Pricing Practices of Resellers in the Airline Industry: Posted Price vs. Name-Your-Own-Price Mechanisms

by

Esther Gal-Or*

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*Mailing address: 222 Mervis Hall, Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, PA 15260. Email: esther@katz.pitt.edu . Phone: 412-648-1722. Fax: 412-648-1552.

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Abstract

We consider a simple, two stage model to capture the main characteristics of demand classes in the airline industry. While leisure travelers learn of their need to fly in the first stage, business travelers become informed of this need in the second stage. Since business class demand is stochastic and has a higher willingness to pay, airlines find it optimal to reserve capacity for sale in the second stage, after offering advance-purchase tickets to some leisure travelers. Airlines can use resellers in the second stage to clear any unsold capacity they experience. We found that when there is a single reseller in the market, airlines prefer that this reseller utilizes the Name Your-Own Price (a la Priceline) instead of the regular Posted Price (a la Hotwire) model. Introducing competition among resellers eliminates the distinction between the two pricing models from the perspective of the airline. Either form of pricing generates the same outcome with competition as vertical integration of the airline with the downstream market of resellers. Interestingly, leisure travelers do not necessarily benefit from such vertical integration, since a larger portion of them is exposed to the risk of not being served at all, when the airline is vertically integrated with the resellers.

1. Introduction

The emergence of commercial websites such as Priceline, Hotwire, Travelocity, and Expedia has had a significant effect on pricing practices of service providers active in travel related industries. Those web portals allow providers in the airline, car rental, and lodging industries to clear excess capacity to a larger extent than they could prior to the proliferation of resellers on the Internet. Moreover, with the aid of those resellers, providers can maintain high prices on their direct marketing channels, while still clearing capacity by permitting the resellers to offer "last minute" special deals. This option can, in fact, be attractive to airlines due to the significant uncertainty they face regarding the extent of demand from business travelers, who normally learn of their need to travel very close, and sometimes on the date of departure. Since this segment of consumers has a relatively high willingness to pay in comparison to leisure travelers, airlines are tempted to reserve capacity for such "last minute" customers. They offer this reserved capacity on their direct websites for very high prices in order to extract the surplus of business travelers, while using the resellers to absorb excess capacity, if it arises. This approach is facilitated by business travelers' reluctance to use resellers in booking trips, given that the availability of seats via the resellers is at the mercy of the airlines, and as a result cannot be guaranteed.[1]

In the present paper we evaluate how the existence of resellers affects the decision of providers in the travel industry concerning the allocation of capacity between advance-purchase and last minute sales. We conduct this evaluation for two different types of pricing models utilized by the reseller: regular Posted Price (PP) and Name-Your-Own-Price (NYOP) models. While Hotwire, Travelocity, and Expedia are examples of portals utilizing the regular take-it-or-leave-it PP mechanism, Priceline collects individual customer offers (price bids guaranteed by credit cards) and communicates the information to participating providers or to their private databases. It operates on a commission plus the difference between the consumer bid and the price it pays the service provider (Dolan and Moon 2000; Maguire 2002.)

We start our investigation by considering a monopolist airline and a single reseller. In such a successive monopoly setting, the airline prefers that the reseller utilizes the NYOP rather than the PP model. Under the NYOP paradigm consumers submit bids in accordance with their

[1] A *USA Today* July 7, 2003 report mentions that *Hotwire* estimates that 8% of its hotel bookings are made by business travelers. While no comparable estimate is available for *Priceline*, its more restrictive bidding model for airline seats is likely to attract even fewer business travelers.

preferences, thus facilitating improved price discrimination in comparison to an environment where all consumers face an identical price. Since price discrimination facilitates extracting more successfully the consumer surplus, the airline can charge the reseller higher wholesale prices under the NYOP model. However, leisure travelers may actually be worse off when the NYOP mechanism is utilized by the reseller. When the level of capacity is fixed and the reseller switches from PP to NYOP, the airline chooses to raise the price of advance-purchase tickets, thus providing higher powered incentives to leisure travelers to postpone their purchase. As a result a bigger number of them are exposed to the risk of not being able to fly at all. Since the airline gives higher priority in allocating his capacity to business travelers buying directly from him rather than to the reseller, leisure travelers who postpone their purchase reduce the likelihood of being able to obtain tickets. When capacity is endogenously selected by the airline to maximize expected profits, the NYOP regime leads to higher investment in capacity than the PP regime and to a smaller number of advance-purchase tickets offered by the airline. As a result, the welfare comparison of the two pricing models from the perspective of the consumers becomes ambiguous. On the positive side, the higher investment under NYOP results in lower wholesale prices charged by the airline, and therefore, to lower bids becoming acceptable to the reseller. On the negative side, the smaller number of advance-purchase tickets still exposes the leisure travelers to a higher risk of not being able to fly. A similar ambiguity in the comparison of the two pricing models exists also from the perspective of the reseller. While the switch from PP to NYOP introduces a disadvantage to the reseller since the airline raises the wholesale prices he charges, it also expands the size of the leisure segment that opts to buy tickets from the reseller.

When competition among resellers is introduced in the model, with at least one of them using the PP mechanism, the difference between the PP and NYOP mechanisms disappears. The exact same allocation of capacity and prices arise at the equilibrium, irrespective of the pricing model adopted by each reseller. The outcome with a competitive resellers' market is equivalent, in fact, to the monopolist airline vertically integrating with the resellers' downstream market. Under vertical integration, the monopolist is indifferent between the two pricing mechanisms. The main results of the paper extend to an environment with two competing airlines. The difference is that competing airlines are more reluctant to reserve capacity for "last minute" demand arising from business travelers. Since competition implies that the two airlines cannot fully internalize the benefits from sales to business travelers, airlines choose to allocate a larger portion of their capacities for advance-purchase sales to leisure travelers.

Consumers in our model are divided into two groups: leisure and business travelers. While the former learn of their need to travel early in advance of the departure date, the latter learn this information closer to the date of travel. Leisure travelers have heterogeneous preferences in terms of their willingness to pay, and the willingness to pay of business travelers exceeds the average reservation price of leisure travelers. The earlier literature dealing with pricing practices in the airline industry as implied by the existence of multiple classes of demand has made similar assumptions. For instance, while considering a model with two possible time slots for flying, the earlier literature assumed that all consumers share a common reservation price for flying at their most preferred time slot. However, business travelers incur higher disutility than leisure travelers when flying at their least-preferred time slot (see Gale and Holmes 1992, 1993, Gale 1993, and Dana 1999). Dana (1998) incorporates additional heterogeneity among consumers by assuming that they can differ both in their reservation prices and in the probability of requiring the service. Firms face aggregate demand uncertainty since they do not know in advance which of the two flights will be the peak-demand flight. Our model is also characterized by aggregate demand uncertainty due to fluctuations in the number of business travelers demanding service. Noche and Peitz (2007) is another study addressing the topic of advance-purchase by consumers. However, in their paper it is not limited capacity and aggregate demand uncertainty that motivates the firm to offer the advance-purchase option. Instead, it is the uncertainty that each consumer faces about her own valuation of the service/product offered by the firm.

As in the previous papers, we also find that airline companies find it optimal to offer advance-purchase tickets to some leisure travelers who have relatively high reservation prices. However, another segment of the population of leisure travelers, having relatively low reservation prices, chooses to postpone the purchase and buy the tickets from resellers. This option is unavailable in the previous papers, where all sales occur only via the direct marketing channel of the airlines. This latter assumption of the earlier literature implies also that airlines may experience excess capacity. In our model, capacity is always cleared by offering unsold capacity via the resellers. The existence of resellers makes it difficult for an airline to pre-commit to leaving some capacity unutilized. As long as it can obtain a positive price for an empty seat it will have an incentive to use resellers to clear capacity. Even if clearing capacity may have a detrimental effect on the prices that the airline can charge for advance-purchase tickets, a commitment to leave some capacity idle is not credible. The issue of credibility in dynamic pricing has been widely discussed in the Durable Good Monopolist literature (see, for instance, Coase 1972, Stokey 1981, and Bulow 1982).

Our model is also related to the topic of clearance sales conducted by retailers that was discussed by Lazear (1986). In Lazear's paper, retailers change their prices over time after updating their expectations of consumers' preferences based upon the length of time that products remain unsold on the shelf. In our model, airlines use the resellers to clear their capacities, thus permitting them to maintain high prices in their direct-marketing channels.

Another strand of literature that is related to our paper is the extensive work on yield or revenue management available in the Operations Research literature (see, for instance, Belobaba 1989, Smith, Leimkuhler, and Darrow 1992, Weatherford and Bodily 1992, and Belobaba and Wilson 1997). Most of this literature, however, treats prices as exogenous, whereas in our model prices are determined endogenously. The literature on priority service pricing under demand uncertainty is also related to the present analysis (see Harris and Raviv 1981 and Chao and Wilson 1987.) We use a much simpler specification than the mechanism design approach of this literature. However, the basic idea of optimally allocating scarce supplies by offering to consumers a choice between high priority and low priority access to the service is preserved in our model. Specifically, leisure travelers can choose between advance-purchase of the service, in which case service is guaranteed, and postponed purchase via resellers, in which case there is a lower probability of gaining access to the service. In an earlier working paper, Wang et-al. (2007) develop a similar framework while restricting attention to the NYOP mechanism. In this paper, the authors derive conditions under which a monopolist service provider finds it optimal to transact with a NYOP reseller. In the current paper, we extend the investigation to allow for a comparison between the NYOP and PP mechanisms, and for competition both in the service and reseller markets.

The rest of this paper is organized as follows. In the next section we describe the main assumptions of the model. In Section 3 we derive the equilibrium for a successive monopoly airline-reseller model when capacity is exogenous and the reseller utilizes the PP mechanism. In Section 4 we do the same when the reseller utilizes the NYOP mechanism. In Section 5 we introduce competition among resellers and in Section 6 we allow the airline to choose capacity optimally. In section 7 we consider competition among airlines, and in Section 8 we conclude the paper. The appendix includes the proofs of all the Lemmas and Propositions.

2. The Model

A monopolist service provider can offer his service via two different channels: his own direct marketing channel and that of an intermediary who serves as a clearinghouse for any excess capacity the provider experiences. While the monopolist can only use a posted-price mechanism in selling his product on the direct marketing channel, we consider two different pricing mechanisms practiced by the intermediary. The first is the posted price mechanism, where the intermediary posts a take-it-or-leave-it retail price to consumers and the second is a Name-Your-Own-Price mechanism, where the intermediary collects bids from consumers interested in the service and accepts any bid that exceeds the wholesale price it has to pay the service provider.[2] We refer to the intermediary's channel as the clearinghouse channel.

The provider faces two different groups of customers. The first group becomes aware of its need for the service early, in advance of the date the service is rendered, and the second group becomes aware of its need very close to the date of service. In the context of the airline industry, those two groups correspond to leisure and business travelers. While leisure travelers have the flexibility to plan their trips well in advance of the actual departure date, business travelers learn of their need to conduct certain business trips very close, and sometimes on the day of departure. We will use the airline industry example in motivating the assumptions of our model throughout the paper.

In order to capture the different time that consumers may become aware of their need for the service we model the environment as consisting of two stages. While leisure travelers learn of their need to travel in the first stage, business travelers become aware of their need in the second stage. Both stages occur prior to the date the service is rendered. We assume that leisure travelers differ in their willingness to pay for the service. For simplicity, we assume that the reservation price of a leisure traveler is uniformly distributed over the unit interval.[3] We denote this reservation price by $v \in [0,1]$. In contrast, we assume that all business travelers share a

[2] Accepting a bid lower than the wholesale price would imply that the retailer incurs losses. On the other hand, any bid in excess of the wholesale price generates a positive surplus for the retailer.

[3] This simple specification of the distribution function can be relaxed without affecting our qualitative results.

common reservation price[4] $r > \frac{1}{2}$, which implies that *r* is higher than the average reservation price of leisure travelers. In particular, if *r>1* even the leisure traveler having the highest possible reservation price is unwilling to pay the prices business travelers might pay. We assume that the number of business travelers who demand service in the second stage is stochastic and unknown to the service provider in the first stage of the game. We denote this number by *y*, and assume that *y* is uniformly distributed over the interval $[0, \overline{y}]$. The service provider learns of the realization of *y* in the second stage of the game. The assumptions we make about the nature of demand in the leisure and business segments of the market reflects the reality of the travel industry. Prices business travelers are willing to pay tend to be much higher than those paid by leisure travelers. However, airlines face uncertainty concerning the number of business travelers that might be interested in flying on a certain date. They experience difficulty, therefore, in deciding on the number of seats to reserve for last minute business travelers. While this segment is lucrative because of its higher willingness to pay, it is also subject to frequent random shocks that cannot be predicted in advance.[5]

We assume that while the direct marketing channel of the airline is active in both stages of the game, the clearinghouse channel becomes available only in the second stage, once the airline is informed of the number of business travelers demanding service. Moreover, we assume that the airline cannot credibly commit in the first stage not to transact with the clearinghouse channel in the second stage. This assumption implies, in particular, that whenever the airline experiences a state of excess capacity he will have an incentive to sell this excess via the reseller provided he can command a positive price for it. The airline can change the posted price he charges in his direct marketing channel over the two stages. We will later derive a sufficient condition to guarantee that the price in the direct channel rises in the second stage in comparison to its level in the first stage. This rising price schedule is consistent with casual observation of

[4] We can easily relax this assumption by assuming a distribution of reservation prices for the business travelers as well. In such an extension, we should still assume that the average reservation price of business travelers is higher than that of the leisure travelers. Since business travelers do not have flexibility concerning the timing of purchase, segmenting this market according to their differing valuations would be pointless, given that all business travelers behave in a similar manner.

