MATH 0031 - ALGEBRA

SAMPLE FINAL EXAM
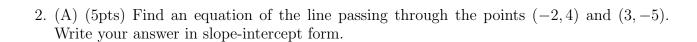
Exam length: 1 hour 50 minutes

<u>INSTRUCTIONS:</u>

1. NO TABLES, BOOKS, NOTES, HEADPHONES, CALCULATORS, OR COMPUTERS MAY BE USED.

2. <u>Show ALL of your calculations and display answers clearly.</u> You may leave your final answers in exact form. Unjustified answers will <u>receive no credit.</u>

3. WRITE YOUR SOLUTIONS in the space provided. EXTRA SPACE is available on the BACKS of the pages. When using these back pages, clearly LABEL the problem, and also clearly indicate on the appropriate front page where your back-page solution (or continuation of a solution) is located.

4. Write neatly. Cross out any work that you do not wish to be considered for grading.

5. Academic Integrity Strictly Applies. Looking at another person's paper is reason to assume cheating and your paper will be taken.

6. All Cell phones and electronic devices must be OFF and put away and hats removed.

1. (A) (5 pts) Find an equation of a circle with center $(3, -4)$ passing through the point $(-2, 1)$.

(B) (5 pts) Given that the coordinates of the midpoint of a line segment are $(-4, 3)$ and one end of the line segment has coordinates $(-6, -2)$, find the coordinates of the other end of the line segment.

2. (A) (5pts) Find an equation of the line passing through the points $(-2, 4)$ and $(3, -5)$. Write your answer in slope-intercept form.

(B) (5 pts) Write a slope-intercept equation for a line through the point $(3, -2)$ that is perpendicular to the line $3x + 4y = 5$.

3. (A) (5 pts) Dimitri's two student loans total \$9,000. One loan is 5% simple interest and the other is at 6%-simple interest. At the end of one year Dimitri owes \$492 in interest. What is the amount of each loan?

(B) (5 pts) Given the piecewise function:

$$f(x) = \begin{cases} x+2, & x \le -3 \\ -5, & -3 < x < 2 \\ x^2 - 1, & x \ge 2 \end{cases}$$

Find the following function values:

(A) $f(-4)$ (B) $f(-3)$ (C) $f(0)$ (D) $f(2)$ (E) $f(3)$

4. (A) (5 pts) Given the functions $f(x) = \sqrt{x-3}$ and $g(x) = x^3 + 8$ find the composition functions $(f \circ g)(x)$ and $(g \circ f)(x)$ and the **DOMAINS** of each.

(B) (5 pts) Describe how the graph of the function $f(x) = -2|x+5| + 3$ can be obtained from the graph of $y = |x|$. **List the transformations** and sketch the graph of $f(x)$.

5. (A) (5 pts) Solve by factoring $6x^3 + x^2 - 12x - 2 = 0$. (**Hint:** Factor by Grouping)

(B) (5 pts) Solve $y^{\frac{2}{3}} + 4y^{\frac{1}{3}} - 5 = 0$ (**Hint:** Let $u = y^{\frac{1}{3}}$).

6. (10 pts) Solve by **Completing the Square** to obtain the exact solutions. Then check by using the Quadratic Formula.

$$5x^2 - 4x - 11 = 0$$

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7. (A) (5 pts) Simplify:
$$\sqrt{-4} \cdot \sqrt{-25}$$

(B) (5 pts) Simplify and write the answer in $a + bi$ form.
$$\frac{6 - i}{-7 + 2i}$$

8. (10 pts) Given the Quadratic Function $f(x) = -x^2 - 6x + 3$

(A) Find the vertex.

(B) Find the axis of symmetry.

(C) Determine whether there is a maximum or minimum value and find that value.

(D) Graph the function.

9. (A) (5 pts) Solve the rational equation:

$$\frac{2}{x+5} + \frac{1}{x-5} = \frac{16}{x^2 - 25}$$

(B) (5 pts) y varies directly with x and inversely with z. $y = 4$ when $x = 2$ and $z = 5$. Find an equation of variation then find y when $x = 4$ and $z = 8$.

10. (10 pts) Solve the Radical Equation. Check your solutions.

$$\sqrt{2x + 3} - \sqrt{x + 2} = 2.$$

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11. (10 pts) Solve the inequalities. Write **Interval Notation for your Solutions** and then **Graph the solutions on the Number Line**.

(A) $|3x + 4| > 13$

(B) $|4x - 2| \leq 1$

12. (10 pts) Given the polynomial function $f(x) = x^3 - x^2 - 6x$

(A) Determine the end behavior of $f(x)$

(B) Find the zeros of the function. State the multiplicity of each zero and whether the graph crosses or just touches the x-axis there.

(C) Use the x-intercepts (zeros) to divide the x-axis into intervals and determine the sign of the function in each interval.

(D) Find the y-intercept.

(E) Draw the graph.

13. (A) (5 pts) Use synthetic division to find $f(-2)$ for $f(x) = x^3 + 7x^2 - 12x - 3$ and then check your answer by direct substitution in $f(x)$.

(B) (5 pts) Find a polynomial function of degree 5 with -4 and 2 as zeros of multiplicity one and -1 as a zero of multiplicity 3. Write the function in **factored form**. **DO NOT MULTIPLY IT OUT!!**

14. (10 pts) For the polynomial function $f(x) = 5x^4 - 4x^3 + 19x^2 - 16x - 4$.

(A) Find all possible rational zeros.

(B) Find all zeros, that is solve $f(x) = 0$.

(C) Factor $f(x)$ into linear factors.

15. (10 pts) Given the rational function $f(x) = \dfrac{x^2 - 9}{x + 1}$

(A) Find and label all asymptotes.

(B) Find the $x-$ and $y-$intercepts and list the domain.

(C) Graph the function.

16. (10 pts) Given the one-to-one function $f(x) = \dfrac{x-4}{x+1}$

(A) Find the inverse function $f^{-1}(x)$.

(B) Given the domain and range of **BOTH** f and f^{-1}.

17. (A) (6 pts) Solve the polynomial inequality $x^2 - x - 5 \geq x - 2$

(B) (4 pts) Find:

(i) $\log_3 \left(\dfrac{1}{27} \right)$

(ii) $\log(1,000)$

18. (A) (5 pts) Solve $4^{x+3} = 3^{-x}$.

(B) (5 pts) Solve $\log_3(x-2) + \log_3 x = 1$

19. (10 pts) Solve the linear system

$$x - y + 2z = -3$$
$$x + 2y + 3z = 4$$
$$2x + y + z = -3$$

(You may use either algebraic elimination, Gaussian elimination, or Cramer's Rule)

20. (10 pts) Graph the system of inequalities and then find the coordinates of the vertices.

$$y \geq x$$

$$y \leq 4 - x$$