1. (30 pts.) Find the derivatives of the following functions (you need not simplify):

(a) $f(x) = e^{-\pi^3} - \dfrac{e^{2-x}}{\ln(1-x)}$

$$f'(x) = 0 - \frac{\ln(1-x)e^{2-x}(-1) - e^{2-x}(\frac{-1}{1-x})}{[\ln(1-x)]^2}$$

(b) $f(x) = (e^{\sqrt{x}})\ln x + \dfrac{4}{e^x}$

$$f'(x) = (e^{\sqrt{x}})(\frac{1}{x}) + (\frac{1}{2\sqrt{x}})(e^{\sqrt{x}})\ln x - \frac{4}{e^x}$$

(c) $f(x) = \sqrt[3]{\ln(1-x^5)} - \dfrac{1}{(x-2)^4}$.

$$f'(x) = \frac{1}{3}[\ln(1-x^5)]^{-\frac{2}{3}}(\frac{-5x^4}{1-x^5}) + \frac{4}{(x-2)^5}.$$

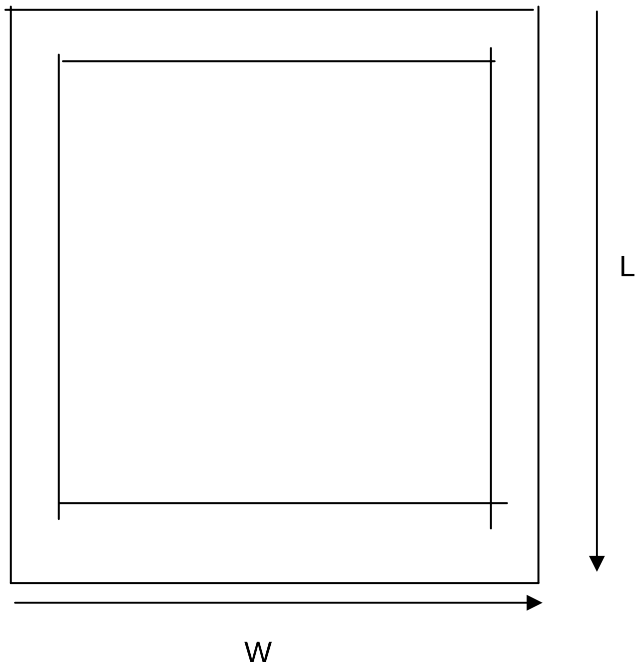
2. (5 pts.) The resale value R (in dollars) of a certain model car after t years is given by $R(t) = 20{,}000\,e^{-0.1t}$. What are the instantaneous and relative rates of change (depreciation) the moment the car is sold (t = 0)?

$R'(t) = 20{,}000\,e^{-0.1t}(-0.1) = -2{,}000.\,e^{-0.1t}$

$R'(0) = -2{,}000$ dollars/year (instantaneous rate of change at t = 0).

$\dfrac{R'(0)}{R(0)} = -\dfrac{2000\ dollars/year}{20000\ dollars} = -\dfrac{.1}{year} = -\dfrac{10\%}{year}$ (relative rate of change at t = 0)

3. (15 pts.) A poster is to have 2-inch margins at the top and bottom and 1-inch margins on the sides. The total area is to be 162 square inches. Find the dimensions that maximize the print area. (The area of a rectangle is given by A=(length)(width)).



Let TA be the total area and PA the print area of the poster. Then

$TA = LW = 162;\ L = \dfrac{162}{W}$

$PA = (L - 4)(W - 2) = LW - 4W - 2L + 8 = 162 - 4W - 2L + 8 = 170 - 4W - 2L$

$PA(W) = 170 - 4W - 2\left(\dfrac{162}{W}\right),\ W > 2$

$PA'(W) = -4 + \dfrac{324}{W^2}$

$PA'(W) = 0$ for $4W^2 = 324$ or $W = 9$ in.

$PA''(W) = -\dfrac{648}{W^3}$

$PA''(9) < 0$, so PA is an absolute maximum for W = 9 in. (by the second derivative test for absolute extreme values)

The dimensions that maximize the print area are W= 9 in. and L $\dfrac{162\ in^2}{9\ in} = 18$ in.

4. (20 pts.) $f(x) = x^4 - 4x^3 = x^3(x - 4)$. $f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$, and $f''(x) = 12x^2 - 24x = 12x(x - 2)$.
(a) Find the critical numbers and the inflection points of f. (b) Construct sign charts for the first and second derivatives. (c) Find all open intervals of increase and decrease and open intervals on which the graph is concave up and concave down. (d) Classify each critical point as a relative maximum, relative minimum or neither. (e) Sketch the graph of $y = f(x)$ by hand, labeling **only** the relative extreme points and the inflection points (Note that $f(-3) = 189$, $f(-2) = 48$, $f(0) = 0$, $f(2) = -16$, and $f(3) = -27$).

(a) The critical numbers are x = 0 and x = 3. The are inflection points at x = 0 and x = 2, (0, 0) and (2, -16).

