**Math 0120**
**Examination #3**
**Sample Solutions**

**Name (Print)** ______________________________ **PeopleSoft #** __________.

**Signature** ____________________________________ **Score** _________.

**TA (Circle one)**

**Instructions:**

1. Clearly print your name and PeopleSoft number and sign your name in the space above.

2. There are 8 problems, each worth the specified number of points, for a total of 100 points. There is also an extra-credit problem worth 5 points.

3. Please work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.

4. Show all calculations and display answers clearly. Unjustified answers will receive no credit.

5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.

6 **No calculators, headphones, tables, books, notes, or computers may be used. All derivatives are to be found by learned methods of calculus.**

1. Let $f(x)=\frac{1}{x}$.

(a) (5 pts.) Approximate the area under the curve y = f(x) from a = 1 to b = 13 using a Riemann sum with 3 left rectangles. (Write the sum; you need not evaluate it.)

$\Delta x=\frac{13-1}{3}=4$. $x_1 = 1$, $x_2 = 5$, $x_3 = 9$

$A \approx R_L(3) = [f(1) + f(5) + f(9)](4) = [1 + \frac{1}{5}+\frac{1}{9}](4) = 4+\frac{4}{5}+\frac{4}{9}$

(b) (5 pts.) Find the exact area under the curve y = f(x), 1 ≤ x ≤ 13, using the Fundamental Theorem of Calculus.

$A = \int_1^{13} \frac{1}{x}\,dx = \ln|x|\Big|_1^{13} = \ln(13) - \ln(1) = \ln(13)$

2. (12 pts.) Find the following integrals:

(a) $\int (x^4 - \frac{3}{e^{-x}} - \frac{4}{x} - \sqrt[4]{x^5} + \frac{2}{\pi})dx = \frac{x^5}{5} - \frac{3}{e^{-x}} - 4\ln|x| - \frac{4}{9}x^{\frac{9}{4}} + \frac{2}{\pi}x + C$

(b) $\int \frac{x^2-1}{x+1}\,dx = \int (x-1)\,dx = \frac{x^2}{2} - x + C$

3. (24 pts.) Use substitution to find the following integrals:

(a) $\int \frac{x^2 - x}{(2x^3 - 3x^2)^3}\, dx = \frac{1}{6}\int \frac{du}{u^3} = -\frac{1}{12}u^{-2} + C = -\frac{1}{12}(2x^3 - 3x^2)^{-2} + C$

Let $u = 2x^3 - 3x^2$.

$du = (6x^2 - 6x)\, dx = 6(x^2 - x)\, dx$

$\frac{du}{6} = (x^2 - x)\, dx$

(b) $\int_0^3 x\sqrt{x^2 + 16}\, dx = \frac{1}{2}\int_{16}^{25} \sqrt{u}\, du = \frac{1}{2}\frac{2}{3}u^{\frac{3}{2}}\Big|_{16}^{25} = \frac{1}{3}(25^{\frac{3}{2}} - 16^{\frac{3}{2}}) = \frac{1}{3}(125 - 64) = \frac{61}{3}$

Let $u = x^2 + 16$

$du = 2x\, dx$

$\frac{du}{2} = x\, dx$

(c) $\int x\sqrt{x-3}\, dx = \int (u+3)\sqrt{u}\, du = \int (u^{\frac{3}{2}} + 3u^{\frac{1}{2}})\, du = \frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + C = \frac{2}{5}(x-3)^{\frac{5}{2}} + 2(x-3)^{\frac{3}{2}} + C$

$u = x - 3$

$du = dx$

$u + 3 = x$

4. (10 pts.)The acceleration of a particle at time t seconds is given by $\mathbf{a(t) = t + e^{-0.5t}}$ ft./sec$^2$. Find **s(t),** the distance of the particle at time t seconds, if its initial velocity (the velocity at time t=0) is 5 ft/sec, and **s(0)** =8 feet.

$v(t) = \int a(t)\, dt = \frac{t^2}{2} - 2e^{-0.5t} + C_1$ ; $v(0) = -2 + C_1 = 5$. $C_1 = 7$; $v(t) = \frac{t^2}{2} - 2e^{-0.5t} + 7$

$s(t) = \int v(t)\, dt = \frac{t^3}{6} + 4e^{-0.5t} + 7t + C_2$ ; $s(0) = 4 + C_2 = 8$. $C_2 = 4$; $s(t) = \frac{t^3}{6} + 4e^{-0.5t} + 7t + 4$

5. (5 pts.) Find the average value of $f(x) = e^x$ on [0, ln4].

$$f_{avg} = \frac{1}{\ln 4 - 0}\int_0^{\ln 4} e^x\, dx = \frac{1}{\ln 4 - 0} e^x \Big|_0^{\ln 4} = \frac{1}{\ln 4}(4-1) = \frac{3}{\ln 4}$$

6. (16 pts.) Set up, but do not evaluate, integrals for the area

(a) Between the curves $y = x$ and $y = 4x - x^2$ on [-1,1].