[5] The model can be extended to allow for an influx of impulse buyers in the second stage who are attracted by the lower prices charged by the reseller. If those impulse buyers are modeled as another segment of consumers whose aggregate demand $D(P)$ is declining with the price charged by the reseller, the basic qualitative results of our paper remain unchanged. Specifically, segmentation of leisure travelers who plan their purchases in the first stage will sill arise. Their likelihood of gaining access to the service when postponing the purchase to the second stage declines, however, when additional impulse buyers may join the market in the second stage.

the pattern of prices in the airline industry, where prices sometimes more than double close to the date of departure in comparison to booking flights three weeks in advance. This trend reflects the attempt of airlines to avoid alienating customers who buy tickets early. American and Delta, for instance, now have guarantees to refund any future price reductions to travelers who book in advance (see Skertic 2005 and the website of Delta Airlines[6].) With such a guarantee in place, lowering prices over time becomes extremely expensive for airlines (see McAfee and Velde (2007) for a discussion of the stickiness of prices in the airline industry.)

Given the rising schedule of prices in the direct channel, only business travelers purchase tickets directly from the provider in the second stage. Leisure travelers who wish to buy tickets directly from the provider find it optimal to do so in the first stage, when the price is lower. As a result, the airline raises his posted price in the second stage to r, in order to extract the entire surplus of business travelers. We designate the posted price charged by the airline in the first stage by P^H. In the second stage, the airline chooses also the wholesale price he charges the intermediary contingent upon the realized level of the demand in the business segment. We designate this wholesale price by $P^W(y)$. Hence, after observing the state of the demand in the business segment, the airline can adjust the wholesale price charged from the intermediary. Note that the airline cannot utilize more sophisticated pricing schemes with the reseller, given our assumption that he is unable to credibly commit not to use the reseller in clearing his capacity. In particular, two part tariff schemes that include a fixed transfer in the first stage from the reseller to the airline for the right to offer the airline's tickets are not feasible. No reseller would agree to make such a fixed payment knowing that the airline has incentives to use his services irrespective of whether the payment had been made.

While business travelers have no flexibility in our model, leisure travelers can either purchase tickets is the first stage for the posted price P^H or they can wait for the second stage and buy tickets at the clearinghouse channel. If the intermediary operates this channel by utilizing the PP mechanism, a leisure traveler chooses whether she wishes to obtain the ticket at the retail price $P^R(y)$ posted by the intermediary. Since the intermediary sets its price after observing the wholesale price charged by the airline and since this wholesale price can be adjusted contingent upon the realized state of business class demand, the posted retail price chosen by the intermediary is also indirectly a function of y. If the intermediary operates the

[6] This guarantee appears at the following link: http://www.delta.com/help/faqs/refunds/index.jsp#refund_adjust

clearinghouse channel by using the NYOP model, a leisure traveler who waits for the second stage places a bid with the intermediary. We designate the bid chosen by a leisure traveler of type v as $B(v)$. A leisure traveler who postpones her purchase to the second stage runs the risk of not obtaining the service at all since the airline gives priority to the demand arising from the business segment in the second stage. Specifically, only if the airline experiences excess capacity because realized demand from the business sector is relatively low (small values of the random variable y) a leisure traveler has some prospect of obtaining a ticket from the clearinghouse channel. Moreover, even in this case, when the intermediary utilizes the NYOP model, the leisure traveler will obtain a ticket only if her bid exceeds the wholesale price set by the airline, namely only if $B(v) > P^W(y)$. Since a leisure traveler cannot normally observe the realization of y, she runs the risk of her bid failing even when the airline experiences a state of excess capacity. We designate the capacity that is available to the airline by K and assume that $K<1$.

Figure 1 summarizes the timeline of our model and Table 1 summarizes our notation.

Figure 1: Timeline of the Model

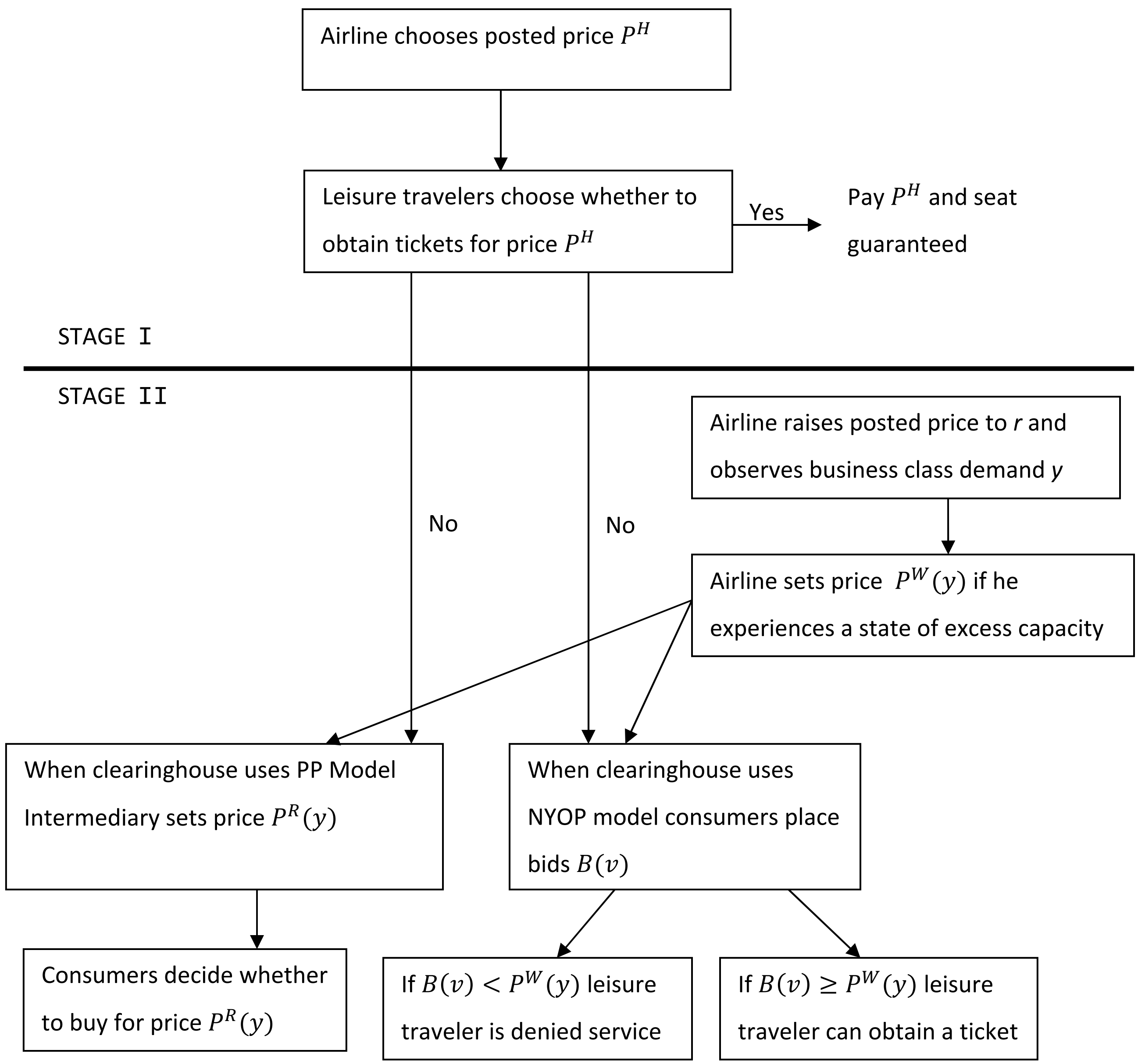


Table 1 Definition of Variables and Parameters in the Model

Variables and Parameters	Definition
$K \in [0,1]$	Capacity of airline
$v \in [0,1]$	Reservation price of leisure traveler
r	Reservation price of business traveler
$y \in [0, \bar{y}]$	Business class demand
P^H	Posted price in first stage
$P^W(y)$	Wholesale price set by airline in second stage
$P^R(y)$	Retail price set by intermediary if it uses PP
$B(v)$	Bid submitted by leisure traveler of type *v* if intermediary uses NYOP

3. <u>Analysis – The Intermediary Utilizes the Posted Price Model</u>

Leisure travelers who plan to participate in the clearinghouse market are aware of the fact that they may not be able to purchase the service at all if the state of business-class demand is sufficiently high. Specifically, if *z* designates the level of capacity sold in the first stage, then only if $y < K - z$, the airline has excess capacity to sell via the intermediary. Hence, the expected surplus of a leisure traveler of type *v* who postpones her purchase to the second stage, designated as $CS^{CH}(v)$, can be derived as follows:

$$CS^{CH}(v) = \frac{1}{\bar{y}}\int_0^{Min\{k-z,\bar{y}\}}\big(v - P^R(y)\big)dy = \begin{cases} \frac{K-z}{\bar{y}}v - \frac{1}{\bar{y}}\int_0^{K-z} P^R(y)dy & if \;\; K - z < \bar{y} \\ v - \frac{1}{\bar{y}}\int_0^{\bar{y}} P^R(y)dy & if \;\; K - z \geq \bar{y} \end{cases} \tag{1}$$

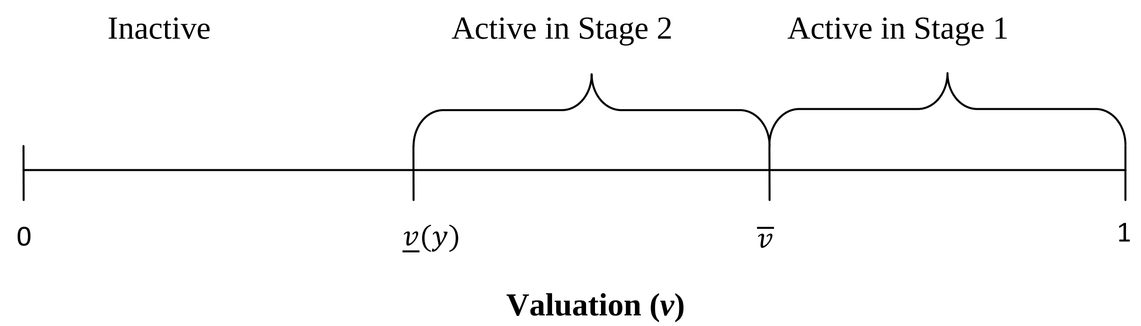
The consumer compares her net payoff when buying in the direct marketing channel, which amounts to $(v - P^H)$, to $CS^{CH}(v)$ in deciding when to obtain the service. In order to identify the group of leisure travelers who find it optimal to postpone their purchase, we define the function $R(v)$ as the additional benefit derived by a leisure traveler of type *v* from purchasing the product in the first stage, namely $R(v) \equiv [v - P^H - CS^{CH}(v)]$. Note that this function is strictly increasing in *v* assuming that $K - z < \bar{y}$. Hence, when the remaining capacity in the second stage falls short of the maximum possible business class demand, the function $R(v)$ is strictly increasing, implying the following potential segmentation of the leisure market.

$$R(v)\begin{cases}\geq 0 & if \quad v \geq \bar{v} \\ < 0 & if \quad v < \bar{v}\end{cases} \tag{2}$$

The regions described in (2) imply that leisure travelers having higher reservation prices purchase tickets in the first stage ($v \geq \bar{v}$), and those having lower reservation prices postopone the purchase to the second stage ($v < \bar{v}$). The leisure traveler of type $\bar{v}$ is just indifferent between the two options since $R(\bar{v}) = 0$. The segmentation described in (2) depends, however, on the assumption that $K - z < \bar{y}$, which implies that postponing the purchase to the second stage reduces the probability of actually obtaining tickets. Given the priority awarded to business travelers in allocating the remaining capacity at the second stage, leisure travelers run the risk of no capacity remaining for them. In our derivation we restrict attention only to this case.[7] Note also that some leisure travelers may withdraw from the market altogether if the retail price charged by the intermediary is sufficiently high. Specifically, let $\underline{v}(y)$ designate the lowest reservation price that is active in the market, then $\underline{v}(y) = P^R(y)$.

Figure 2 depicts the segmentation of the leisure market that is implied by the above discussion.

