Angle modulation

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Electronic Communications II

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Objectives of this lecture

- Define and mathematically describe angle modulation
- Explain the difference between FM and PM
- Describe FM and PM waveforms
- Define frequency deviation and modulation index
- Frequency analysis of an FM waveform
- Determine the BW requirements
- Explain the FM modulator/demodulator
- Describe the output noise at FM demodulator and how to compensate

Outline

Part A: Background

- Introduction to phased lock loop (PLL)

Part B: Angle modulation

1. Introduction
2. Angle modulation mathematical description
3. Frequency deviation and modulation index
4. Frequency analysis of FM
5. FM modulator/demodulator
6. Noise at FM demodulator and Preemphasis/Deemphasis

Issues that are not covered in this lecture

- Oscillator
- Frequency synthesizer
- AM modulator/demodulator
- Digital modulation

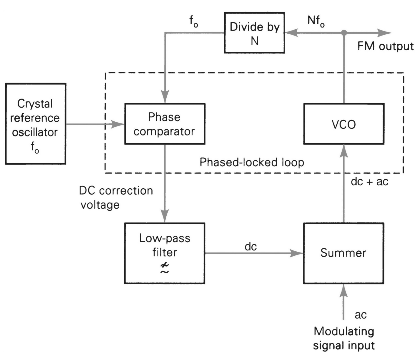
Part A outline (PLL part)

- PLL applications
- Definition of PLL
- PLL components
- PLL loop operation
- Output/input relationship of PLL
  - Output frequency of VCO
  - Output voltage of phase comparator
- PLL loop gain

PLL applications in electronic communications

- Modulation/demodulation
- Signal processing
- Carrier and clock recovery
- Frequency generation
- Frequency synthesis
- etc.

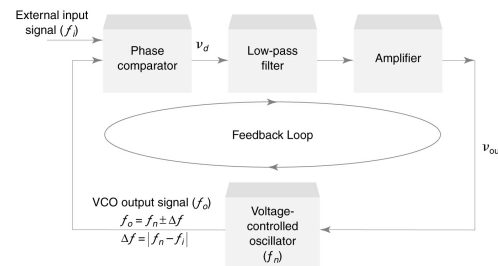
Example: Phased lock loop in FM transmitter



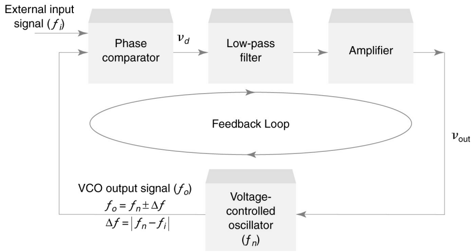
PLL FM transmitter

Definition of PLL

- PLL is a closed loop feedback control system in which either the frequency or the phase of the feedback signal is the parameter of interest.



PLL components

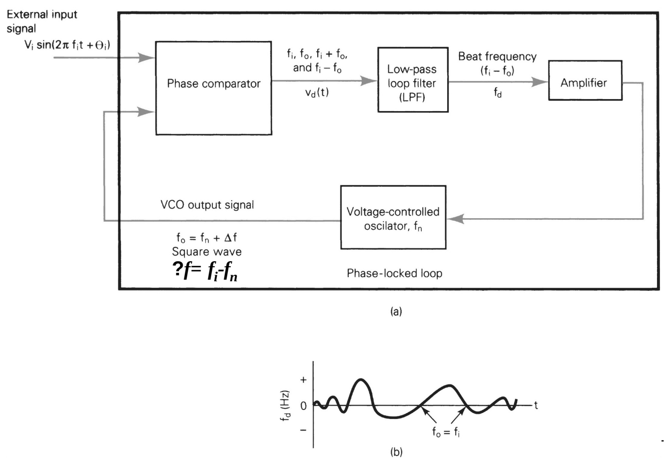


- Phase comparator is a nonlinear device that performs the multiplication of two signals (i.e. balanced modulator or product modulator or product detector).
- Voltage controlled oscillator (VCO) generates signal that has a frequency varied based on the input voltage.

Three operating states of PLL

- Free running state
  - No external input frequency or the feedback loop is open.
- Capture state
  - The PLL is in the process of acquiring frequency lock with the external input signal by using the feedback loop.
  - The time required to achieve lock is called acquisition time or pull-in time.
- Lock state
  - This is the state where the VCO output frequency is locked onto (equal to) the frequency of the external input signal.

