Math 0031 **Sample Exam 1**

Spring 2017 **S o l u t i o n s**

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1. Consider the relation $\{(-2,3),(0,4),(3,3),(4,3),(6,-1)\}$.

(a) Determine whether the relation is a function.

Solution: Yes, it is a function because no two ordered pairs have the same first coordinate and different second coordinates.

(b) Identify the domain and the range.

Solution: The domain is the set of all first coordinates: $\{-2,0,3,4,6\}$.

The range is the set of all second coordinates: $\{-1,3,4\}$.

2. Find a point-slope equation of the line that has y intercept -8 and is parallel to the line $y+2x=3$.

Solution: $y+2x=3 \quad \Leftrightarrow \quad y=-2x+3 \quad \Rightarrow$ the slope of both lines is $m=-2$.

The point-slope equation of the line is $y-(-8)=-2(x-0) \quad \Leftrightarrow \quad y+8=-2x$.

3. Solve the compound inequality $x-2 \le 3+2x < 9-x$.

Solution: We have to solve two inequalities $x-2 \le 3+2x$ and $3+2x < 9-x$ and find the intersection of the corresponding solution sets.

$x-2 \le 3+2x \quad \Leftrightarrow \quad -x \le 5 \quad \Leftrightarrow \quad x \ge -5$. The solution set is $[-5,\infty)$.

$3+2x < 9-x \quad \Leftrightarrow \quad 3x < 6 \quad \Leftrightarrow \quad x < 2$. The solution set is $(-\infty,2)$.

The solution set of the compound inequality is $[-5,2)$.

4. A distance between a town A and a town B is 275 miles. At 1 pm a car leaves town A and goes to town B. At the same time a bus leaves town B and goes to town A. The car and the bus meet at 3:30 pm. The car runs 10 mph faster than the bus. Find the speed of the car.

Solution: Let x be the speed of the car, in mph. Then the speed of the bus is $x-10$ mph.

Before they met they had been traveled for 2.5 hours. Then $2.5x+2.5(x-10)=275$.

$\Leftrightarrow \quad 5x - 25 = 275 \quad \Leftrightarrow \quad 5x = 300 \quad \Leftrightarrow \quad x = 60.$

Answer: The the speed of the car is 60 mph.

5. For the function $f(x) = x^3 + 5$ find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

Solution: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{((x+h)^3 + 5 - (x^3 + 5)}{h}$

$= \dfrac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3 - 5}{h} = \dfrac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2.$

6. Simplify. Write answers in the form $a + bi$ where a and b are real numbers.

(a) $(5 + 3i) - (8 - 3i)$.

Solution: $(5 + 3i) - (8 - 3i) = 5 + 3i - 8 + 3i = -3 + 6i$

(b) $(-2 + 5i)(3 - 2i)$.

Solution: $(-2 + 5i)(3 - 2i) = -6 + 4i + 15i - 10i^2 = -6 + 19i - 10 = -16 + 19i$

(c) i^{33}.

Solution: $i^{33} = i^{32+1} = i^{4 \cdot 8 + 1} = i^1 = i$

(d) $\dfrac{-2 + 5i}{3 - 2i}$.

Solution: $\dfrac{-2 + 5i}{3 - 2i} = \dfrac{-2 + 5i}{3 - 2i} \cdot \dfrac{3 + 2i}{3 + 2i} = \dfrac{(-2 + 5i)(3 + 2i)}{3^2 + 2^2} = \dfrac{-6 - 4i + 15i - 10}{13}$

$= \dfrac{-16 + 11i}{13} = -\frac{16}{13} + \frac{11}{13}i$

7. $f(x) = -\frac{2}{3}x^2 + 4x - 4$.

(a) By completing the square write $f(x)$ in the form $a(x - h)^2 + k$.

Solution: $f(x) = -\frac{2}{3}x^2 + 4x - 4 = -\frac{2}{3}(x^2 - 6x) - 4 = -\frac{2}{3}(x^2 - 2 \cdot 3x + 9 - 9) - 4$

$= -\frac{2}{3}(x^2 - 2 \cdot 3x + 9) - \frac{2}{3}(-9) - 4 = -\frac{2}{3}(x^2 - 2 \cdot 3x + 9) + 6 - 4$

$f(x) = -\frac{2}{3}(x - 3)^2 + 2$

(b) Find the vertex and the axis of symmetry of $f(x)$.