Figure 2: Market Segmentation of Leisure Travelers



Consumers with valuations below $\underline{v}(y)$ withdraw from the market. Those in the interval $[\underline{v}(y), \overline{v}]$ purchase tickets from the intermediary (in Stage 2), and those in the interval $[\overline{v}, 1]$

[7] This is the only interesting case to consider since otherwise, if $K - z > \bar{y}$ all leisure travelers purchase tickets at the same time. They either all buy in the direct channel in the first stage or via the intermediary in the second stage contingent upon whether P^H is lower or higher than the expected retail price EP^R of the intermediary. No segmentation of leisure travelers is possible, as a result.

purchase tickets in the direct marketing channel (in Stage 1). At the market clearing equilibrium, therefore, $z = 1 - \overline{v}$, namely the allocated capacity in the first stage equals the mass of leisure travelers who find it optimal to obtain tickets early. Note also, that the threshold consumer who is active in the market depends upon the realized state of business class demand, since the retail price charged by the intermediary depends upon y. In particular, if $P^R(y)$ is an increasing function of y, fewer leisure travelers are active in the market when the state of business-class demand is higher.

Given the segmentation depicted in Figure 2, in Lemma 1 we derive the expressions of the price schedules $P^W(y)$, $P^R(y)$ as well as P^H as functions of the threshold consumer $\overline{v}$. In the derivation of $P^W(y)$ and $P^R(y)$, we restrict attention to y realizations that guarantee that the clearinghouse market is active, namely $y < K - (1 - \overline{v})$. We also implicitly assume that prices are always positive.

Lemma 1

To guarantee that the allocated capacity matches the optimal choice of leisure travelers,

$$P^W(y) = 2(1-K) - \overline{v} + 2y, \quad for \quad y < K - (1 - \overline{v}). \tag{3}$$

$$P^R(y) = 1 - K + y, \quad for \quad y < K - (1 - \overline{v}). \tag{4}$$

$$P^H = \overline{v} - \frac{[K-(1-\overline{v})]^2}{2\overline{y}}. \tag{5}$$

Note that the wholesale and retail prices established in the second stage rise when the state of business-class demand is improved. With higher business-class demand the airline has less excess capacity to allocate to the intermediary, and finds it optimal, therefore, to raise his wholesale price. This price is positive for all values of y provided that $2(1-K) - \overline{v} > 0$. We will check whether this inequality holds once we solve for the optimal value of $\overline{v}$.[8] As well, from (5) we can derive a sufficient condition to guarantee that[9] $P^H < r$. If $K > 1 - r + \frac{\overline{y}}{2}$, posted prices definitely rise over time. Note that this condition is sufficient rather than necessary, implying that a weaker condition on K will also support the outcome $P^H < r$. Nevertheless, we will restrict attention to capacity values in the range $K \epsilon \left[1 - r + \frac{\overline{y}}{2}, 1\right]$.

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[8] It turns out to be valid as long as K, r, $\overline{y}$ are sufficiently small.

[9] Using (5) to express the inequality $r - P^H > 0$ as a quadratic expression in $\overline{v}$ and requiring that the discriminant is negative, yields the condition $K > 1 - r + \frac{\overline{y}}{2}$. This condition, in turn, guarantees that the inequality $r > P^H$ is always valid.

In the first stage of the game, the airline chooses the price P^H to maximize his expected profits over both stages. Those expected profits can be expressed as follows:

$$E\pi = P^H(1-\overline{v}) + \frac{r}{\overline{y}}\left[\int_0^{K-(1-\overline{v})} y\,dy + \int_{K-(1-\overline{v})}^{\overline{y}} [K-(1-\overline{v})]dy\right. \\ + \frac{1}{\overline{y}}\int_0^{K-(1-\overline{v})} P^W(y)[K-(1-\overline{v})-y]dy, \qquad (6)$$

where P^H and $P^W(y)$ are given in (5) and (3), respectively.

The first term of (6) corresponds to the airline's revenues from leisure travelers buying in the first stage. The second term corresponds to the expected revenues from business travelers, and the last term corresponds to the expected revenues that accrue from selling excess capacity via the intermediary. Instead of optimizing over P^H, it is simpler to formulate the airline's maximization problem as an optimal choice of the threshold consumer $\overline{v}$. The optimization over $\overline{v}$ yields the solution characterized in Proposition 1.

<u>Proposition 1</u>

(i) When the intermediary utilizes the PP model, the threshold leisure consumer who is indifferent between buying the ticket from the airline in the first stage or the intermediary in the second stage satisfies the equation:

$$\overline{v}_{PP} = \frac{1+r}{2} + \overline{y} - \sqrt{(K-\frac{1-r}{2})^2 + \overline{y}^2} < \frac{1+r}{2}. \qquad (7)$$

The expected profits of the airline are given as:

$$E\pi_{PP} = (1-\overline{v}_{PP})\overline{v}_{PP} + r[K-(1-\overline{v}_{PP})][1-\frac{K-(1-\overline{v}_{PP})}{2\overline{y}}] \\ + \frac{1}{6\overline{y}}[K-(1-\overline{v}_{PP})]^2[1+2\overline{v}_{PP}-4K]. \qquad (8)$$

(ii) The optimal value of $\overline{v}_{PP}$ is an increasing function of r and $\overline{y}$ and a decreasing function of K.

(iii) To guarantee that some leisure travelers buy tickets in the first stage (i.e., $\overline{v}_{PP} < 1$) and that the wholesale price is always positive (i.e., $\overline{v}_{PP} < 2(1-K)$), the following conditions should hold:

(a) $r < 1$ and $K < \frac{3-r}{4}$ or $r < 1$, $K \geq \frac{3-r}{4}$ but $\overline{y} < \frac{(1-K)[3K-(2-r)]}{[4K-(3-r)]}$.

(b) $r \geq 1, K < \frac{1}{2}$ and $\overline{y} < \frac{K(K+r-1)}{(r-1)}$

$r \geq 1, K \geq \frac{1}{2}$ and $\overline{y} < \frac{(1-K)[3K-(2-r)]}{[4K-(3-r)]}$.

According to part (*ii*) of Proposition 1, the segment of the leisure market that is active in the first stage is smaller when the willingness to pay of the business segment is higher, when the potential size of the business segment is larger; and/or when capacity is relatively small. Part (iii) of the Proposition states that the interior equilibria of the kind we consider exist as long as *r* and/or *K*, and/or $\overline{y}$ are sufficiently small.

4. Analysis – The Intermediary Utilizes the NYOP Model

Leisure travelers who plan to participate in the clearinghouse channel are aware that the airline can adjust his wholesale price P^W contingent upon the state of the business class demand in the second stage. They also know that if this demand is sufficiently high, the airline will sell his entire capacity in his direct channel, leaving no available capacity for the intermediary. Let $\hat{y}$ denote the highest value *y* can assume for the intermediary to be active. When $\hat{y} < \overline{y}$, the demand over the two periods in the direct channel can exceed capacity if $\hat{y} < y \leq \overline{y}$; and when $\hat{y} = \overline{y}$, there is always sufficient capacity to sell in the clearinghouse channel.

A consumer chooses her bid *B* to maximize her expected payoff as follows:

$$\text{Max}_B\, CS(B) = (v - B)Pr(B \geq P^W(y)\ and\ y \leq \hat{y}). \quad (9)$$

Designating by $\overline{P}^W$ the highest wholesale price that can arise in the market, namely $\overline{P}^W = P^W(\hat{y})$, then objective (9) reduces to:[10]

$$\text{Max}_B\, CS(B) = \begin{cases} (v - B)\frac{(P^W)^{-1}(B)}{\overline{y}} & if\ B < \overline{P}^W \\ (v - B)\frac{\hat{y}}{\overline{y}} & if\ B \geq \overline{P}^W \end{cases} . \quad (10)$$

Objective (10) is implied by the fact that *y* is uniformly distributed on $[0,\overline{y}]$. Note that if the bid of the leisure travelers fails, she cannot return to the direct marketing channel of the airline to obtain the ticket, since capacity has already been fully allocated at this point either to leisure travelers who purchase tickets in advance, or to those who submit successful bids with the intermediary[11], or to business travelers.

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[10] If $B < \overline{P}^W$, $Pr(B \geq P^W(y)$ and $y \leq \hat{y})$ reduces to $Pr\,(y < (P^W)^{-1}(B))$ since $(P^W)^{-1}(B) < \hat{y}$. When $B \geq \overline{P}^W$, $Pr(B \geq P^W(y)$ and $y \leq \hat{y})$ reduces to $Pr(y \leq \hat{y})$ since $(P^W)^{-1}(B) \geq \hat{y}$.

[11] If an extended model consisting of more than two periods were considered, the airline would adjust his posted price and the allocation of his capacity more gradually over time. Leisure travelers submitting failed bids would be able to return to the direct channel in such a setting.

The second part of objective (10) implies that a consumer will never submit a bid higher than $\overline{P}^W$, given that her objective is strictly diminishing in B, when $B \geq \overline{P}^W$. Further, let $B(v)$ designate the solution of the first part of objective (10), then the functional form of this part of the objective implies that $B(v)$ is a strictly increasing function of the consumer's valuation v. Hence there may be consumers of very low valuation levels whose bids fall short of the threshold minimum acceptable bid $P^W(y)$. Since $B\big(\underline{v}(y)\big) = P^W(y)$, it follows that:

$$(P^W)^{-1}\left(B\big(\underline{v}\big)\right) = y. \tag{11}$$

The expected payoff of a consumer active in the clearinghouse channel is obtained by substituting the optimal bid back into (10) as follows:

$$CS^{CH}(v) = \begin{cases} CS\big(B(v)\big) & if \quad B(v) < \overline{P}^W \\ \left(v - \overline{P}^W\right)\frac{\hat{y}}{\bar{y}} & if \quad B(v) \geq \overline{P}^W. \end{cases} \tag{12}$$

In deciding on whether to submit a bid at the NYOP channel, each leisure traveler compares the above expected payoff to the one she can expect by purchasing directly from the airline. When purchasing at the posted price P^H, the net payoff of the consumer is equal to $v - P^H$, since tickets' availability is assured in this case. The threshold consumer $\underline{v}(y)$ submits a bid $B\big(\underline{v}\big)$ just equal to the wholesale price $P^W(y)$. We implicitly assume that this consumer, as well as those who have higher valuations, derive non-negative net payoff when transacting with the intermediary. It is easy to show that if $P^W(y)$ is increasing, this implicit assumption is, indeed, valid.

As we have pointed out in the previous section, since the salvage value of unsold capacity is zero, the only credible mode of behavior on the part of the airline is to set the wholesale price $P^W(y)$ in a manner that guarantees full utilization of his capacity (as long as $P^W(y) > 0$). A commitment to any other price is not credible because the airline always has an incentive to sell any remaining (unsold) capacity in the second stage. Since the combined demand in the direct channel of the airline over the two periods amounts to $(1 - \bar{v}) + y$, where $\bar{v}$ is the leisure consumer who is indifferent between buying early or postponing the purchase, the wholesale price should be chosen so that:

$$\bar{v} - \underline{v}(y) = K - (1 - \overline{v}) - y \quad \text{for} \quad y \leq \hat{y}. \tag{13}$$

In Lemma 2 we derive the schedule of $P^W(y)$ that is implied by (13).

Lemma 2

The wholesale price is a linear and increasing schedule of the realization of the business class demand, as follows:

$$P^W(y) = (1 - K) + \frac{y}{2}. \tag{14}$$

The upper bound of the price schedule depends on the capacity level. In particular, if $K < (1 - \overline{v}) + \overline{y}$, $\hat{y} = K - (1 - \overline{v}) < \overline{y}$ and the airline does not allocate any capacity to the clearinghouse channel when $y > K - (1 - \overline{v})$. In contrast, if $K > (1 - \overline{v}) + \overline{y}$, $\hat{y} = \overline{y}$ and the clearinghouse channel is active for every possible realization of *y*.

The wholesale price schedule derived in Lemma 2 allows us to characterize the bid submitted by and the expected payoff of leisure travelers who consider being active in the NYOP channel, as follow:

Lemma 3

(i) If $K < (1 - \overline{v}) + \overline{y}$, the bid submitted by a leisure traveler of type *v* is given as follows:

$$Bid(v) = \begin{cases} \frac{(v+1-K)}{2} & if\ 0 \leq v \leq \overline{v} \\ \frac{(\overline{v}+1-K)}{2} & if\ \ \overline{v} < v \leq 1. \end{cases} \tag{15}$$

The expected payoff of this traveler is:

$$CS^{CH}(v) = \begin{cases} \frac{(v-(1-K))^2}{2\overline{y}} & 0 \leq v \leq \overline{v} \\ \left[v - \frac{(\overline{v}+1-K)}{2}\right]\frac{(\overline{v}-(1-K))}{\overline{y}} & \overline{v} < v \leq 1. \end{cases}$$

(ii) If $K > (1 - \overline{v}) + \overline{y}$, the bid submitted by a leisure traveler of type *v* is given as follows:

$$Bid(v) = \begin{cases} \frac{(v+1-K)}{2} & if\ 0 \leq v \leq 1 - K + \overline{y} \\ (1 - K) + \frac{\overline{y}}{2} & 1 - K + \overline{y} < v \leq 1. \end{cases}$$

The expected payoff of this traveler is:

$$CS^{CH}(v) = \begin{cases} \frac{(v-(1-K))^2}{2\overline{y}} & 0 \leq v \leq 1 - K + \overline{y} \\ (v - (1 - K) - \frac{\overline{y}}{2}) & 1 - K + \overline{y} < v \leq 1. \end{cases}$$

According to Lemma 3, the threshold leisure traveler who is just indifferent between purchasing a ticket in the direct and in the clearinghouse channels($v = \overline{v}$) submits a bid that is equal to the highest possible wholesale price that can be selected by the airline. Such a choice guarantees this traveler that her bid will be accepted as long as the clearinghouse channel is active (i.e., $y < K - (1 - \overline{v})$). Part *(i)* of the Lemma asserts that if excess demand can arise (i.e., when $\hat{y} < \overline{y}$), considering the range of valuations $v \leq \bar{v}$, the consumer with valuation $\overline{v}$ is the only one who submits this high bid and all other consumers who are active in the clearinghouse channel submit lower bids. Part *(ii)* asserts that when there is always access capacity to be sold in the clearinghouse channel (i.e., $\hat{y} = \overline{y}$), there is a range of consumers active in this secondary market with valuation levels lying in the upper tail of the distribution (in the range $1 - K + \overline{y} < v < \overline{v}$), who submit the bid $\overline{P}^W = P^W(\overline{y})$.

Defining, as in the previous section, $R(v)$as the added benefit derived by a leisure traveler of type v from purchasing the ticket early, yields from Lemma 3 that $R'(v)$ is strictly positive for all v values only if $K < (1 - \overline{v}) + \overline{y}$. According to part *(ii)* of the Lemma, if $K > (1 - \overline{v}) + \overline{y}$, $R'(v) = 0$ for all $(1 - K) + \overline{y} < v \leq 1$. In particular, since $(1 - K) + \overline{y} < \bar{v}$, in this case, the function $R(v)$ is flat around $\overline{v}$ and the segmentation of the market is impossible. Hence, like in the previous section, we restrict attention only to the case that $K < (1 - \overline{v}) + \overline{y}$, an assumption that guarantees that the segmentation depicted in Figure 2 is still valid when the intermediary uses the NYOP model (since $R(v)$ is a strictly increasing function of *v*.) From Lemma 3 we can also express the price P^H as a function of $\overline{v}$ to guarantee that $R(\overline{v}) = 0$. Specifically, when $K < (1 - \overline{v}) + \overline{y}$

$$P^H = \overline{v} - \frac{(\overline{v} - (1-K))^2}{2\overline{y}} . \tag{16}$$

The relationship between $P^H$and $\bar{v}$ under NYOP, given by (16), is the same as (5) under PP. Hence, the condition $K > 1 - r + \frac{\bar{y}}{2}$ still guarantees the rising schedule of posted prices over time (i.e., $P^H < r$).

The objective function of the airline is still expressed by equation (6). Substituting the expressions we have derived for $P^W(y)$ and P^H in (14) and (16), respectively, yields the following objective of the airline as a function of the threshold leisure consumer $\overline{v}$.

$$E\pi_{NYOP} = \overline{v}(1 - \overline{v}) + r[K - (1 - \overline{v})]\left[1 - \frac{K-(1-\overline{v})}{2\overline{y}}\right] + \frac{(K-(1-\overline{v}))^2}{12\overline{y}}[5(1 - K) + 7\overline{v} - 6]. \tag{17}$$

The airline chooses $\overline{v}$ (or indirectly, P^H) to maximize his expected profits. We characterize the solution to the maximization in Proposition 2.

Proposition 2

(i) When the intermediary utilizes the NYOP model, the threshold leisure consumer who is indifferent between buying the ticket from the airline is the first stage or the intermediary in the second stage satisfies the equation:

$$\bar{v}_{NYOP} = \frac{1+r}{2} + \frac{[8\bar{y}-6K+3(1-r)]}{14} - \sqrt{\frac{(K-\frac{1-r}{2})^2}{7} + \left[\frac{8\bar{y}-6K+3(1-r)}{14}\right]^2} < \frac{1+\mathrm{r}}{2} \qquad (18)$$

(ii) The optimal value of $\bar{v}_{NYOP}$ is an increasing function of r and $\bar{y}$ and a decreasing function of K.

(iii) To guarantee that some leisure travelers choose to obtain the service in the first stage (i.e., $\bar{\mathrm{v}}_{\mathrm{NYOP}} < 1$), the following condition should hold:

$r < 1$ or

$r \geq 1$ and $\bar{y} < K + \frac{K^2}{4(r-1)}$.

(iv) The wholesale price charged from the intermediary is positive for all values of y.

Before conducting a comparison of the two pricing models from the perspective of the airline, reseller, and consumers, in Proposition 3, we investigate, for a fixed $\bar{v}$, which pricing model generates, on average, higher proceeds from wholesale prices for the airline and retail revenues for the reseller.

Proposition 3

For a fixed value of $\bar{v}$ (or alternatively, fixed P^H), the expected proceeds of the airline from wholesale prices charged from the reseller are higher under NYOP than under PP. In contrast, the expected retail gross proceeds of the reseller either from consumer bids under NYOP or from retail prices under PP are the same.

The personalized bidding by consumers under NYOP facilitates improved price discrimination and surplus extraction in comparison to a policy that charges the same price from all consumers (under PP). As a result, the airline can raise the average wholesale price it charges from the reseller. The reseller is still able to retain a positive surplus due to his ability to observe privately the bids of the consumers under NYOP and to establish a markup above the wholesale price under PP. However, on average, the gross proceeds the reseller generates are the same either way. It is the airline that can better take advantage of the improved extraction of the consumer surplus under NYOP due to his first move in the game.

Given that the NYOP regime can support higher proceeds from wholesale prices, in Proposition 4 we demonstrate that the airline is more inclined to transfer sales to the second stage under NYOP. Specifically, when P^H (or alternatively,$\overline{v}$) is chosen optimally, the airline chooses to raise the price of the advance-purchase tickets under NYOP so that fewer leisure travelers find it optimal to buy tickets in the first stage. The Proposition summarizes also the comparison of the two pricing models from the perspective of the airline, reseller, and consumers.

Proposition 4

(i) The airliner prefers that the intermediary uses the NYOP model instead of the PP model in the clearinghouse channel. Moreover, more leisure travelers choose to postpone their purchase to the second stage if the intermediary utilizes the NYOP paradigm. Specifically, $\mathrm{E}\pi_{\mathrm{NYOP}} > \mathrm{E}\pi_{\mathrm{PP}}$; $\overline{\mathrm{v}}_{\mathrm{NYOP}} > \overline{\mathrm{v}}_{\mathrm{PP}}$; and $\underline{\mathrm{v}}_{\mathrm{NYOP}}(\mathrm{y}) = \underline{\mathrm{v}}_{\mathrm{PP}}(\mathrm{y}) = 1 - \mathrm{K} + \mathrm{y}$.

(ii) The expected payoff of leisure travelers is higher if the intermediary utilizes the PP instead of the NYOP mechanism.

(iii) The expected payoff of the intermediary may or may not increase when it switches from the NYOP to the PP model. Specifically, if w_{PP} and $\mathrm{w}_{\mathrm{NYOP}}$ designate the intermediary's expected payoff with PP and NYOP, respectively, then:

$$w_{PP} - w_{NYOP} = \frac{[K-(1-\overline{v}_{PP})]^3}{4\overline{y}} - \frac{(\overline{v}_{NYOP}-\overline{v}_{PP})}{4\overline{y}}\left[\left(K-\left(1-\frac{(v_{PP}+v_{NYOP})}{2}\right)\right)^2 + \frac{(\overline{v}_{NYOP}-\overline{v}_{PP})^2}{12}\right] \quad (19)$$

(iv) The total producer surplus, that consists of the sum of the airline's and intermediary's expected payoffs, is higher with the NYOP than with the PP mechanism.

According to Proposition 4 the airline is unambiguously better off if the intermediary utilizes the NYOP mechanism. Leisure travelers prefer, however, the PP model over the NYOP mechanism since the latter mechanism implies that a bigger portion of leisure travelers ends up postponing their purchase to the second stage. Hence, more leisure travelers are exposed to the risk of not being able to obtain tickets at all.[12] The comparison of the two pricing mechanisms from the perspective of the intermediary is ambiguous. On one hand, the intermediary pays a higher wholesale price to the airline under NYOP, implying that its margin is smaller under this

[12] This comparison from the perspective of consumers may change when capacity can be endogenously chosen. We investigate this possibility in Section 6.

regime. On the other hand, a bigger number of leisure customers is active in the second stage since $\overline{v}_{NYOP} > \overline{v}_{PP}$, implying that the NYOP model results in an expansion of the intermediary's market. While the first term of (19) corresponds to the benefit from the increased margin that the PP paradigm supports, the second term corresponds to the loss due to the shrinkage of the market that arises with the PP model. Part (*iv*) of the Proposition reports, however, that the total producer surplus is unambiguously higher with NYOP due to the improved extraction of the consumer surplus.

5. Vertical Integration or Competition in the Clearinghouse Channel

In the present section we consider the characterization of the equilibrium if the airline vertically integrates with the clearinghouse channel. If the vertically integrated entity utilizes the PP model, then $P^R(y) = P^W(y) = 1 - K + y$ in order to guarantee that capacity is fully cleared in the second stage. If the vertically integrated entity utilizes the NYOP model Lemma 2 still characterizes the wholesale price schedule that clears capacity and Lemma 3 still characterizes the optimal bidding of leisure travelers who are active in the second stage. However, with vertical integration the airline obtains the full bid of each leisure traveler instead of only the market clearing wholesale price. As in the previous section, we restrict attention to the case that $K < 1 - \overline{v} + \overline{y}$. In Lemma 4 we report that irrespective of the pricing model utilized in the secondary channel, the objective function facing the airline in the first stage, when P^H(or alternatively $\overline{v}$) is selected, remains the same.

Lemma 4

Irrespective of whether the clearinghouse channel utilizes the posted price or NYOP model, the expected payoff of a vertically integrated airline as a function of the threshold consumer $\overline{v}$ can be expressed as follows:

$$E\pi_{PP} = E\pi_{NYOP} = (1-\overline{v})\overline{v} + r[K-(1-\overline{v})]\left[1 - \frac{K-(1-\overline{v})}{2\overline{y}}\right] + \frac{[K-(1-\overline{v})]^2}{6\overline{y}}(4\overline{v} - 2K - 1) \quad (20)$$

In the first stage, the vertically integrated airline chooses $\overline{v}$ to maximize (20).

With vertical integration, the airline internalizes the complete added surplus generated by the secondary channel, irrespective of the type of pricing model utilized by this channel. Hence the objective function of the airline when choosing $\overline{v}$ (or alternatively P^H) remains the same. As a result, the characterization of the equilibrium is identical under both pricing models, as reported in Proposition 5.

Proposition 5

(i) Irrespective of whether the clearinghouse channel utilizes the posted price or NYOP models, when the airline is vertically integrated with the clearinghouse channel the threshold leisure traveler who is indifferent between buying in stage 1 or stage 2 satisfies the equation:

$$\bar{v} = \begin{cases} 1 & if \quad r \geq 1 \\ \frac{1+r}{2} & if \quad r < 1. \end{cases}$$

The posted price chosen by the airline in the first stage is given, therefore, as:

$$P^H = \begin{cases} > 1 & if \;\; r \geq 1 \\ \frac{1+r}{2} - \frac{\left(K - \frac{1-r}{2}\right)^2}{2\bar{y}} & if \quad r < 1 \end{cases}$$

(ii) The equilibrium profits of the vertically integrated airline are given as:

$$E\pi = \begin{cases} rK - \frac{K^2}{6\bar{y}}(3r + 2K - 3) & if \;\; r \geq 1 \\ \frac{1-r^2}{4} + r\left[K - \frac{1-r}{2}\right] - \frac{\left(K - \frac{1-r}{2}\right)^3}{3\bar{y}} & if \; r < 1. \end{cases}$$

Hence equilibrium profits increase with r and $\bar{y}$.

(iii) To guarantee that some leisure travelers are active in the first stage and that $K < 1 - \bar{v} + \bar{y}$, $r < 1$ and $K < \frac{1-r}{2} + \bar{y}$.

According to Proposition 5, the direct channel of the airline is active in the first stage only if *r<1*. Otherwise, if the willingness to pay of business travelers is higher than the highest possible reservation price of leisure travelers, the airline reserves his entire capacity for the second stage, by charging a very high price in the first stage (i.e., P^H>1). After answering the entire business class demand (if possible), the airline sells in this case any remaining excess capacity in the leisure market, but only via the clearinghouse channel.

In Proposition 6, we compare the solution with vertical integration to the one we obtained in the previous section when the airline and the clearinghouse channel are separate entities.

Proposition 6

(i) The segment of the leisure market that is active in the first stage is smaller with vertical integration than with separation.

(ii) The profits of the airline are higher and the consumer surplus is smaller with vertical integration than with separation.