PLL Operations



VCO output frequency (f_o)

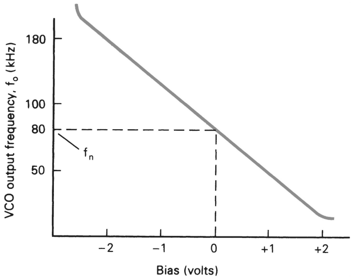
- VCO is an oscillator (more specifically, a free running multivibrator) with a stable frequency of oscillation that depends on an external bias voltage.
- When there is no external input signal or when the feedback loop is open, the VCO operates at a preset frequency called its natural or free running frequency (f_n).
  - f_n is determined by external R and C components.

Voltage controlled oscillator (VCO)

- When a dc or slowly changing ac voltage is applied to the VCO input, the output frequency deviates from f_n proportionally.
  - This deviation is called ?f
- Mathematically, the VCO output frequency (f_o) is

$$f_o = f_n \pm ?f$$

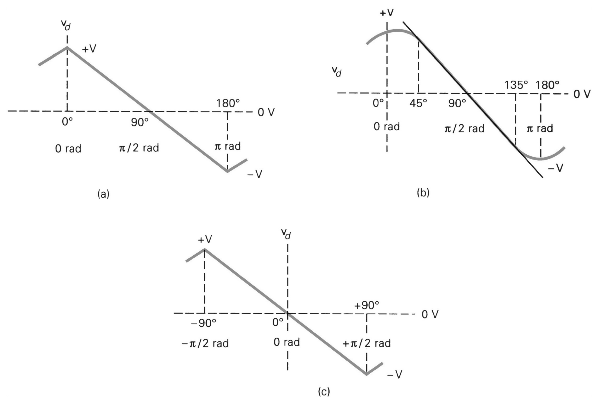
  - ?f= f_i-f_n
  - f_i = external input frequency



Phase comparator

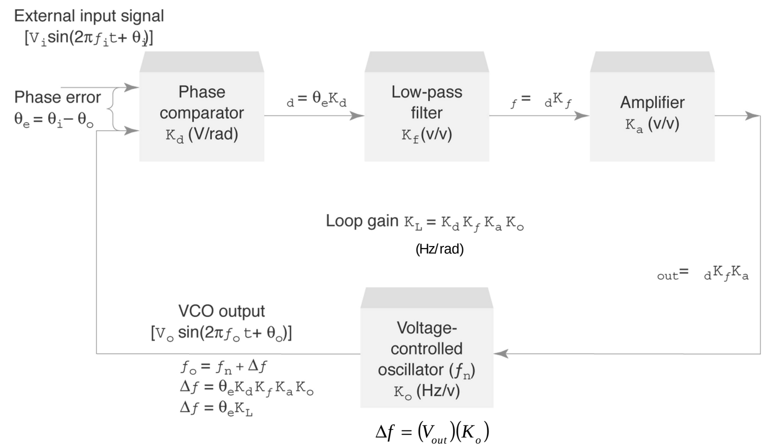
- Phase comparator is a nonlinear device with two input signals. Its output is the product of the two signals of frequencies f_i and f_o and, therefore, contains the sum and difference frequencies.
- The phase comparator is a frequency comparator until the frequency acquisition is achieved (f_o=f_i).
- Once the loop is frequency locked, the phase difference between the input signals of the phase comparator is converted to a dc voltage and fed back to the VCO to hold lock.

Phase comparator output voltage (V_d) and phase difference (?$_e$)



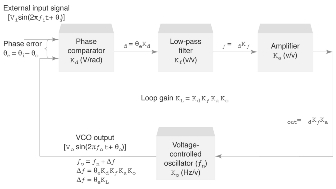
(a) square-wave inputs; (b) sinusoidal inputs; (c) square-wave inputs, phase bias reference

PLL Loop gain



Example of PLL loop gain

- For the PLL shown in the previous slide, a VCO natural frequency f_n=200 kHz, an external input frequency f_i=210 kHz, and the transfer function K_d=0.2 V/rad, K_f=0.2, K_a=5, and K_o=20 kHz/V determine
  - PLL loop gain in Hz/rad
  - Change in VCO frequency necessary to achieve lock (?f)
  - PLL output voltage V_{out}
  - Phase detector output voltage V_d
  - Static phase error ?$_e$



