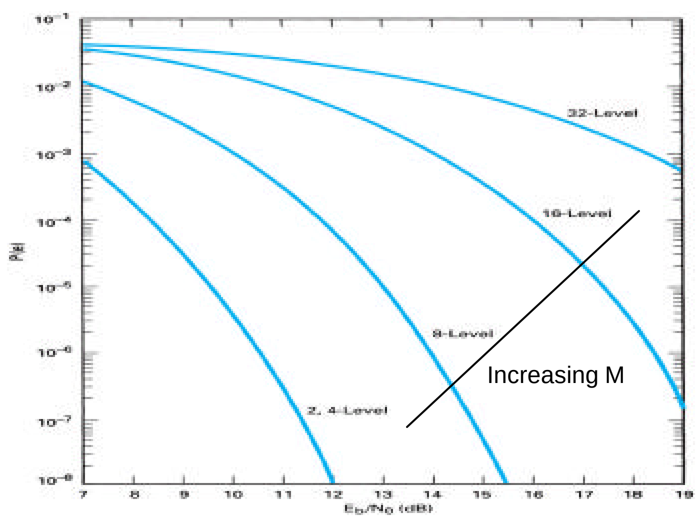


M-ary Error Performance

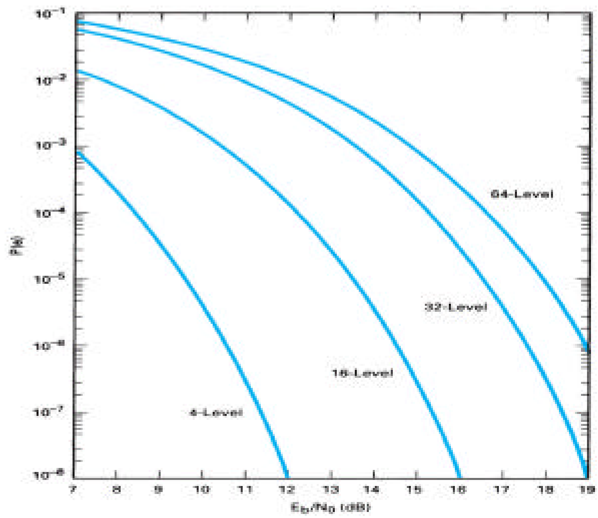


M-PSK Error Rate Performance

Increasing M increases error rate and data rate



QAM Error Performance



Performance Comparison of Various Digital Modulation Schemes (BER = 10^{-6})

Modulation Technique	C/N Ratio (dB)	E_b/N_0 Ratio (dB)
BPSK	10.6	10.6
QPSK	13.6	10.6
4-QAM	13.6	10.6
8-QAM	17.6	10.6
8-PSK	18.5	14
16-PSK	24.3	18.3
16-QAM	20.5	14.5
32-QAM	24.4	17.4
64-QAM	26.6	18.8

Tradeoffs between BER, power and bandwidth

- (1) Trade BER performance for power – fixed data rate
- (2) Trade data rate for power – fixed BER
- (3) Trade BER for data rate – fixed power