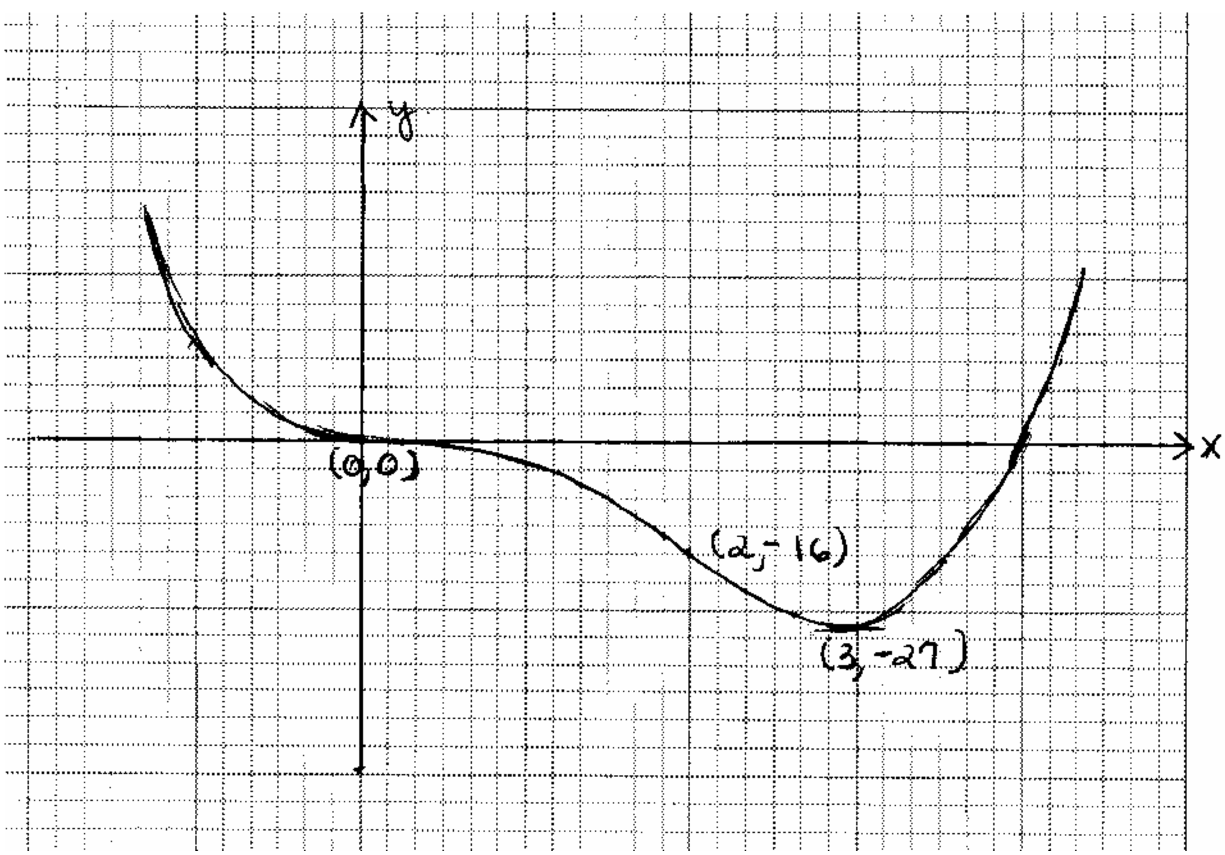
(b) – – + f'(x) + – + f''(x)

_______0______________3________ _______0__________2_____________

(c) Increasing $(3, \infty)$; Decreasing $(-\infty, 0)$, $(0, 3)$; Concave up $(-\infty, 0)$, $(2, \infty)$; Concave down (0,2)

(d) x = 0 or (0, 0) is neither; x = 3 or (3, -27) is a relative minimum

(e)



5. (10 pts.) A cherry tree will yield 100 pounds of cherries now, which will sell for 60 cents a pound. Each week that the farmer waits will increase the yield by 5 pounds, but the selling price will decrease by 2 cents per pound. How long should the farmer wait to pick the fruit in order to maximize his revenue?

Let x be the number of weeks that the farmer should wait.

Then his yield (in pounds per tree) is $q(x) = 100 + 5x$ and

his selling price (in cents per pound)is $p(x) = 60 – 2x$.

His revenue (in cents per tree) is $R(x) = (60 – 2x)(100 + 5x) = 6000 + 100x – 10x^2$.

$R'(x) = 100 – 20x$

$R'(x) = 0$ for $x = 5$

$R''(x) = – 20 < 0$, so there is an absolute maximum at $x = 5$.

The farmer should wait 5 weeks.

6. (10 pts.) A demand function is given by $D(p) = 12{,}000 – 10p^2$. Find the elasticity of demand, E(p), at p = 20. Determine whether the demand is elastic, inelastic, or unitary at p = 20. Should the price be raised, lowered, or left the same, in order to increase revenue?

$D'(p) = -20p$

$$E(p) = -\frac{pD'(p)}{D(p)} = \frac{20p^2}{12000 - 10p^2}$$

$$E(20) = \frac{20(20^{2)}}{12000 - 10(20^2)} = \frac{8000}{12000 - 4000} = \frac{8000}{8000} = 1.$$ The demand is unitary, so the price should be left the same.

7. (10 pts.) $xy^2 = 2y + x^2$. Find $\frac{dy}{dx}$ and an equation of the tangent line at x = 2 and y = -1.

$$2xy\frac{dy}{dx} + y^2 = 2\frac{dy}{dx} + 2x$$
$$2xy\frac{dy}{dx} - 2\frac{dy}{dx} = 2x - y^2$$
$$(2xy - 2)\frac{dy}{dx} = 2x - y^2$$
$$\frac{dy}{dx} = \frac{2x - y^2}{2xy - 2}$$

$$\frac{dy}{dx}\Big|_{(2,-1)} = \frac{2(2) - (-1)^2}{2(2)(-1) - 2} = \frac{4 - 1}{-4 - 2} = \frac{3}{-6} = -\frac{1}{2}$$

L_T: $(y + 1) = -\frac{1}{2}(x - 2) = -\frac{1}{2}x + 1$

$y = -\frac{1}{2}x$ is an equation of the tangent line at x = 2 and y = -1.

Extra credit (5 pts.) If $\log_a 2 = W$, $\log_a 3 = X$, $\log_a 5 = Y$, and $\log_a 7 = Z$, find $\log_a(\frac{9}{14})$.