Example of PLL loop gain (cont)

$$f_n = 200 \text{ kHz}, \; f_i = 210 \text{ kHz}, \; K_d = 0.2 \text{ V/rad},$$

$$K_f = 1, K_a = 5, K_o = 20 \text{ kHz/V}$$

Thus,

$$K_L = 0.2 \times 1 \times 5 \times 20 \text{ k} = 20 \text{ kHz/rad}$$

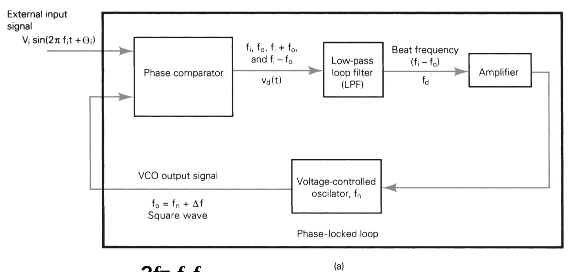
$$\Delta f = f_i - f_n = 210 - 200 = 10 \text{ kHz}$$

$$V_{out} = \frac{\Delta f}{K_o} = \frac{10}{20} = 0.5 \text{ V}$$

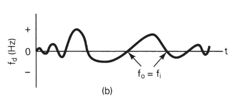
$$V_d = \frac{V_{out}}{K_f K_a} = \frac{0.5}{1 \times 5} = 0.1 \text{ V}$$

$$q_e = \frac{V_d}{K_d} = \frac{0.1}{0.2} = 0.5 \text{ rad or } 28.65^\circ$$

PLL summary



$?f = f_i - f_n$



Part B: Angle modulation

1. Introduction
2. Angle modulation mathematical description
3. Frequency deviation and modulation index
4. Frequency analysis of FM
   – Bessel function
   – BW calculation
5. FM modulator/demodulator
6. Noise at FM demodulator and Preemphasis/Deemphasis

Introduction

- Angle modulation was first introduced in 1931 as an alternative to AM.
  - It could improve the performance of radio communication because it is less susceptible to noise than AM
- Angle modulation is used extensively for
  - Commercial radio broadcasting
  - Television sound transmission
  - Two way mobile radio
  - Cellular radio
  - Microwave and satellite communication systems

Angle modulation: Basic form

- Let $?_i(t)$ denote the angle of a modulated sinusoidal carrier
  - $?_i(t)$ is a function of the message signal
- Express the resulting angle modulated wave as

$$s(t) = A_c \cos[q_i(t)]$$

Phase modulation (PM)

- PM is that form of angle modulation in which the angle $?_i(t)$ is varied linearly with the message signal $m(t)$

General form of angle modulation

$$s(t) = A_c \cos[q_i(t)]$$

Phase modulation

Carrier frequency

$$q_i(t) = 2pf_c t + k_p m(t)$$

$$s(t) = A_c \cos(2pf_c t + k_p m(t))$$

k_p is the phase sensitivity (rad/volt) of the modulator

Frequency modulation (FM)

- FM is that form of angle modulation in which the instantaneous frequency $f_i(t)$ is varied linearly with the message signal $m(t)$

General form of angle modulation

$$s(t) = A_c \cos[q_i(t)]$$

Frequency modulation

$$f_i(t) = f_c t + k_f m(t)$$

$$q_i(t) = 2pf_c t + 2pk_f \int_0^t m(t)dt$$

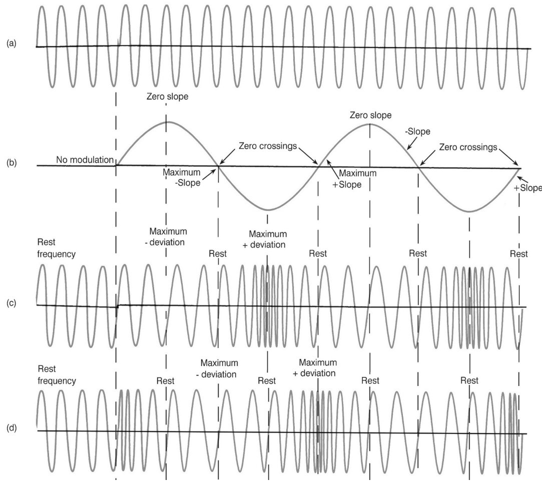
$$s(t) = A_c \cos\left[2pf_c t + 2pk_f \int_0^t m(t)dt\right]$$

k_f is the frequency sensitivity (Hz/volt) of the modulator

PM and FM expressions

Type of Modulation	Modulating Signal	Angle-Modulated Wave, $m(t)$
(a) Phase	$v_m(t)$	$V_c \cos[\omega_c t + K v_m(t)]$
(b) Frequency	$v_m(t)$	$V_c \cos[\omega_c t + K_1 \int v_m(t)\, dt]$
(c) Phase	$V_m \cos(\omega_m t)$	$V_c \cos[\omega_c t + K V_m \cos(\omega_m t)]$
(d) Frequency	$V_m \cos(\omega_m t)$	$V_c \cos\left[\omega_c t + \frac{K_1 V_m}{\omega_m} \sin(\omega_m t)\right]$

PM and FM waveforms



(a) unmodulated carrier

(b) modulating signal

It is impossible to distinguish an FM waveform from a PM waveform without knowing the modulating signal

(c) FM

With FM, the maximum frequency deviation occurs during the maximum positive and negative peaks of the modulating signal

(d) PM

The frequency deviation is proportional to the first derivative of the modulating signal

Phase and frequency modulators and demodulators

1. PM modulator = differentiator followed by an FM modulator
2. PM demodulator = FM demodulator followed by an integrator
3. FM modulator = integrator followed by a PM modulator
4. FM demodulator = PM demodulator followed by a differentiator

- We may thus deduce all the properties of PM signals from those of FM signals and vice versa. Henceforth, we concentrate our attention on FM signals.

Frequency modulation analysis

$$m(t) = A_m \cos(2pf_m t)$$

$$f_i(t) = f_c + k_f A_m \cos(2pf_m t)$$

$$f_i(t) = f_c + \Delta f \cos(2pf_m t)$$

Frequency deviation

$$q_i(t) = 2p \int_0^t f_i(t) dt$$

$$q_i(t) = 2pf_c t + \frac{\Delta f}{f_m} \sin(2pf_m t)$$

$$q_i(t) = 2pf_c t + b \sin(2pf_m t)$$

Modulation index

$$s(t) = A_c \cos[2pf_c t + b \sin(2pf_m t)]$$

Physical meaning

- Frequency deviation is the change in frequency that occurs in the carrier when it is acted on by a modulating signal frequency.
  - Frequency deviation is typically given as a peak frequency shifts in hertz (?*f*)

- Modulation index (*ß*) represents the phase deviation of the FM signal, that is, the maximum departure of the angle $?_i$(t) from the angle $?_c t$ of the unmodulated carrier. *ß* is measured in radians.

In this lecture slide, ß and m are used interchangeably.

Example of frequency deviation and modulation index

- Example 7.1: Determine the peak frequency deviation (?*f*) and modulation index (*m*) for an FM modulator with a deviation sensitivity K=5kHz/V and a modulating signal $v_m(t)$=2cos(2p 2000t)

  - ?*f* =2*5=10 kHz
  - m=10/2=5

Narrow or wide band FM

- Narrow band FM, for which ß is small compared to one radian
- Wide band FM, for which ß is large compared to one radian

Spectrum illustration is provided after explaining the Bessel function.

Frequency analysis of FM waves

- The frequency components of the angle modulated wave are much more complexly related to the frequency components of the amplitude modulated signal.

- In FM, a single frequency modulating signal produces an infinite number of pairs of side frequencies, i.e. infinite bandwidth.
  - However, most of the side frequencies are negligibly small in amplitude and can be ignored.

Frequency analysis: FM Modulation by a single frequency sinusoid

$$s(t) = A_c \cos[2pf_c t + m\cos(w_m t)]$$

The individual frequency components of this FM signal is not obvious in this equation. Using the Bessel function identity as shown below can show all side frequencies of the FM signal.

$$\cos(a + m\cos b) = \sum_{n=-\infty}^{\infty} J_n(m)\cos\left(a + nb + \frac{np}{2}\right)$$

$$J_n(m) = \left(\frac{m}{2}\right)^n \left[\frac{1}{n} - \frac{(m/2)^2}{1!(n+1)!} + \frac{(m/2)^4}{2!(n+2)!} - \frac{(m/2)^6}{3!(n+3)!} + \ldots\right]$$

Frequency analysis: FM Modulation by a single frequency sinusoid (cont.)

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(m)\cos\left(w_c t + n w_m t + \frac{np}{2}\right)$$

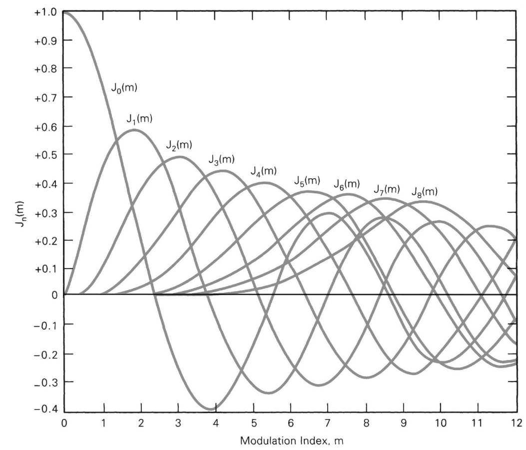
$$s(t) = V_c \left\{ \begin{array}{l} J_0(m)\cos(w_c t) + J_1(m)\cos\left(w_c t + w_m t + \frac{p}{2}\right) - J_1(m)\cos\left(w_c t - w_m t - \frac{p}{2}\right) \\ + J_2(m)\cos(w_c t + 2w_m t) - J_2(m)\cos(w_c t - 2w_m t) + \cdots \end{array} \right\}$$

- *m*: modulation index
- $J_0(m)$: carrier component
- $J_n(m)$: *n*th set of side frequencies displaced from the carrier by nf_m

Bessel function $J_n(m)$

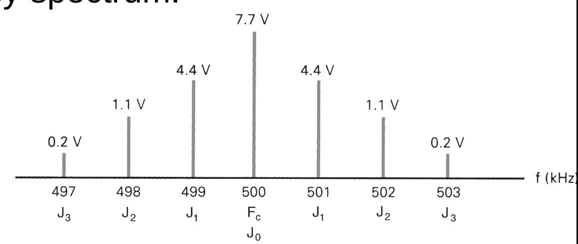
Modulation Index	Carrier	Side Frequency Pairs													
m	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—
2.4	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—
5.45	0	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—
10.0	−0.25	0.05	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01

$J_n(m)$ versus m



Example of FM frequency analysis

- Example 7-2: For an FM modulator with a modulation index *m*=1, a modulating signal $v_m(t)=V_m\sin(2p\ 1000t)$ and an unmodulated carrier $v_c(t)=10\sin(2p\ 500*10^3t)$. Draw the frequency spectrum.

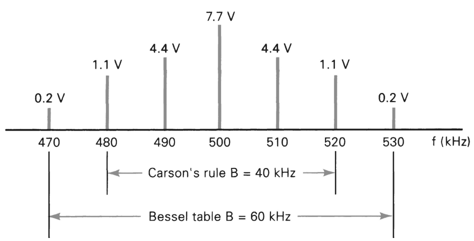


FM bandwidth calculation

- Actual BW=2(n+f_m), where n=number of significant sidebands from the Bessel function table
- BW approximation
  - For m<1, BW=2f_m
  - For m>10, BW=2?f
  - Or using Carson's rule: BW=2(?f+f_m)

Example of BW calculation

- ?f = 10 kHz, f_m=10 kHz, V_c=10 V and f_c=500 kHz

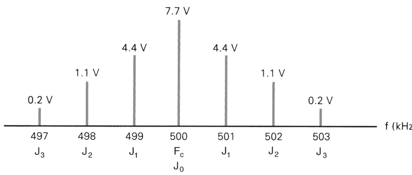


A system that was designed using a Carson's rule would have a poorer performance than a system designed using the Bessel table.

For m>5, Carson's rule is a close approximation to the actual BW required

Example of average power calculation

- Determine the average power of the unmodulated carrier $v_c(t)$=10sin(2p 500*10^{3}t) (assume RL=50 O) .
  - P=10^2/(2*50)=1 W
- The total power of the FM wave is
  - P=7.7^2/(2*50)+2(4.4)^2/(2*50)+ 2(1.1)^2/(2*50)+ 2(0.2)^2/(2*50)=1.0051 W



an FM modulator with a modulation index m=1, a modulating signal $v_m(t)=V_m$sin(2p 1000t) and an unmodulated carrier $v_c(t)$=10sin(2p 500*10^{3}t).

Commercial broadcast band FM

- The FCC has assigned the commercial FM broadcast service a 20 MHz band of frequencies that extends from 88 MHz to 108 MHz.
  - The 20 MHz band is divided into 100 channels of 200 kHz BW.
  - The maximum allowable frequency deviation is 75 kHz with the maximum modulating frequency of 15 kHz.

Commercial broadcast band FM (cont)

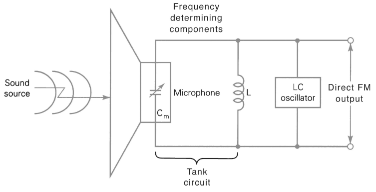
- The modulation index is 75 kHz/15 kHz=5.
  - Referring to the Bessel table, eight pairs of significant side frequencies are produced.
  - The actual BW = 2*8*15 kHz=240 kHz (exceed the FCC allocated BW by 40 kHz)
    - Cause adjacent channel interference
  - The seventh and eighth sets of side frequencies have little power in them, and it is also highly unlikely that maximum frequency deviation is ever obtained at the maximum modulating signal frequency.

FM modulator

- Direct FM modulator
  - The oscillator frequency is changed directly by the modulating signal
- Indirect FM modulator
  - The oscillator frequency is not directly changed. The phase of the output waveform is deviated directly proportional to the modulating signal.

FM modulator/demodulator

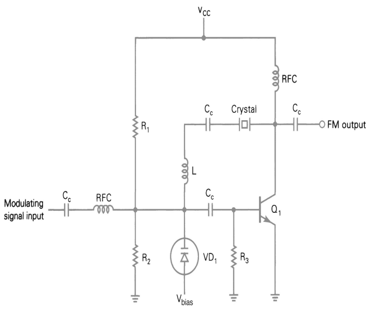
Simple direct FM modulator



impractical

- L and C_m are the frequency determining components for a standard LC oscillator.
- The distance between the plates of C_m is varied according to the acoustic energy. Thus, the value of C_m is varied. Consequently, the resonant frequency is also varied. The output signal is therefore the FM signal.

Varactor diode direct FM modulator



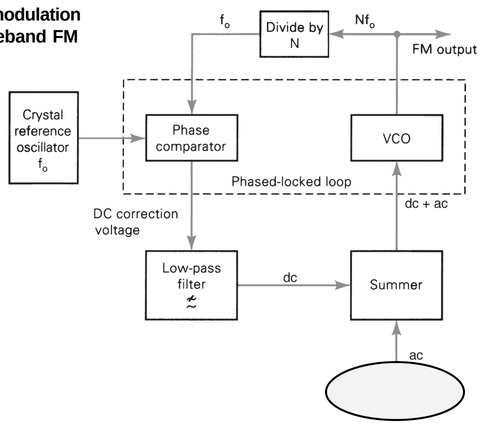
- R1 and R2 develop a dc voltage that reverse biases diode VD1 and determines the rest frequency of the oscillator.
- The external modulating signal voltage adds to and subtracts from the DC bias, which changes the capacitance of the diode and thus the frequency of oscillation.

Varactor diode direct FM modulator

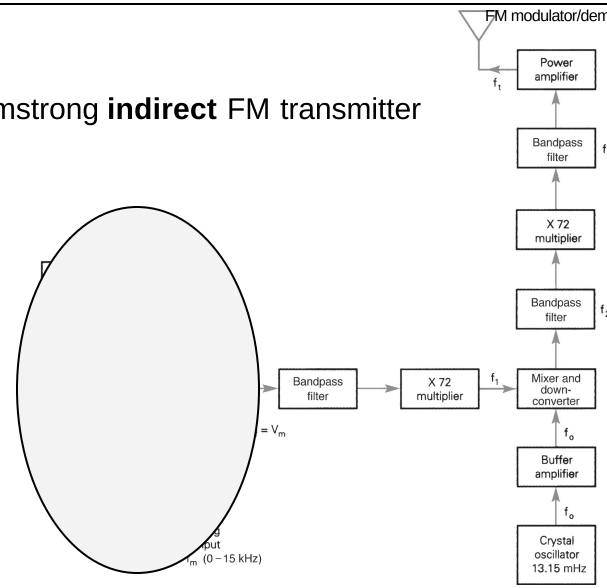
- Because a crystal is used, the peak frequency deviation is limited to small values. So they are used primarily for low modulation index applications.