Since with vertical integration the airline can extract the entire revenues generated from consumers active in the clearinghouse market, without having to share any portion of those revenues with a different entity, he is more inclined to increase the number of leisure customers who are active in the second stage. His profits increase, as a result, but leisure customers are worse off since a bigger fraction of them is exposed to the risk of not being served at all, given that business customers are awarded higher priority in the second stage.[13]

Next, we demonstrate that if there are multiple competing resellers operating in the clearinghouse channel, the vertically integrated outcome can be achieved even if the airline does not integrate with any of them. To illustrate this argument, consider first the case that there are two resellers competing in the clearinghouse channel, with each using the posted price model. If the airline chooses his wholesale price according to (4), Bertrand competition between the resellers forces them to set their retail price equal to marginal cost which coincides with the wholesale price set by the airline. As a result, the airline implements the vertically integrated outcome characterized in Proposition 5.

Next, consider the case that the secondary market consists of two resellers one of whom utilizes the posted price model and the other the NYOP model. Consumers who choose to submit bids with the NYOP reseller can now observe the retail price chosen by the PP reseller before submitting their bids (most consumers who plan to submit bids with Priceline, visit first the Hotwire site in order to gather information about the state of demand). Since the retail price is selected contingent upon the wholesale price charged by the airline, rational consumers are able to fully infer the state of business class demand and submit bids just equal to the wholesale price charged by the airline (namely, $Bid(v) = P^W(y)$). The reseller that utilizes the NYOP mechanism is then equivalent, from the perspective of consumers, to a reseller who posts the price $P^W(y)$ to consumers. Bertrand competition implies, once again, that the other reseller is forced to set his retail price equal to the wholesale price in order to attract consumers.

The above argument implies that as long as the secondary channel consists of at least two resellers, one of whom utilizing the posted price mechanism, the airline is able to implement the

[13] If the airline could utilize two part tariff pricing schemes and charge, in particular, a fixed payment from the reseller in the first stage, the vertically integrated outcome could be implemented even under vertical separation. Recall, however, that in our setting the airline does not have the commitment power to withhold sales via the intermediary. If the airline experiences excess capacity he has incentives to transact with the intermediary in the second stage irrespective of promises made in the first stage. Charging a fixed payment in such a setting is not feasible.

equilibrium with vertical integration characterized in Proposition 5. Note that this outcome critically depends upon at least one of the resellers' using the PP mechanism. If both resellers use the NYOP model, consumers are unable to infer the state of business class demand before submitting bids. As a result, the equilibrium is still characterized as in Proposition 2 for the vertically separated case. We summarize the above discussion in the following Corollary.

Corollary 1

When the clearinghouse channel is vertically separated but consists of at least two resellers, one of whom utilizing the Posted Price model, the airline is able to implement the vertically integrated outcome characterized in Proposition 5.

6. Capacity is Endogenously Chosen

Up until now we assumed that the level of capacity that is available to the airline is exogenous. We now relax this assumption and allow the airline to choose K optimally prior to the two stages described in the previous sections. We denote the unit cost of capacity by c. If the clearinghouse channel is a separate entity, the airline chooses K to maximize (8) under PP and (17) under NYOP, net of the investment cost cK. If the clearinghouse channel is vertically integrated with the airline or if this secondary market of resellers is competitive, the airline chooses K to maximize (20), net of the investment cost. The result of this maximization is reported in Proposition 7. In stating the optimal values, we use the subscripts VI, NYOP, and PP to designate the solutions with vertical integration (VI) and with the NYOP and PP models under vertical separation, respectively.

Proposition 7

(i) When the level of capacity is endogenously selected, it can be expressed as a function of the optimal value of $\bar{v}$ as follows:

$$K_x^* = (1-\bar{v}_x^*) + \sqrt{\bar{y}(2\bar{v}_x^* - 1 - c)}, \qquad \text{where } x\text{=VI, NYOP, and PP.}$$

(ii) The optimal value of the posted price P^H is equal to $\frac{1+c}{2}$ irrespective of the pricing model or the vertical structure of the clearinghouse channel.

(iii) $K_{VI}^* > K_{NYOP}^* > K_{PP}^*$ and $\bar{v}_{VI}^* > \bar{v}_{NYOP}^* > \bar{v}_{PP}^*$.

According to Proposition 7, if the airline can optimize both with respect to $\bar{v}$ and K, it ends up setting the price of the advance-purchase ticket equal to that chosen by a monopolist who serves exclusively the leisure market only. However, in contrast to such a monopolist, the airline offers advance-purchase tickets to fewer than $\frac{1-c}{2}$ leisure travelers in the first stage, since $\bar{v}^* > \frac{1+c}{2}$. As well, part *(iii)* of the proposition asserts that a vertically integrated airline invests in the largest capacity but offers the smallest number of advance-purchase tickets. When the clearinghouse channel is a vertically separated entity, the airline invests in a larger capacity if the reseller uses the NYOP rather than the PP models. In the former case, the airline offers also fewer advance-purchase tickets.

Given the simple expression derived for $\bar{v}_{VI}^*$ in Proposition 5, we use Proposition 7 to derive the optimal level of investment in capacity under vertical integration, which we do in Corollary 2.

Corollary 2

(i) When the airline and clearinghouse markets are vertically integrated or when the clearinghouse market consists of multiple competing resellers:

$$K_{VI}^* = \frac{1-r}{2} + \sqrt{(r-c)\bar{y}} \quad . \tag{21}$$

(ii) To guarantee that the optimal value of K_{VI}^* supports the interior equilibria we consider (i.e., $K < (1-\bar{v}) + \bar{y}$ and $K < 1$), the value of the parameter $\bar{y}$ should be restricted as follows:

$$r - c < \bar{y} < \frac{(1+r)^2}{4(r-c)} \quad , \tag{22}$$

and to guarantee a rising schedule of the posted price overtime $\frac{1+c}{2} < r$.

The optimal investment of the airline as expressed in (21), is an increasing function of r and $\bar{y}$. This optimal capacity level is bigger than $\frac{1-c}{2}$, which corresponds to the optimal investment of an airline that serves exclusively only the leisure market.

It is noteworthy that the welfare comparison we conducted in the previous section between the NYOP, PP, and VI models may yield different conclusions when the airline can choose the capacity level endogenously. In particular, under vertical separation, the consumer surplus of leisure travelers is not necessarily lower under NYOP than under PP. Since the NYOP regime leads to higher investment in capacity than PP, the airline charges the reseller lower wholesale prices in the second stage. Consumers can expect therefore to obtain tickets by submitting lower bids. This positive effect on consumer surplus can outweigh the negative effect that is implied by the reduction in the number of advance-purchase tickets that are offered under NYOP (i.e., the increased risk of not being able to obtain tickets from the reseller.) Similarly, vertical integration can lead to higher welfare of leisure travelers for the same reason.

7. Competition Among Airlines

In this section we extend our analysis to allow for competition between two airlines that are considered homogenous by consumers. As in our original formulation, each airline can offer tickets via his direct marketing channel (available in both stages) or via an intermediary (available only in the second stage) that serves as a clearinghouse for any excess capacity experienced by the airlines. We maintain our assumption that the price in the direct marketing channel of the airline rises over time. More specifically, we still assume that each airline raises his price in the second stage to the level of the reservation price of business travelers (i.e., *r*). Hence, airlines can avoid competing in price for business travelers, thus permitting them to extract the entire surplus of this segment of consumers.[14] Casual observation of the airline industry illustrates, indeed, that business class ticket prices are still very high in spite of the fierce competition that exists in this industry for leisure travelers.

We denote the capacity that is available to airline *i* by K_i, where $K_i < 1$, and $1 - K_1 - K_2 < r$. Hence, if both airlines sell their entire capacity in the first stage the price they can charge falls short of the price they can command from business travelers. This inequality guarantees that airlines have an incentive to reserve some capacity for business class demand that is realized in the second stage.

We focus on the derivation of the Cournot equilibrium, where each airline chooses the number of advance-purchase tickets to sell to leisure travelers in the first stage, denoted by x_i for airline *i*, and prices that are established in order to clear this allocated capacity. Specifically, the posted price in the first stage, P^H, and the wholesale price charged from the intermediary in the second stage, $P^W(y)$, are determined to guarantee that $(1 - \overline{v}) = x_1 + x_2$. Hence, the mass of leisure travelers who find it optimal to obtain tickets early coincides with the level of aggregate capacity that is allocated by the airlines to be sold in the first stage.

[14] We make this assumption for simplicity. We will show that our results actually strengthen if competition for business travelers in the second stage lowers the price in the direct marketing channel of each airline below *r*.

We can use a similar approach to that of the previous sections to derive the schedules of P^H and $P^W(y)$ in terms of the number of advance-purchase tickets allocated by the airlines to serve leisure travelers in the first stage. In Lemma 5 we derive those schedules.

Lemma 5

For fixed capacities x_1, x_2 that are allocated by the airlines to serve leisure travelers in the first stage, the schedules of the market clearing prices can be expressed as follows:

(i) When the intermediary uses the PP model:

$$P^W(y) = 2(1 - K_1 - K_2) - (1 - x_1 - x_2) + 2y$$

$$P^R(y) = 1 - K_1 - K_2 + y$$

$$P^H = (1 - x_1 - x_2) - \frac{[(K_1+K_2)-(x_1+x_2)]^2}{2\overline{y}}$$

(ii) When the intermediary uses the NYOP model:

$$P^W(y) = (1 - K_1 - K_2) + \frac{y}{2}$$

$$P^H = (1 - x_1 - x_2) - \frac{[(K_1+K_2)-(x_1+x_2)]^2}{2\overline{y}}$$

Note that the expressions for the price schedules are equivalent to those derived with a single airline (in (3), (4), (5), (14), and (16)), with the exception that $\overline{v}$ is replaced by $(1 - x_1 - x_2)$, and the aggregate capacity K is replaced by $(K_1 + K_2)$.

Before deriving the objective function of each airline, we should first explain how business class demand is split between the airlines. Since the airlines are considered homogeneous by consumers, business travelers are indifferent between them, given our assumption that both charge the reservation price *r* in the second stage. Hence, it is sensible to assume that the airlines split business class demand equally, as long as they have sufficient capacity to answer this equal share. Specifically, without any loss of generality, assume that Airline 2 has a larger capacity available in the second stage than Airline 1, namely $(K_2 - x_2) \geq (K_1 - x_1)$, then the airlines share business class demand as follows:

$$s_1 = Min\left\{\frac{y}{2}, K_1 - x_1\right\},$$
$$s_2 = Min\left\{Max\left\{\frac{y}{2}, y - K_1 + x_1\right\}, K_2 - x_2\right\}. \quad (23)$$

According to (23), the airlines share equally business class demand. However, if Airline 1 is unable to serve half of the realized business demand Airline 2, who has the larger remaining capacity in the second stage, serves the entire residual demand that is left unserved by Airline 1 (assuming he has sufficient remaining capacity to do so).

Given the sharing rule defined in (23), we derive in the Appendix the objective functions of the possibly asymmetric airlines. In Proposition 8 we characterize the equilibrium with two competing airlines and compare it to the result obtained with a monopoly airline.

Proposition 8

(i) When $K_2 \geq K_1$, the airlines choose their allocated capacities for sale in the first stage so that $K_2 - x_2^* \geq K_1 - x_1^*$.

(ii) At the symmetric equilibrium, when $K_1 = K_2$, $x_1^* = x_2^* = x^*$. Moreover, $x_{PP}^* > x_{NYOP}^*$, implying that a larger share of each airline's capacity is sold in the first stage if the intermediary utilizes the posted price instead of the NYOP model. The expected profits of each airline are higher when the NYOP model is utilized by the intermediary.

(iii) For a fixed aggregate capacity that is available in the industry irrespective of the number of competing airlines, at the symmetric equilibrium a larger portion of aggregate capacity is sold in the second stage if a monopoly airline rather than two competing airlines operate in the industry. Specifically, $\overline{v}_{monopoly} > \overline{v}_{duopoly}$.

The intuition for the results reported in Proposition 8 is quite straightforward. Part *(i)* states the sensible result that the airline that has the larger capacity ends up allocating a larger portion of it to the second stage in order to take advantage of the more lucrative business class demand. Part (*ii*) of the Proposition is simply an extension of the similar result derived for a monopoly airline. Part (*iii*) of the Proposition states that competition between airlines reduces the profitability of reserving capacity for the second stage since unlike a monopolist, competitors are unable to fully internalize the expected benefit from selling to business class travelers. It is noteworthy that this last result is likely to only strengthen if we relax our assumption that competing airlines can avoid price competition in the business class segment. If competition for business travelers forces the airlines to lower the business class fare below *r*, the attractiveness of reserving capacity for the second stage is further eroded, thus making it more advantageous to sell capacity to leisure travelers in the first stage.

Finally in Proposition 9 we characterize the equilibrium when there are multiple intermediaries competing in the clearinghouse channel, with at least one of them utilizing the posted price model. Recall from our discussion in an earlier section that if at least one of the competing intermediaries utilizes the posted-price model, Bertrand competition among them eliminates their entire profits (i.e., $P^R(y) = P^W(y)$ for those using the posted price model and $Bid(v) = P^W(y)$ for every consumer who submits a bid with an intermediary using the NYOP model).