Solution: The vertex is $(3, 2)$. The axis of symmetry is $x = 3$.

(c) Find the interval on which $f(x)$ is increasing and the interval on which it is decreasing.

Solution: The interval of increasing is $(-\infty, 3)$. The interval of decreasing is $(3, \infty)$.

(d) Sketch the graph of the function $f(x)$. Mark the vertex and draw the axis of symmetry.

Solution:

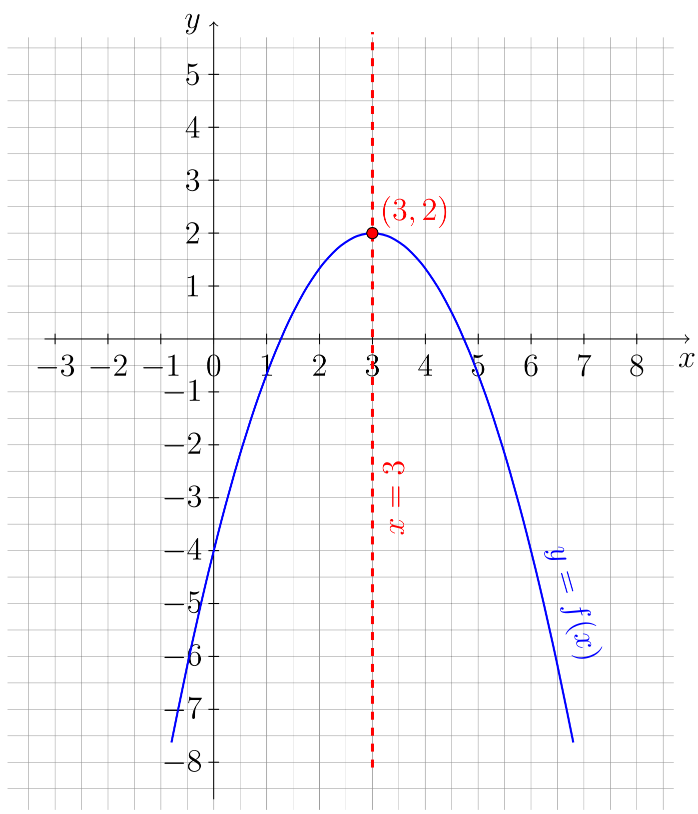

Figure 1: Graph of the function $f(x) = -\frac{2}{3}x^2 + 4x - 4$.

8. Solve the equation $\dfrac{2x}{x+3} - \dfrac{1}{x+1} = 0$.

Solution: $\dfrac{2x}{x+3} - \dfrac{1}{x+1} = 0 \iff \dfrac{2x(x+1)-(x+3)}{(x+3)(x+1)} = 0 \iff \dfrac{2x^2+x-3}{(x+3)(x+1)} = 0$

$\iff 2x^2+x-3=0, \ x \neq -3, \ x \neq -1$

$\iff x = \dfrac{-1 \pm \sqrt{1+24}}{4}, \ x \neq -3, \ x \neq -1$

$\iff x = \dfrac{-1 \pm 5}{4}, \ x \neq -3, \ x \neq -1$

$\iff x = -\frac{3}{2}$ or $x = 1, \ x \neq -3, \ x \neq -1$

Answer: $-\frac{3}{2}$ and 1.

9. Solve the equation $|3x+1| - 4 = -1$.

Solution: $|3x+1| - 4 = -1 \iff |3x+1| = 3 \iff 3x+1 = -3$ or $3x+1 = 3$

$\iff 3x = -4$ or $3x = 2 \iff x = -\frac{4}{3}$ or $x = \frac{2}{3}$

Answer: $-\frac{4}{3}$ and $\frac{2}{3}$.

10. (a) Solve the inequality $|x-4| > 1$ and write interval notation for solution set.

Solution: $|x-4| > 1 \iff x-4 < -1$ or $x-4 > 1 \iff x < 3$ or $x > 5$.
The solution set is $(-\infty, 3) \cup (5, \infty)$.

(b) Graph the solution set.

Solution: