Math 0031 - Final Exam

December 15, 2015

Name:

Question	Points	Possible
1		20
2		20
3		10
4		10
5		20
6		20
7		20
8		20
9		10
10		10
11		20
12		20
Total		200

NO books, notes, calculators, computers, phones or friends may be used during this exam. Remember to carefully read each question, and show all of your work. Partial credit may be awarded for showing your work or explaining how to solve the problem (even for incomplete or incorrect answers). The point value for each question is given in the table above.

1. (20 points) Given the points $A = (-3, 2)$ and $B = (1, 4)$:

(a) Find the midpoint of the line segment connecting A and B.

(b) Write the equation of the circle passing through the points A and B and centered at their midpoint.

(c) Find the equation of the line that passes through the midpoint and is *perpendicular* to the line passing through points A and B.

2. (20 points) Let $f(x) = x^2 - 3\,x - 7$

(a) Use the Intermediate Value Theorem to show f has a zero (root) in the interval $(-2, 0)$.

(b) Write $f(x)$ in the form

$$f(x) = (x + 1)\,q(x) + R\ ,$$

where $q(x)$ is the quotient and R is the remainder.

(c) Find and simplify the difference quotient,

$$\frac{f(-1+h) - f(-1)}{h}\ .$$

3. (10 points)

(a) Solve

$$t^2 - 6t + 13 = 0.$$

(b) Write the following complex numbers in the form $a + b\,i$, where a, b are real numbers.

(i)

$$\frac{2 - i}{1 - 3i}\ .$$

(ii)

$$(1 + 4i)(2 - i)\ .$$

4. (10 points) Determine the polynomial function $f(x)$ of degree 4 that has zeroes at $x = -3$ and $x = 2$, each with multiplicity 1, a zero at at $x = 1$ with mulitplicity 2, and is such that $f(0) = 12$.

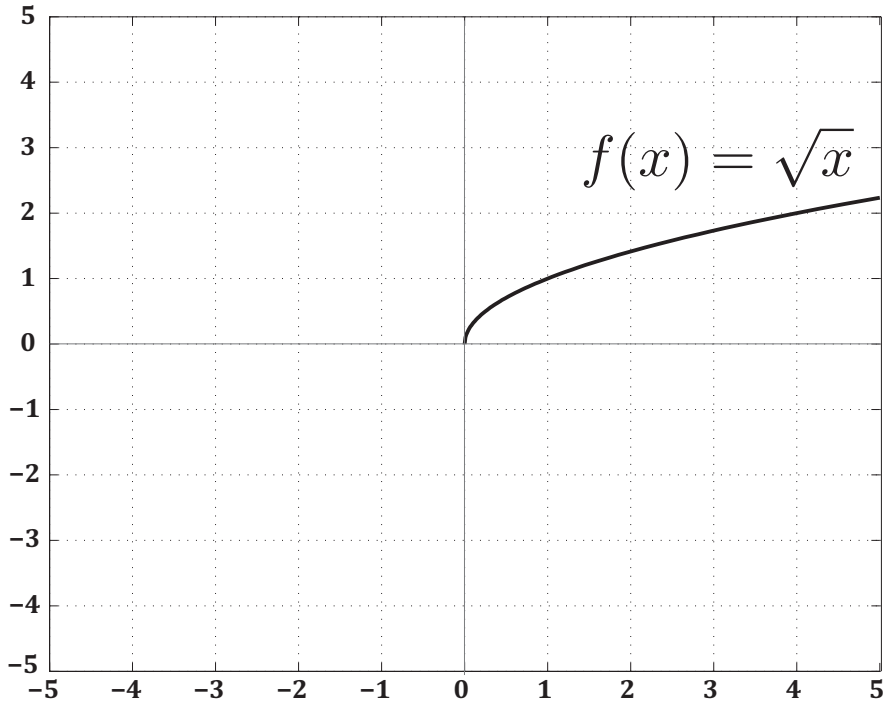
5. (20 points) Let $f(x) = \sqrt{x}$, $g(x) = 1 - x$, and $h(x) = x - 2$.

(a) Write

$$k(x) = \sqrt{1 - x} - 2\,,$$

as a composition of $f(x)$, $g(x)$, and $h(x)$.

(b) Given the graph of $f(x) = \sqrt{x}$, sketch the graph of $k(x)$.