PLL FM direct transmitter

For high modulation index wideband FM



Armstrong **indirect** FM transmitter



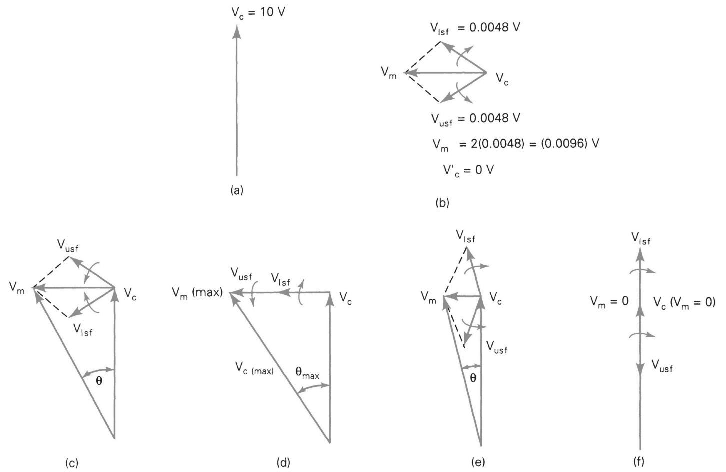
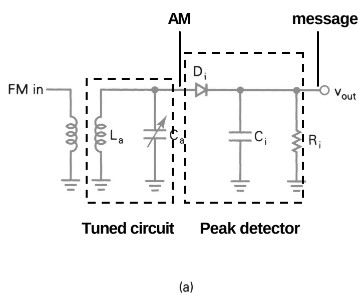


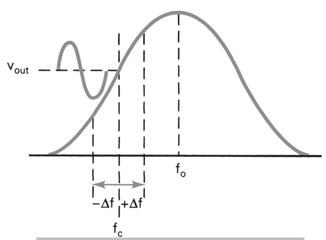
FIGURE 7-28 Phasor addition of V_c, V_{usf}, and V_{lsf}: (a) carrier phasor; (b) sideband phasors; (c) - (f) progressive phasor addition. Part (d) shows the peak phase shift.

FM demodulator

- Slope detector
- PLL FM demodulator

Slope detector

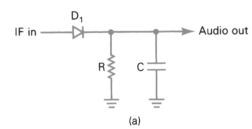




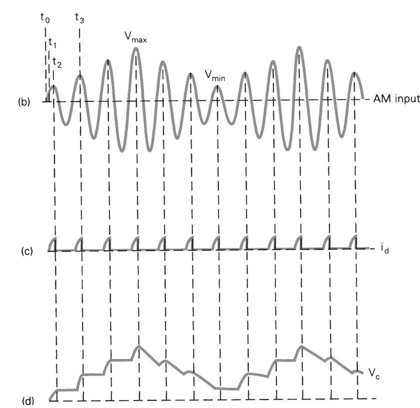
Output voltage of the tuned circuit is proportional to the input frequency of FM signal

Seldom used but its circuit operation is basic to all tuned circuit frequency discriminators

FIGURE 5-26 Peak detector: (a) schematic diagram; (b) AM input waveform ; (c) diode current waveform; (d) output voltage waveform



Peak detector



PLL FM demodulator

- The natural frequency of the VCO is set at the IF center frequency of the FM signal
- The PLL input is a deviated FM signal
- The correction voltage at the output of the phase comparator that is fed back to the VCO is proportional to the frequency deviation and is, thus, the demodulated information signal.