<u>Proposition 9</u>

When there are multiple intermediaries competing in the clearinghouse channel, with at least one of them utilizing the posted price model:

(i) The level of capacity sold in the first stage by each of two identical airlines is equal to $\frac{1-r}{3}$. This level is lower than first stage sales of each airline in the absence of competition in the clearinghouse channel.

(ii) For a fixed aggregate capacity in the industry, total sales in the first stage are higher with two competing airlines than with a monopolist airline in the industry. Specifically, $\overline{v}_{monopoly} = \frac{1+r}{2}$ and $\overline{v}_{duopoly} = \frac{1+2r}{3}$.

Part *(i)* of Proposition 9 is implied by the fact that competition in the clearinghouse channel permits the airlines to extract the entire revenues generated in this channel, without having to share any of them with the intermediaries. Hence, airlines are less concerned about reserving capacity for the second stage, when some sales will have to occur via the intermediaries (if business class demand is relatively low). The intuition for part *(ii)* of Proposition 9 is similar to that used to explain part *(iii)* of Proposition 8. Since competing airlines cannot internalize the benefit from selling to business travelers to the extent that a monopolist can, they choose to increase their sales to leisure travelers in the first stage, even when the clearinghouse channel is competitive.

8. <u>Concluding Remarks</u>

We considered a simple, two stage model to capture the main characteristics of demand classes in the airline industry. While leisure travelers were assumed to learn of their need to fly in the first stage, business travelers become informed of this need in the second stage. We found that since business class demand is stochastic and has a higher willingness to pay, airlines find it optimal to reserve capacity for sale in the second stage, after offering a certain portion of it in the first stage, in the form of advance-purchase tickets. In contrast to the earlier literature on this topic, we introduced resellers in the model that allow airlines to clear capacity. Specifically, we assumed that airlines can charge high prices on their own marketing channels to extract surplus from business travelers, while offering any remaining, unsold capacity to the resellers at wholesale prices that clear the market.

We found that when there is a single reseller in the market, airlines prefer that the reseller utilizes the Name Your-Own Price (a la Priceline) instead of the Posted Price (a la Hotwire) model. Essentially, airlines can better extract the surplus of the reseller if power over pricing is in the hands of numerous consumers, each bidding according to her preferences, instead of being concentrated in the hands of the reseller. Introducing competition among resellers eliminates the distinction between the two pricing models from the perspective of the airline. Either form of pricing generates the same outcome with competition as vertical integration of the airline with the downstream market of resellers. Interestingly, when the level of capacity is exogenously given, consumers do not necessarily benefit from such vertical integration, since a larger portion of them is exposed to the risk of not being served at all, when the airline vertically integrates with the resellers.

In order to focus attention on the main forces at play, we made several simplifying assumptions concerning the distribution of preferences of consumers. We do not believe that our qualitative results depend upon this simplification. However, a very welcome extension of our analysis would be to consider a more general formulation of the distribution of willingness to pay of leisure and business travelers, as well as a stochastic formulation of the timing at which each type of customer becomes informed of her need to fly. A more sophisticated, mechanism-design approach may be necessary, if such an extension is pursued.

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Appendix

Proof of Lemma 1

Given the wholesale price $P^W(y)$, the intermediary chooses its retail price to maximize its profits as follows:

$$\text{Max}_{P^R}(\overline{v} - P^R)(P^R - P^W(y)). \quad \text{(A.1)}$$

The above objective is implied by the fact that the mass of the leisure traveler buying from the intermediary is equal to $(\overline{v} - \underline{v})$, where $\underline{v} = P^R$. Optimizing (A.1) yields the solution that:

$$P^R(y) = \frac{\overline{v} + P^W(y)}{2}. \quad \text{(A.2)}$$

In order to clear the capacity in the second stage, $K = 1 - \underline{v}(y) + y$. However, since $P^R(y) = \underline{v}(y)$ it follows from (A.2) that:

$$P^W(y) = 2(1 - K) - \overline{v} + 2y. \quad \text{(A.3)}$$

Note that $P^W(y) > 0$ for all values of y if $2(1 - K) - \overline{v} > 0$. We will check for this condition once we solve for $\overline{v}$. Substituting for $P^W(y)$ back into (A.2) yields the expression for $P^R(y)$ as well.

To find the expression for P^H, recall that the threshold consumer $\overline{v}$ should be indifferent between buying the service in Stage 1 or Stage 2, namely $R(\overline{v}) = 0$. Substituting the expression for $P^R(y)$ into (1) and integrating the second term, yields the following expected payoff of a leisure traveler of type v who postpones her purchase to the second stage:

$$CS(v) = \frac{[K - (1 - \overline{v})]}{\overline{y}}\left[\frac{K + 1 - \overline{v}}{2} - (1 - v)\right].$$

Hence, for $\overline{v}$,

$$CS(\overline{v}) = \frac{[K - (1 - \overline{v})]^2}{2\overline{y}}.$$

Since $\overline{v} - P^H = CS(\overline{v})$, the expression for P^H in terms $\overline{v}$ is given in (5). Q.E.D.

Proof of Proposition 1

(i) Substituting for P^H and $P^W(y)$ into the objective function of the airline and conducting the integration, yields the following expression:

$$E\pi = (1 - \overline{v})\overline{v} + r[K - (1 - \overline{v})]\left[1 - \frac{K - (1 - \overline{v})}{2\overline{y}}\right] + \frac{1}{6\overline{y}}[K - (1 - \overline{v})]^2[1 + 2\overline{v} - 4K]. \quad \text{(A.4)}$$

Differentiating (A.4) with respect to $\overline{v}$ and setting the derivative equal to zero yields the following quadratic equation for $\overline{v}$:

$$\frac{\overline{v}^2}{\overline{y}} - \overline{v}\left[2 + \frac{1+r}{\overline{y}}\right] + \left[1 + r\left(1 + \frac{1-K}{\overline{y}}\right) + \frac{K(1-K)}{\overline{y}}\right] = 0$$

The root that satisfies also the second order condition that $\frac{\partial E^2\pi}{\partial \overline{v}^2} < 0,$ is given by (7). The above solution is always nonnegative and satisfies also the condition that $K - (1 - \overline{v}) < \overline{y}$, as is implicitly assumed in objective (6).

$$(ii)\ \frac{\partial \overline{v}}{\partial r} = \frac{1}{2}\left[1 - \frac{\left(K - \frac{1-r}{2}\right)}{\sqrt{\left(K - \frac{1-r}{2}\right)^2 + \overline{y}^2}}\right] > 0$$

$$\frac{\partial \overline{v}}{\partial \overline{y}} = 1 - \frac{\overline{y}}{\sqrt{\left(K - \frac{1-r}{2}\right)^2 + \overline{y}^2}} > 0$$

$$\frac{\partial \overline{v}}{\partial K} = -\frac{\left(K - \frac{1-r}{2}\right)}{\sqrt{\left(K - \frac{1-r}{2}\right)^2 + \overline{y}^2}} < 0\ .$$

(iii) The condition that $\overline{v} < 1$ always holds if $r < 1$. If $r \geq 1,$ it hold if $\overline{y} < \frac{K(K+r-1)}{(r-1)}$. The condition that $\overline{v} < 2(1 - K)$ is more binding than the condition that $\overline{v} < 1$ if $K \geq \frac{1}{2}$. When it is more binding, it is satisfied if $\overline{y} < \frac{(1-K)[3K-(2-r)]}{[4K-(3-r)]}$. The regions reported in the proposition are implied by this reasoning and the conditions that guarantee that the upper bound on $\overline{y}$ is positive.

Q.E.D.

Proof of Lemma 2

Optimizing the first part of objective (10) with respect to B yields the following first and second order conditions:

$$CS'(B) = -(P^W)^{-1}(B) + \frac{(v-B)}{(P^W)'((P^W)^{-1}(B))} = 0, \qquad and$$
$$CS''(B) = -\frac{1}{(P^W)'((P^W)^{-1}(B))} - \frac{\left(P^W\right)^{-1}(B)\left(P^W\right)''\left(\left(P^W\right)^{-1}(B)\right)}{2[(P^W)'((P^W)^{-1}(B))]^2} < 0. \tag{A.5}$$

Since $B\left(\underline{v}(y)\right) = P^W(y)$ it follows that $(P^W)^{-1}\left(B\left(\underline{v}\right)\right) = y$, implying from (A.5) that:

$\underline{v}(y) = P^W(y) + yP^{W\prime}(y)$.

From (13) $\underline{v}(y) = 1 - K + y$. Substituting for $\underline{v}(y)$ into the above equation, yields a first order differential equation for P^W. Solving this equation with the initial condition that $P^W(0)$ is finite, yields the solution in (14). Q.E.D.

Proof of Lemma 3

(i) If $K < 1 - \overline{v} + \overline{y}$, the highest possible realization for which the NYOP retailer is active is $\hat{y} = K - (1 - \overline{v})$. Hence, no leisure traveler will ever submit a bid higher than $P_L(\hat{y}) = \frac{(\overline{v}+1-K)}{2}$. From the expression obtained for $P^W(y)$ in Lemma 2, $(P^W)^{-1}(B) = 2[B - (1-K)]$ and $P^{W\prime}(y) = \frac{1}{2}$ substituting into (A.5) yields that $B(v) = \frac{(v+(1-K))}{2}$. This bid is less than $P^W(\hat{y})$ if $v < \overline{v}$. Substituting the optimal bid back into the consumer's objective (12) yields the expected payoff of the consumer.

(ii) If $K > 1 - \overline{v} + \overline{y}$, the NYOP is active for all values of y and $\hat{y} = \overline{y}$. Hence, no leisure traveler will ever submit a bid higher than $P^W(\overline{y}) = (1-K) + \frac{\overline{y}}{2}$. Since the optimal bid from (A.5) is equal to $B(v) = \frac{(v+(1-K))}{2}$, $B(v) \leq (1-K) + \frac{\overline{y}}{2}$ when $v \leq 1 - K + \overline{y}$, and we obtain the two regions reported in part (*ii*) of the Lemma. Substituting into (12) yields the expected payoff for the two regions. Q.E.D.

Proof of Proposition 2

(i) Differentiating (17) with respect to $\overline{v}$ and setting the derivative equal to zero, yields the following quadratic function of $\overline{v}$:

$$H(\overline{v}, \overline{y}, r\ K) \equiv\equiv 7\overline{v}^2 - \overline{v}(8\overline{y} + 4r + 10 - 6K) + [4\overline{y}(1+r) + (1-K)(4r + K + 3] = 0. \qquad \text{(A.6)}$$

The solution to this quadratic equation that satisfies second order condition is given by (18). This solution satisfies the condition that $\overline{v} > 0$ and $\overline{v} < \overline{y} + (1-K)$. The latter inequality is implicitly assumed when stating objective (6).

(ii) To prove the comparative statics, note that for the feasible region of parameter values $\frac{\partial H}{\partial \overline{y}} > 0, \frac{\partial H}{\partial r} > 0$, and $\frac{\partial H}{\partial K} < 0$ in (A.6). As well, to conduct comparative statics with respect to any parameter x, total differentiation of (A.6) yields that:

$$\frac{\partial \overline{v}}{\partial x} = -\frac{\frac{\partial H}{\partial x}}{\frac{\partial H}{\partial \overline{v}}}.$$

However, from second order condition $\frac{\partial H}{\partial \overline{v}} < 0$, implying that the sign of $\frac{\partial \overline{v}}{\partial x}$ is determined by the sign of $\frac{\partial H}{\partial x}$.

(iii) Evaluating (A.6) at $\overline{v} = 1$ yields:

$$T(K, r, \overline{y}) \equiv 4K(1-r) - K^2 - 4\overline{y}(1-r).$$

To guarantee that $\overline{v} < 1$, the above expression has to be negative. As well, for second order condition to hold, $\frac{dH}{d\overline{v}}_{|\overline{v}=1} < 0$. This last inequality holds provided that $\overline{y} > \frac{3}{4}K + \frac{1-r}{2}$. For this region of values of $\overline{y}$, $T(K, r, \overline{y})$ is definitely negative when $r < 1$. When $r > 1$, it is negative if $\overline{y} < K + \frac{K^2}{4(r-1)}$. Q.E.D.

Proof of Proposition 3

(i) Irrespective of the pricing regime, the reseller's market is active only when $y < K - (1 - \overline{v})$ and $\underline{v}(y) = 1 - K + y$ in order to clear capacity. From (3) and (14), therefore:

$$EP_w^{PP} = \frac{1}{\overline{y}} \int_0^{K-(1-\overline{v})} \int_{1-K+y}^{\overline{v}} [2(1-K) - \overline{v} + 2y] \partial v \partial y = \frac{[K-(1-\overline{v})]^2}{3\overline{y}} \left[2(1-K) - \frac{\overline{v}}{2} \right].$$

$$EP_w^{NYOP} = \frac{1}{\overline{y}} \int_0^{K-(1-\overline{v})} \int_{1-K+y}^{\overline{v}} \left[(1-K) + \frac{y}{2} \right] \partial v \partial y = \frac{[K-(1-\overline{v})]^2}{3\overline{y}} \left[\frac{5}{4}(1-K) + \frac{\overline{v}}{4} \right].$$

It follows that $EP_w^{NYOP} > EP_w^{PP}$ since $K > (1 - \overline{v})$.

