

Math 0120 Final Examination
Sample

Name (Print) _____

ID # _____

Signature _____

Instructor (circle one):

Lecture time (circle one):

Instructions:

1. Show your University of Pittsburgh ID if requested.
2. Clearly print your name and PeopleSoft number and sign your name in the space above. Circle the name of your lecturer and the time of your lecture.
3. Work each problem in the space provided. Extra space is available on the back of each exam sheet. Clearly identify the problem for which the space is required when using the backs of sheets.
4. Show all calculations and display answers clearly. Unjustified answers will receive no credit.
5. Write neatly and legibly. Cross out any work that you do not wish to be considered for grading.
6. **No tables, books, notes, headphones, calculators, or computers may be used. All derivatives and integrals are to be found by learned methods of calculus.**

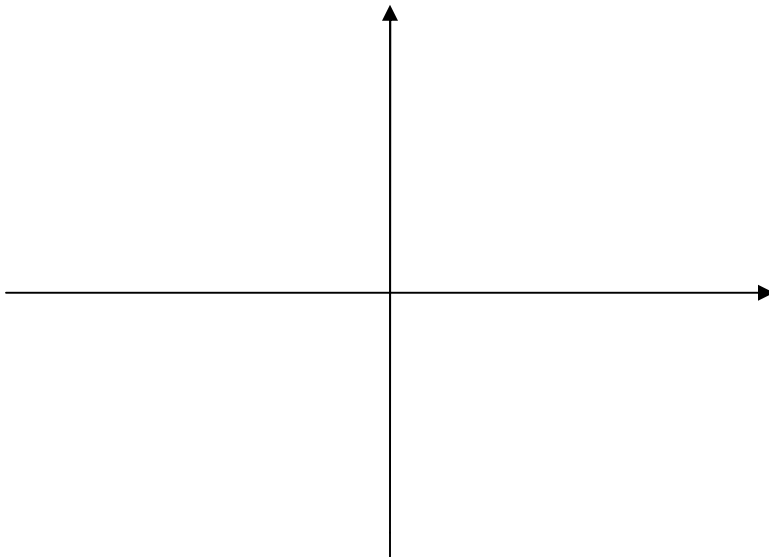
DO NOT WRITE BELOW THIS LINE

Problem	Points	Score	Problem	Points	Score
1	10		6	20	
2	32		7	40	
3	24		8	22	
4	12		9	12	
5	13		10	15	
			Total	200	

1. (a) (3 pts.) Find an equation of the line that passes through the point (1, 2) and has slope $m = -2$.

(b) (3 pts.) Find the vertex of the parabola given by $y = f(x) = x^2 - 2x$.

(c) (4 pts.) Sketch the line and the parabola on the same set of axes, labeling the vertex of the parabola and the x- and y- intercepts of both.



2. (32 pts.) Find $f'(x)$. **You need not simplify.**

(a) $f(x) = e^{x^2} - \frac{1}{x} + e + \sqrt[4]{x^3}$

(b) $f(x) = \left(\frac{1}{2x} + 3\right)^5$

(c) $f(x) = \frac{2x^3 - x}{1 - 2x}$

(d) $f(x) = x \ln(1 - x)$

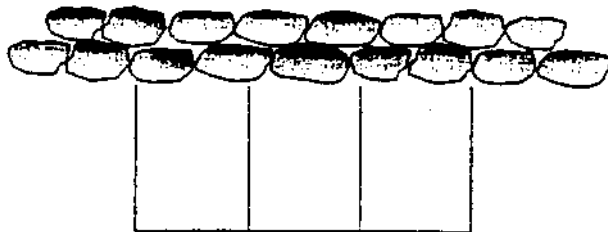
3. (a) (8 pts.) $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$. Use this definition to find the derivative of $f(x) = \frac{1}{x^2}$.

(b) (8 pts.) A snowball is melting so that the radius is decreasing at a rate of 2 inches per hour. How fast is the volume decreasing at the moment when the radius is 10 inches? (The volume of a sphere in terms of the radius is $V = \frac{4}{3}\pi R^3$).

(c) (8 pts.) Find an equation of the tangent line to $f(x) = \ln(x^3)$ at $x = 1$.

4. (12 pts.) City Computers Incorporated finds that it costs \$800 to manufacture each PC, and fixed costs are \$500 per day. The price function is $p(x) = 1600 - 10x$, where $p(x)$ is the price (in dollars) at which exactly x PCs will be sold. Find the number of PCs that City Computers should produce and the price it should charge to maximize profit.

5. (13 pts.) A homeowner wishes to enclose three adjacent rectangular pens of equal size along a straight wall, as in the diagram. If the side along the wall needs no fence, what is the largest total area that can be enclosed using 320 feet of fencing?



6. (20 pts.) Given $f(x) = x^3 - 12x$, do the following:

(a) Make a sign diagram for the first derivative of $f(x)$.

(b) Make a sign diagram for the second derivative of $f(x)$.

(c) State the open intervals on which $f(x)$ is increasing, decreasing, concave up and concave down.

(d) Sketch the graph of $y = f(x)$ by hand, labeling all relative extreme points, inflection points and intercepts.

7. (40 pts.) Find the following integrals:

(a) $\int (5x^3 + \pi^2 - \sqrt[3]{x^4}) dx$

(b) $\int \frac{\sqrt{x+1}}{x} dx$

(c) $\int x e^{0.2x} dx$

(d) $\int (x^2 + 1)\sqrt{x^3 + 3x} dx$

8. (a) (12 pts.) Find the **area bounded by** the curves $y = x^3$ and $y = 4x$.

(b) (10 pts.) Find the average value of $f(x) = e^x$ on $[0, \ln 8]$

9. (12 pts.) Find all critical points of $f(x,y) = 6xy - x^3 - 3y^2$ and classify each as a relative maximum, relative minimum, or saddle point.

10. (15 pts.) Use the method of Lagrange multipliers to maximize **and** minimize $f(x,y) = 2x + y$ subject to the constraint $x^2 + 2y^2 = 72$. (Both extreme values exist.)