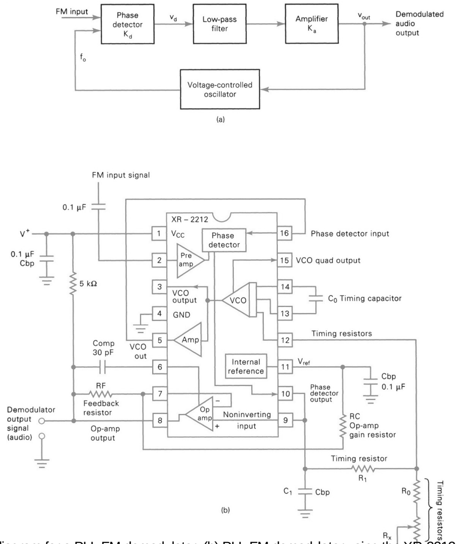
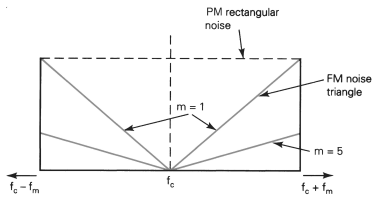


FIGURE 8-7 (a) Block diagram for a PLL FM demodulator; (b) PLL FM demodulator using the XR-2212 PLL

Output noise at FM demodulator

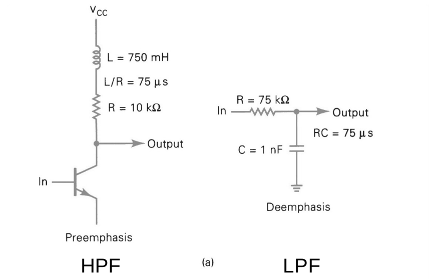
- The noise (both interference and thermal noise) voltage at the output of an FM demodulator increases linearly with frequency.



This is commonly called FM noise triangle

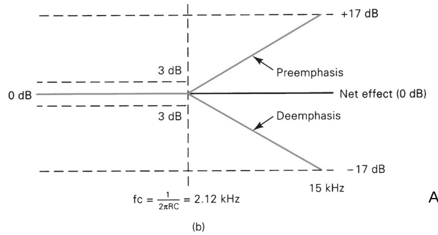
Preemphasis/Deemphasis

- To compensate for the FM noise triangle, the high frequency modulating signals are boosted in amplitude in the transmitter prior to performing modulation.
  - This process is called preemphasis
- To compensate for this boost, the high frequency signals are attenuated in the receiver after demodulation.
  - This process is named deemphasis



$$f_b = \frac{1}{2\pi RC} = \frac{1}{2\pi L / R}$$

The frequency where preemphasis begins



Attenuation curve of FM broadcast

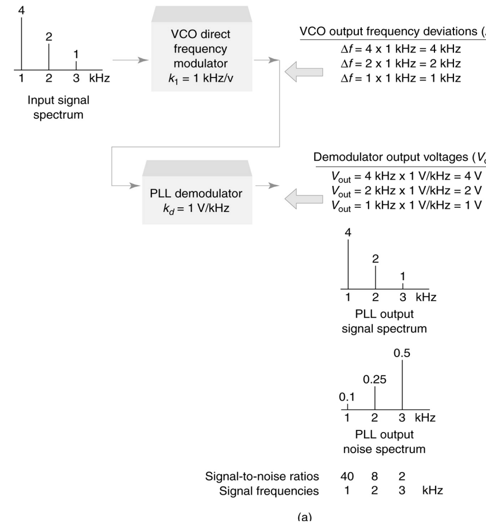


FIGURE 7-15 Figure for Example 7-8: (a) S/N without preemphasis and deemphasis; (*Continued*)

Noise at FM demodulator

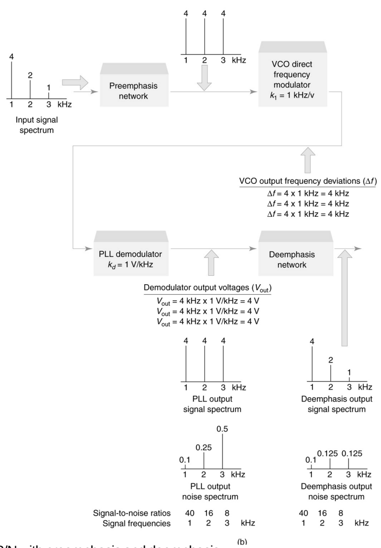


FIGURE 7-15 (*Continued*) (b) S/N with preemphasis and deemphasis

Summary

- Angle modulation is mathematically described
- The difference between FM and PM is shown
- The frequency deviation and the modulation index are defined
- The BW requirements of an FM waveform is determined
- FM modulators/demodulators are illustrated
- The output noise at FM demodulator and how to compensate are described