6. (20 points) Consider the quadratic function

$$f(x) = x^2 - 4x + 3 \ .$$

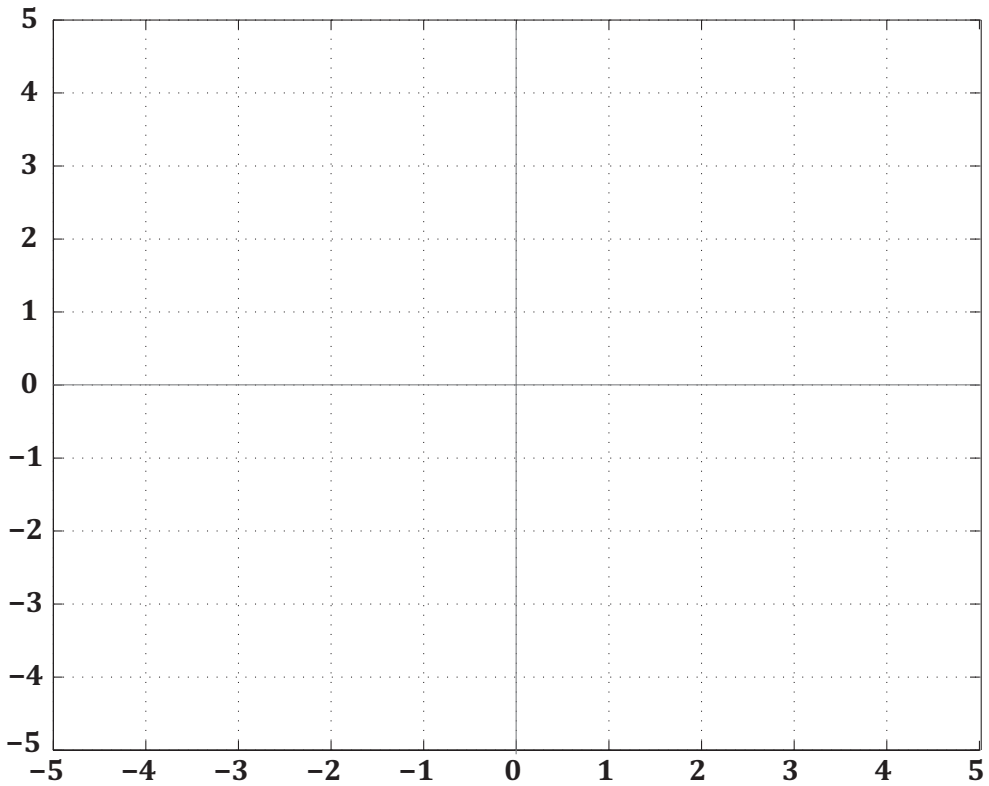
(a) Find the coordinates of the vertex.

(b) Find the equation of the axis of symmetry.

(c) Specify if there is a minimum or maximum at the vertex and find the value of f there.

(d) Find the range of f.

(e) Sketch the graph of f.



7. (20 points) Consider the function $f(x) = \frac{x-3}{x^2-2x-3}$.

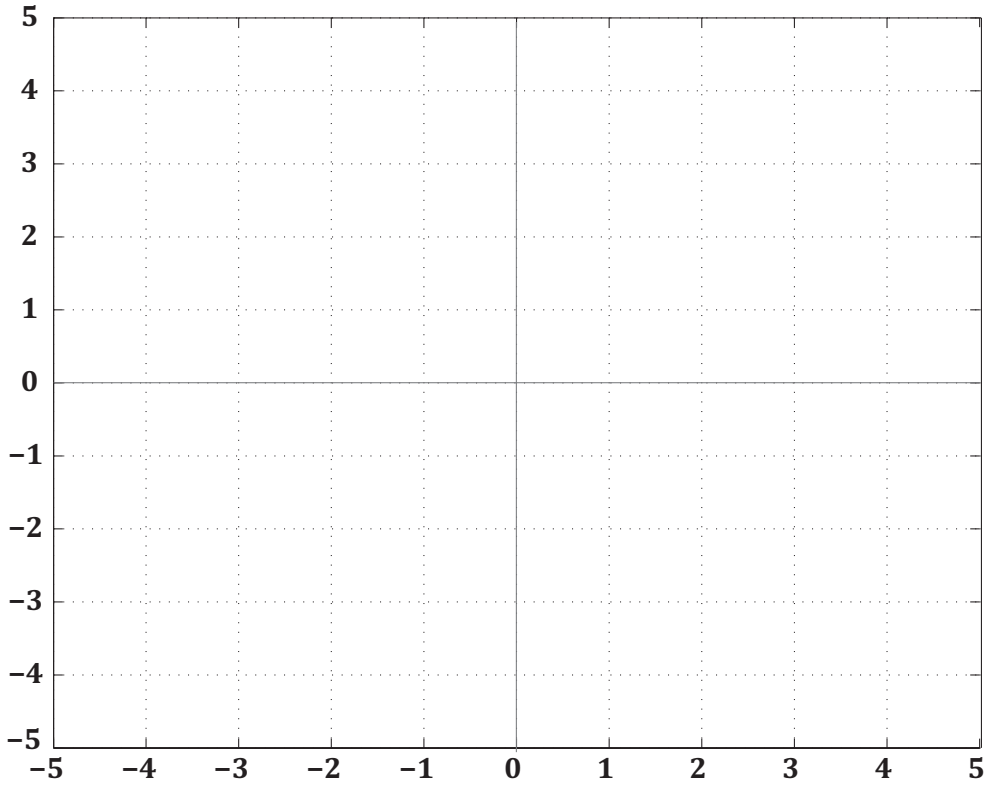
(a) Find the domain and asymptotes of the rational function $f(x)$.

Domain:

Vertical asymptote:

Horizontal asymptote:

(b) Sketch a graph of $f(x)$ below.



(c) Solve the rational inequality $f(x) = \frac{x-3}{x^2-2x-3} \geq 0$.

8. (20 points) Let $f(x) = \frac{x+4}{x-3}$.

(a) Find the domain and range of $f(x)$.

(b) Find the inverse function $f^{-1}(x)$.

(c) Find the domain and range of $f^{-1}(x)$.

9. (10 points) Solve

(a) $9^{(x+2)} = 27^x$

(b) $\log_2(x) + \log_2(x-2) = 3$

10. (10 points) Elena ordered concert tickets that cost \$7.50 for children and \$12.00 for adults. She ordered 8 more children's tickets than adult tickets. If her total bill was \$138.00, how many of each type of ticket did she order?

11. (20 points) Solve

$$\begin{array}{rrrl} 3\,x & +4\,y & +2\,z & = 3 \\ 2\,x & & +z & = 4 \\ x & -5\,y & -3\,z & = 0 \end{array}$$

12. (20 points) For the matrices

$$A = \begin{bmatrix} -2 & 4 \\ 5 & 1 \\ -1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -6 \\ -1 & 4 \end{bmatrix}$$

either compute the following, or explain why it is not possible.

(a) $A - 3\,B$

(b) AB

(c) BA