(ii) From (4) and (15)

$$EP^R = \frac{1}{\overline{y}} \int_0^{K-(1-\overline{v})} \int_{1-K+y}^{\overline{v}} (1-K+y) \partial v \partial y = \frac{[K-(1-\overline{v})]^2}{3\overline{y}} \left[\frac{\overline{v}}{2} + (1-K) \right].$$

$$EBid = \frac{1}{\overline{y}} \int_0^{K-(1-\overline{v})} \int_{1-K+y}^{\overline{v}} \frac{(v+1-K)}{2} \partial v \partial y = \frac{[K-(1-\overline{v})]^2}{3\overline{y}} \left[\frac{\overline{v}}{2} + (1-K) \right].$$

Hence, $EP^R = EBid$. Q.E.D.

Proof of Proposition 4

(i) For a common value of $\overline{v}$ utilized under both the PP and NYOP regimes, a comparison of equations (17) and (8) yields that:

$$E\pi_{NYOP_{|\overline{v}}} - E\pi_{PP_{|\overline{v}}} = \frac{[K-(1-\overline{v})]^3}{4\overline{y}} > 0, \tag{A.7}$$

$$\frac{\partial E\pi_{NYOP}}{\partial \overline{v}}_{|\overline{v}} - \frac{\partial E\pi_{PP}}{\partial \overline{v}}_{|\overline{v}} = \frac{3[K-(1-\overline{v})]^2}{4\overline{y}} > 0. \tag{A.8}$$

The above inequalities hold in particular if $\overline{v} = \overline{v}_{PP}$. Since under the NYOP regime, the airline further adjusts the value of $\overline{v}$ to maximize his expected profits, (A.8) implies that $\overline{v}_{NYOP} > \overline{v}_{PP}$. Moreover,

$\left(E\pi_{NYOP_{|\overline{v}_{NYOP}}} - E\pi_{PP_{|\overline{v}_{PP}}} \right) > \left(E\pi_{NYOP_{|\overline{v}_{PP}}} - E\pi_{PP_{|\overline{v}_{PP}}} \right) = \frac{[K-(1-\overline{v}_{PP}]^3}{4\overline{y}} > 0$, implying that the NYOP regime is more beneficial to the airline.

Under the posted price regime, the lowest valuation leisure traveler active in the second stage satisfies the equation that $\underline{v}(y) = P^R(y)$, which from (4) implies that $\underline{v}(y) = 1 - K + y$. Under the NYOP regime, the lowest valuation consumer submits a bid just equal to the wholesale price chosen by the airline. From (14) and (15), therefore, $\frac{\underline{v}(y)+1-K}{2} = 1 - K + \frac{y}{2}$, implying, once again, that $\underline{v}(y) = (1 - K) + y$. Since $\overline{v}_{NYOP} > \overline{v}_{PP}$, it follows that $\left(\overline{v}_{NYOP} - \underline{v}(y)\right) > \left(\overline{v}_{PP} - \underline{v}(y)\right)$, and a bigger mass of leisure travelers postpones the purchase to the second stage if the intermediary utilizes the NYOP mechanism.

(ii) The expected consumer surplus under a posted price regime can be derived as follows:

$$ECS_{PP} = \int_{\overline{v}}^{1}(v - P^H)\partial v + \frac{1}{\overline{y}}\int_0^{K-(1-\overline{v})}\int_{1-K+y}^{\overline{v}}[v - (1 - K + y)]\partial v \partial y.$$

Her surplus under NYOP is:

$$ECS_{NYOP} = \int_{\overline{v}}^{1}(v - P^H)\partial v + \frac{1}{\overline{y}}\int_0^{K-(1-\overline{v})}\int_{1-K+y}^{\overline{v}}[v - Bid(v)]\partial v \partial y.$$

Using the expressions derived for P^H and $Bid(v)$ in (5), (16), and (15) yields that for a common $\overline{v}$:

$$ECS_{PP_{|\overline{v}}} = E_{CS_{NYOP_{|\overline{v}}}} = \frac{(1-\overline{v})^2}{2} + \frac{[K-(1-\overline{v})]^2}{6\overline{y}}[2(1 - \overline{v}) + K] \equiv G(\overline{v}).$$

Since $\frac{\partial G}{\partial \overline{v}} = -(1 - \overline{v})\left[1 - \frac{K-(1-\overline{v})}{\overline{y}}\right] < 0$ the consumer surplus declines for bigger values of $\overline{v}$. Since $\overline{v}_{NYOP} > \overline{v}_{PP}$, the consumer benefits if the intermediary utilizes the posted price instead of the NYOP mechanism.

(iii) For a fixed $\overline{v}$, the expected payoff of the intermediary under the posted price regime can be derived as follows:

$$w_{PP} = \frac{1}{\overline{y}}\int_0^{K-(1-\overline{v})}\int_{1-K+y}^{\overline{v}}[P^R(y) - P^W(y)]\partial v \partial y,$$

where $P^R(y)$ and $P^W(y)$ are given by (4) and (3), respectively: Hence for $\overline{v} = \overline{v}_{PP}$,

$$w_{PP} = \frac{[K-(1-\overline{v}_{PP})]^3}{3\overline{y}}. \qquad \text{(A.9)}$$

For a fixed $\overline{v}$, the expected payoff of the intermediary with NYOP can be expressed as:

$$w_{NYOP} = \frac{1}{\overline{y}}\int_0^{K-(1-\overline{v})}\int_{1-K+y}^{\overline{v}}[Bid(v) - P^W(y)]\partial v \partial y,$$

where $Bid(v)$ and $P^W(y)$ are given by (15) and (14), respectively. Hence for $\overline{v} = \overline{v}_{NYOP}$,

$$w_{NYOP} = \frac{[K-(1-\overline{v}_{NYOP})]^3}{12\overline{y}}. \qquad \text{(A.10)}$$

The expression for the difference is obtained by subtracting (A.10) from (A.9) and manipulating the expression. The first term of this difference is positive since $K > (1-\overline{v}_{PP})$, given the assumption that a positive capacity is reserved for the second stage. The second term is negative given that $\overline{v}_{NYOP} > \overline{v}_{PP}$.

(iv) For a fixed $\overline{v}$, the expression for the aggregate producer surplus is the same for either the posted price or the NYOP models. Specifically, $PS_{PP} = PS_{NYOP} = PS = (1-\overline{v})\overline{v} + r\big(K-(1-\overline{v})\big)\left[1-\frac{K-(1-\overline{v})}{2\overline{y}}\right] + \frac{1}{6\overline{y}}[K-(1-\overline{v})]^2[4\overline{v}-2K-1]$.

This surplus is maximized when $\overline{v} = \frac{1+r}{2}$. Since $\overline{v}_{PP} < \overline{v}_{NYOP} < \frac{1+r}{2}$, the producer surplus is bigger with NYOP. Q.E.D.

<u>Proof of Lemma 4</u>

The pricing model utilized in the secondary channel does not affect the expression for revenues derived in the direct marketing channel of the airline. Those revenues, denoted by *M*, comprise of proceeds from leisure travelers in the first stage and business travelers in the second stage as follows:

$$M = P^H(1-\overline{v}) + \frac{r}{\overline{y}}\left[\int_0^{K-(1-\overline{v})} y\,dy + \int_{K-(1-\overline{v})}^{\overline{y}} (K-(1-\overline{v}))\,dy\right], \text{ where } P^H = \overline{v} - \frac{[K-(1-\overline{v})]^2}{2\overline{y}} \quad \text{(A.11)}$$

The expression for the expected revenues from the secondary market, denoted by *F*, depends, however, on the pricing model as follows:

With posted pricing it is equal to:

$$F_{PP} = \frac{1}{\overline{y}}\int_0^{K-(1-\overline{v})} P_{PP}^W(y)[K-(1-\overline{v})-y]\partial y, \text{ where } P_{PP}^W(y) = (1-K+y). \quad \text{(A.12)}$$

With NYOP it is equal to:

$$F_{NYOP} = \frac{1}{\overline{y}}\int_0^{K-(1-\overline{v})}\int_{\underline{v}(y)}^{\overline{v}} Bid(v)\partial v\partial y, \text{ where } Bid(v) = \frac{v+1-K}{2} \text{ and } \underline{v}(y) = 1-K+y. \quad \text{(A.13)}$$

The expression for $Bid(v)$ and $\underline{v}(y)$ are implied by the results reported in Lemma 3 and the fact that $Bid\big(\underline{v}(y)\big) = P_{NYOP}^W(y) = (1-K) + \frac{y}{2}$.

Integrating (A.12) and (A.13) yields that:

$$F_{PP} = F_{NYOP} = \frac{[K-(1-\overline{v})]^2}{6}[\overline{v} + 2(1-K)]. \quad \text{(A.14)}$$

Combining the expected revenues from both channels as given in (A.11) and (A.14) yields the objective function (20). Q.E.D.

Proof of Proposition 5

(i) Differentiating (20) with respect to $\overline{v}$ and setting the derivative equal to zero, yields $\overline{v} = \frac{1+r}{2}$. This solution is less than 1 if $r < 1$. The expression for P^H follows from (5) and (16). When $r > 1$, $\overline{v} = 1$, and the airline sets a very high price in the first stage (i.e., $P^H > 1$) to guarantee that no leisure traveler is active in the first stage.

(ii) Obtained by substituting the optimal solution of $\overline{v}$ back into (20). The comparative statics with respect to *r*, $\overline{y}$, and *K* are obtained by differentiating $E\pi^*$.

*(iii)*Follows from the expression derived for $\overline{v}$ in part (*i*).

Proof of Proposition 6

(i) From the expression obtained with vertical separation in (7) and (18)

$\overline{v}_{PP} < \overline{v}_{NYOP} < \frac{1+r}{2}$.

Hence, a smaller segment of leisure travelers is active in the first stage.

(ii) In the proof of part (*iv*) of Proposition 3, the expression derived for the aggregate producer surplus coincides with (20). It is maximized when $\overline{v} = \frac{1+r}{2}$. Hence, any other value of $\overline{v}$ that is obtained with vertical separation reduces the total producer surplus. Since the profit of the airline is a fraction of this surplus, vertical separation definitely reduces the airline's profit. From the proof of part (*ii*) of Proposition 3, the expression derived for the expected consumer surplus as a function of $\overline{v}$ is given by $G(\overline{v})$, where $G'(\overline{v}) < 0$. Since vertical integration results in a bigger value of $\overline{v}$, consumer surplus is smaller with integration. Q.E.D.

Proof of Proposition 7

(i) Differentiating the objectives (8), (17), and (20), net of the investment cost in capacity cK with respect to *K* for the PP, NYOP, and VI cases, and combining the expressions with the corresponding first order conditions for $\overline{v}$, yields the expressions for K_x^* in terms of $\overline{v}_x^*$.

(ii) Substituting for K_x^* in terms of $\overline{v}_x^*$ into (5) and/or (16), yields $P^H = \frac{1+c}{2}$.

(iii) The derivatives of objectives (8), (17), and (20) net of the investment cost with respect to $\overline{v}$ and K yield the following inequalities for fixed values of $\overline{v}$ and K.

$$\frac{\partial E\pi_{PP}}{\partial \overline{v}} < \frac{\partial E\pi_{NYOP}}{\partial \overline{v}} < \frac{\partial E\pi_{VI}}{\partial \overline{v}} \quad \text{and} \quad \frac{\partial E\pi_{PP}}{\partial K} < \frac{\partial E\pi_{NYOP}}{\partial K} < \frac{\partial E\pi_{VI}}{\partial K}.$$

Given that the objectives are concave functions of $\overline{v}$ and *K*, the results follow.Q.E.D.

Proof of Corollary 2

(i) Obtained by substituting $\bar{v}_{VI} = \frac{1+r}{2}$ into the expression derived in Proposition 7.

(ii) To guarantee that $K_{VI}^{*} < 1$ it follows that $\sqrt{(r-c)\bar{y}} < \frac{1+r}{2}$, which implies that $\bar{y} < \frac{(1+r)^2}{4(r-c)}$. To guarantee that $K_{VI}^{*} < 1 - \bar{v} + \bar{y}$, it follows that $\sqrt{(r-c)\bar{y}} < \bar{y}$ or that $\bar{y} > (r-c)$. Q.E.D.

Proof of Lemma 5

Obtained by substituting in the expressions derived in Lemma 1-3, $\bar{v} = 1 - x_1 - x_2$ and $K = K_1 + K_2$.

Q.E.D.

Objective Functions of Airlines – $(K_2 - x_2) \geq (K_1 - x_1)$.