$x = 4x - x^2$
$x^2 - 3x = x(x - 3) = 0$: $x = 0$, $x = 3$

$$A = \int_{-1}^{0} (x^2 - 3x)\, dx + \int_0^1 (3x - x^2)\, dx$$

(b) Bounded by the curves $y = x$ and $y = x^3$.

$x = x^3$
$x^3 - x = x(x - 1)(x + 1) = 0$: $x = -1, 0, 1$

$$A = \int_{-1}^{0} (x^3 - x)\, dx + \int_0^1 (x - x^3)\, dx$$

7. (7 pts.) Given a demand function of **$d(x) = 400 - 0.3x$** and a supply function of **$s(x) = 200 + 0.2x$**.

(a) Find the market demand (the positive value of x at which the demand function intersects the supply function).

$400 - .3x = 200 + 0.2x$
$200 = .5x$
$x = 400 = A$ (market demand)

(b) Set up, but do not evaluate a definite integral for the **producers' surplus** at the market demand.

$B = d(400) = s(400) = 280$

$$\text{C.S.} = \text{P.S.} = \int_0^{400} ((400 - .3x) - 280)\, dx = \int_0^{400} (120 - .3x)\, dx$$

$$\text{P.S.} = \int_0^{400} (280 - (200 + .2x))\, dx = \int_0^{400} (80 - .2x)\, dx$$

8. (16 pts.) Use integration by parts to find:

(a) $\int (x^{-2} \ln x)\, dx = -x^{-1}\ln x + \int x^{-2}\, dx = -x^{-1}\ln x - x^{-1} + C$

$u = \ln x \qquad dv = x^{-2}\, dx$

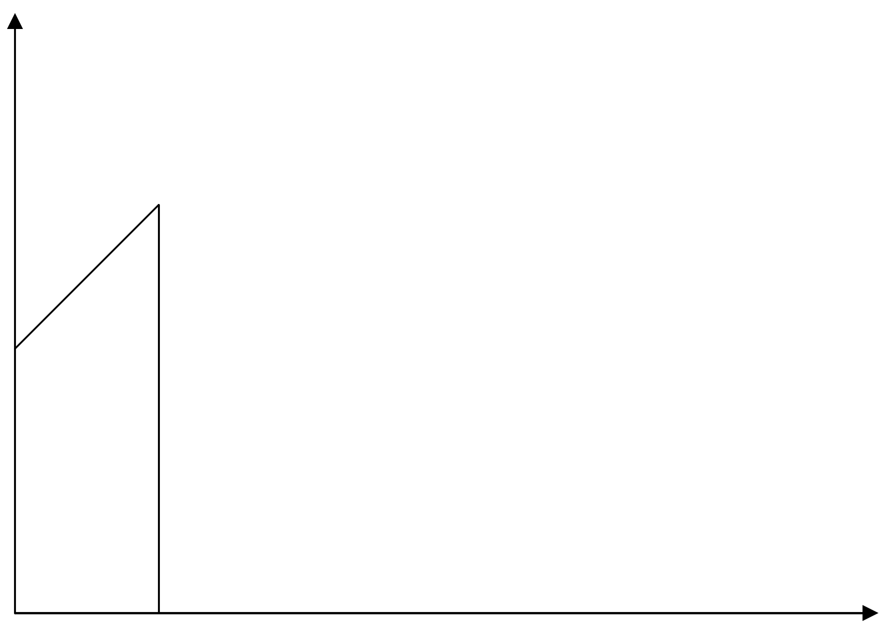
$du = \dfrac{dx}{x} \qquad v = -x^{-1}$

(b) $\int xe^{-2x}\, dx = -\dfrac{x}{2}e^{-2x} + \dfrac{1}{2}\int e^{-2x}\, dx = -\dfrac{x}{2}e^{-2x} - \dfrac{1}{4}e^{-2x} + C$

$u = x \qquad dv = e^{-2x}\, dx$

$du = dx \qquad v = -\dfrac{e^{-2x}}{2}$

**<u>(5 pts) Extra-Credit</u> :** You may earn an extra 5 points by evaluating $\int_0^2 (x+4)\, dx$ <u>without using the Fundamental Theorem of Calculus</u>.



$\int_0^2 (x+4)\, dx = A_{trapezoid} = \dfrac{1}{2}(4 + 6)\,(2) = 10$