1) The revenues that accrue to Airline *i* from sales in the first stage are

$$TR_L^i = P^H x_i = \left[1 - x_1 - x_2 - \frac{[(K_1+K_2)-(x_1+x_2)]^2}{2\bar{y}}\right] x_i.$$

2) The expected revenues that accrue from business travelers are:

For Airline 1:

$$TR_B^1 = \frac{r}{\bar{y}}\left[\int_0^{2(K_1-x_1)} \frac{y}{2} dy + \int_{2(K_1-x_1)}^{\bar{y}} (K_1 - x_1) dy\right] = r(K_1 - x_1)\left[1 - \frac{K_1 - x_1}{\bar{y}}\right]$$

For Airline 2:

$$TR_B^2 = \frac{r}{\bar{y}}\left[\int_0^{2(K_1-x_1)} \frac{y}{2} dy + \int_{2(K_1-x_1)}^{K_2-x_2+K_1-x_1} (y - (K_1 - x_1)) dy + \int_{K_2-x_2+K_1-x_1}^{\bar{y}} (K_2 - x_2) dy\right] =$$
$$\frac{r}{\bar{y}}\left[\frac{(K_1-x_1)^2}{2} - \frac{(K_2-x_2)^2}{2} - (K_1 - x_1)(K_2 - x_2) + \bar{y}(K_2 - x_2)\right]$$

3) The expected revenues that accrue from the clearinghouse channel are:

For Airline 1:

$$TR_{CL}^1 = \int_0^{2(K_1-x_1)} \left[(K_1 - x_1) - \frac{y}{2}\right] P^W(y) dy$$

For Airline 2:

$$TR_{CL}^2 = \int_0^{2(K_1-x_1)} \left[(K_2 - x_2) - \frac{y}{2}\right] P^W(y) dy + \int_{2(K_1-x_1)}^{K_2-x_2+K_1-x_1} [(K_2 - x_2 + K_1 - x_1) - y]\, P^W(y) dy,$$

where $P^W(y)$ is given as follows:

$$P^W(y) = \begin{cases} 2(1-K_1-K_2)-(1-x_1-x_2)+2y & \textit{if intermediary uses PP} \\ 1-K_1-K_2+\frac{y}{2} & \textit{if intermediary uses NYOP} \end{cases}$$

The objective of Airline *i* is the sum $\pi_i = \left(TR_L^i + TR_B^i + TR_{CL}^i\right)$ and each chooses x_i to maximize this objective.

<u>Calculations of Marginal Profits</u>

1) Intermediary Uses Posted Price

<u>Marginal Profits of Airline 1</u>

$$\frac{\partial \pi_1^{PP}}{\partial x_1} = 1 - x_1 - x_2 - \frac{[K_2-x_2+K_1-x_1]^2}{2\overline{y}} - x_1\left[1-\frac{K_2-x_2+K_1-x_1}{\overline{y}}\right] - r\left(1-\frac{2(K_1-x_1)}{\overline{y}}\right) - \frac{(K_1-x_1)}{\overline{y}}[2(1+x_1+x_2)-4(K_1+K_2)+3(K_1-x_1)].$$

<u>Marginal Profits of Airline 2</u>:

$$\frac{\partial \pi_2^{PP}}{\partial x_2} = 1 - x_1 - x_2 - \frac{[K_2-x_2+K_1-x_1]^2}{2\overline{y}} - x_2\left[1-\frac{K_2-x_2+K_1-x_1}{\overline{y}}\right] - r\left(1-\frac{K_2-x_2+K_1-x_1}{\overline{y}}\right) - \frac{(K_2-x_2+K_1-x_1)}{2\overline{y}}[2+x_1+x_2-3(K_1+K_2)] - \frac{(K_1-x_1)^2}{\overline{y}}$$

2) Intermediary Uses NYOP

<u>Marginal Profits of Airline 1</u>:

$$\frac{\partial \pi_1^{NYOP}}{\partial x_1} = 1 - x_1 - x_2 - \frac{[K_2-x_2+K_1-x_1]^2}{2\overline{y}} - x_1\left[1-\frac{K_2-x_2+K_1-x_1}{\overline{y}}\right] - r\left(1-\frac{2(K_1-x_1)}{\overline{y}}\right) - \frac{(K_1-x_1)}{\overline{y}}[(K_1-x_1)+2(1-K_1-K_2)]$$

<u>Marginal Profits of Airline 2</u>:

$$\frac{\partial \pi_2^{NYOP}}{\partial x_2} = 1 - x_1 - x_2 - \frac{[K_2-x_2+K_1-x_1]^2}{2\overline{y}} - x_2\left[1-\frac{K_2-x_2+K_1-x_1}{\overline{y}}\right] - r\left(1-\frac{K_2-x_2+K_1-x_1}{\overline{y}}\right) - \frac{(K_1-x_1)}{\overline{y}}[(K_1-x_1)+2(1-K_1-K_2)] - \frac{[(K_2-x_2)-(K_1-x_1)]}{\overline{y}}\left[(1-K_1-K_2)+\frac{3}{4}(K_1-x_1)+\frac{1}{4}(K_2-x_2)\right].$$

<u>Proof of Proposition 8</u>

(i) Evaluating the partial derivative of Airline 2 at $x_2 = x_1^* + K_2 - K_1$ where x_1^* is the optimal solution of Airline 1 that solves the condition $\frac{\partial \pi_1}{\partial x_1} = 0$, yields:

$$\frac{\partial \pi_2}{\partial x_2}_{|x_2 = x_1^*+K_2-K_1} = -(K_2-K_1)\left[1-\frac{2(K_1-x_1^*)}{\overline{y}}\right] < 0.$$

Since the above marginal benefit expression for Airline 2 is negative, it follows that $x_2^* < x_1^* + K_2 - K_1$, or $K_2 - x_2^* > K_1 - x_1^*$.

(ii) It is easy to show that when $K_1 = K_2$, setting the expressions of the marginal benefits equal to zero yields that $x_1^* = x_2^* = x^*$. At the symmetric equilibrium:

$$\frac{\partial \pi_i^{PP}}{\partial x_i}_{|x_1=x_2=x} = 1 - 2x - \frac{2(K-x)^2}{\overline{y}} - x\left(\frac{2(K-x)}{\overline{y}}\right) - r\left(1 - \frac{2(K-x)}{\overline{y}}\right) - \frac{(K-x)}{\overline{y}}[2 + x - 5K],$$

$$\frac{\partial \pi_i^{NYOP}}{\partial x_i}_{|x_1=x_2=x} = 1 - 2x - \frac{2(K-x)^2}{\overline{y}} - x\left(\frac{2(K-x)}{\overline{y}}\right) - r\left(1 - \frac{2(K-x)}{\overline{y}}\right) - \frac{(K-x)}{\overline{y}}[2 - x - 3K].$$

Hence, $\frac{\partial \pi_i^{NYOP}}{\partial x_i}_{|x_1=x_2=x} = \frac{\partial \pi_i^{PP}}{\partial x_i}_{|x_1=x_2=x} - \frac{2(K-x)^2}{\overline{y}}$.

Given that the marginal benefit expression under NYOP is smaller than under PP, it follows that $x^{*NYOP} < x^{*PP}$. For a common level of capacity allocated by each airline under either NYOP or PP:

$$\pi_i^{NYOP}{}_{|x_1=x_2=x} = \pi_i^{PP}{}_{|x_1=x_2=x} + \frac{2(K-x)^3}{\overline{y}} > \pi_i^{PP}{}_{|x_1=x_2=x}.$$

Hence, the above inequality holds, in particular, for x^{*PP}. Since according to part (*iii*) of the Proposition $(1 - \overline{v}_{monopoly}) < 2x^{*NYOP} < 2x^{*PP}$, the adjustment of the allocated capacity when moving from PP to NYOP increases the joint profits of the airlines. Hence, $\pi_i^{NYOP}(x^{*NYOP}) > \pi_i^{NYOP}(x^{*PP})$and the NYOP model generates higher profits for each airline than the PP model.

(iii) Designate by *z* the capacity allocated by a monopolist for sale to leisure travelers in the first stage, and let $\overline{K}$ be the monopolist's capacity, then the value of *z* satisfies the following FOC:

$$1 - 2z - r\left(\frac{\overline{K}-z}{\overline{y}}\right) - \frac{(\overline{K}-z)}{\overline{y}}\left(1 - z - \overline{K}\right) = 0 \quad \text{with PP}$$

$$1 - 2z - r\left(1 - \frac{\overline{K}-z}{\overline{y}}\right) - \frac{(\overline{K}-z)}{4\overline{y}}\left(4 - 7z - \overline{K}\right) = 0 \quad \text{with NYOP.}$$

Suppose that with two competing airlines $K_1 = K_2 = \frac{\overline{K}}{2}$, so that aggregate capacity in the industry remains fixed. Substituting into the expressions derived for the marginal benefit with competing airlines $x_1 = x_2 = \frac{z}{2}$, we obtain:

$$\frac{\partial \pi_i^{PP}}{\partial x_i}_{|x_1=x_2=\frac{z^{PP}}{2}} = 1 - \frac{3}{2}z^{PP} - r\left(1 - \frac{\overline{K}-z^{PP}}{\overline{y}}\right) - \frac{(\overline{K}-z^{PP})}{2\overline{y}}\left(2 - \frac{3}{2}z^{PP} - \frac{3}{2}K\right)$$

$$\frac{\partial \pi_i^{NYOP}}{\partial x_i}_{|x_1=x_2=\frac{z^{NYOP}}{2}} = 1 - \frac{3}{2}z^{NYOP} - r\left(1 - \frac{\overline{K}-z^{NYOP}}{\overline{y}}\right) - \frac{(\overline{K}-z^{NYOP})}{2\overline{y}}\left(2 - \frac{5}{2}z^{NYOP} - \frac{K}{2}\right).$$

Given the first order conditions satisfied by $z^{PP}$and z^{NYOP}, it follows that:

$$\frac{\partial \pi_i^{PP}}{\partial x_i}_{|x_1=x_2=\frac{z^{PP}}{2}} = \left[\frac{z^{PP}}{2} - \frac{(\overline{K}-z^{PP})}{\overline{y}}\frac{(\overline{K}+z^{PP})}{4}\right] > 0 \quad if \quad r < 1$$

$$\frac{\partial \pi_i^{NYOP}}{\partial x_i}_{|x_1=x_2=\frac{z^{NYOP}}{2}} = \frac{z^{NYOP}}{2}\left[1 - \frac{(\overline{K}-z^{NYOP})}{\overline{y}}\right] > 0.$$

Recall that the assumption $r < 1$ is necessary to guarantee an active first stage leisure market when a monopolist airline is vertically integrated. Since the marginal benefit expressions evaluated at $x_i = \frac{z}{2}$ are positive, each of the two airlines allocates a larger capacity than $\frac{z}{2}$ for sale in the first stage. This implies that a smaller overall capacity is reserved for business class demand when two airlines compete in the market. Q.E.D.

Proof of Proposition 9

(i) When multiple intermediaries compete in the market, with at least one using the PP model, $B(v) = P^W(y) = 1 - K + y$. Substituting this schedule of the wholesale price into the revenues that accrue from the clearinghouse channel and differentiating the objective of Airline 1 with respect to x_1 yields:

$$\frac{\partial \pi_1}{\partial x_1} = 1 - x_1 - x_2 - \frac{(K_2-x_2+K_1-x_1)^2}{2\overline{y}} - \left[1 - \frac{(K_2-x_2+K_1-x_1)}{\overline{y}}\right]x_1 - r\left(1 - \frac{2(K_1-x_1)}{\overline{y}}\right) - \frac{2(K_1-x_1)}{\overline{y}}(1 - K_2 - x_1).$$

At the symmetric equilibrium, when $K_1 = K_2 = K, x_1 = x_2 = x$ and the expression for the marginal benefit of each airline reduces to:

$$\frac{\partial \pi_i}{\partial x_i}_{|x_1=x_2=x} = (1 - 3x - r)\left(1 - \frac{2(K-x)}{\overline{y}}\right).$$

Setting the marginal benefit equal to zero yields that $x = \frac{1-r}{3}$. Substituting this value into the marginal benefit expression in the absence of competition for the NYOP model yields:

$$\frac{\partial \pi_i^{NYOP}}{\partial x_i}_{|x_1=x_2=\frac{1-r}{3}} = (1 - 3x - r)\left[1 - \frac{2(K-x)}{\overline{y}}\right] + \frac{(K-x)^2}{\overline{y}} = \frac{(K-x)^2}{\overline{y}} > 0.$$

Hence, in the absence of competition $x^{*NYOP} > \frac{1-r}{3}$ and since $x^{*PP} > x^{*NYOP}, x^{*PP} > \frac{1-r}{3}$ as well.

(ii) In the previous section we found that $x^*_{monopoly} = \frac{1-r}{2}$ when the clearinghouse market is competitive. With two competing airlines the aggregate capacity allocated to the first stage is $\frac{2(1-r)}{3}$. Hence,

$\overline{v}_{monopoly} = \frac{1+r}{2} > \frac{1+2r}{3} = \overline{v}_{duopoly}$. Q.E.D.