Math 0031 **Midterm Exam 1**

Spring 2017 **S o l u t i o n s**

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1. Consider the relation $\{(2,3),(0,4),(3,-2),(2,0),(5,1)\}$.

(a) Determine whether the relation is a function.

Solution: No, it is not a function because ordered pairs $(2,3)$ and $(2,0)$ have the same first coordinate and different second coordinates.

(b) Identify the domain and the range.

Solution: The domain is the set of all first coordinates: $\{0,2,3,5\}$.

The range is the set of all second coordinates: $\{-2,0,1,3,4\}$.

2. Find a slope-intercept equation of the line that passes through the point $(-2,1)$ and is perpendicular to the line $y=-\frac{1}{3}x+2$.

Solution: The slope of the line $y=-\frac{1}{3}x+2$ is $m=-\frac{1}{3}$.

The slope m_2 of the line perpendicular to it is negative reciprocal to $m \quad \Rightarrow \quad m_2=3$.

The point-slope equation of the line is $y-1=3(x-(-2))$.

Then $y-1=3x+6$, and the slope-intercept equation is $y=3x+7$.

3. Solve the compound inequality $x-11\le 1-2x$ or $5<9-2x$.

Solution: $x-11\le 1-2x \quad \Leftrightarrow \quad 3x\le 12 \quad \Leftrightarrow \quad x\le 4$. The solution set is $(-\infty,4]$.

$5<9-2x \quad \Leftrightarrow \quad 2x+5<9 \quad \Leftrightarrow \quad 2x<4 \quad \Leftrightarrow \quad x<2$. The solution set is $(-\infty,2)$.

The solution set of the compound inequality is the union of sets found above.

It is $(-\infty,4]$.

4. The width of the soccer field recommended for players under the age 12 is 35 yd less than the length. The perimeter of the field is 330 yd. Find the dimensions of the field.

Solution: Let x be the width of the field, in yards. Then its length is $x + 35$ yd.

The formula for the perimeter gives $x + (x + 35) + x + (x + 35) = 330$.

$\Leftrightarrow \quad 4x + 70 = 330 \quad \Leftrightarrow \quad 4x = 260 \quad \Leftrightarrow \quad x = 65$ and $x + 35 = 100$.

Answer: The width of the field is 65 yards and its length is 100 yards.

5. For the function $f(x) = x^2 - x$ find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$.

Solution: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{((x+h)^2 - (x+h)) - (x^2 - x)}{h}$

$= \dfrac{x^2 + 2xh + h^2 - x - h - x^2 + x}{h} = \dfrac{2xh + h^2 - h}{h} = 2x - 1 + h.$

6. Simplify. Write answers in the form $a + bi$ where a and b are real numbers.

(a) $(-3 + i) - (4 - 2i)$.

Solution: $(-3 + i) - (4 - 2i) = -3 + i - 4 + 2i = -7 + 3i$

(b) $(2 + 3i)(1 - 2i)$.

Solution: $(2 + 3i)(1 - 2i) = 2 - 4i + 3i - 6i^2 = 2 - i + 6 = 8 - i$

(c) i^{55}.

Solution: $i^{55} = i^{52+3} = i^{4 \cdot 13 + 3} = i^3 = -i$

(d) $\dfrac{2 + 3i}{1 + 2i}$.

Solution: $\dfrac{2 + 3i}{1 + 2i} = \dfrac{2 + 3i}{1 + 2i} \cdot \dfrac{1 - 2i}{1 - 2i} = \dfrac{(2 + 3i)(1 - 2i)}{1^2 + 2^2} = \dfrac{2 - 4i + 3i + 6}{5}$

$= \dfrac{8 - i}{5} = \frac{8}{5} - \frac{1}{5}i$

7. The purpose of this problem is to draw the graph of the function $f(x) = \frac{1}{3}x^2 - 2x + 4$ by transforming the graph of the function $g(x) = x^2$.

(a) By completing the square write $f(x)$ in the form $a(x-h)^2 + k$.

Solution: $$f(x) = \tfrac{1}{3}x^2 - 2x + 4 = \tfrac{1}{3}(x^2 - 6x + 9 - 9) + 4 = \tfrac{1}{3}(x^2 - 2\cdot 3x + 9) - 3 + 4$$
$$= \tfrac{1}{3}(x^2 - 2\cdot 3x + 9) + 1$$

Alternatively, $$f(x) = \tfrac{1}{3}x^2 - 2x + 4 = \tfrac{1}{3}(x^2 - 6x + 12) = \tfrac{1}{3}(x^2 - 2\cdot 3x + 9 + 3)$$
$$= \tfrac{1}{3}(x^2 - 2\cdot 3x + 9) + 1$$

Therefore, $f(x) = \frac{1}{3}(x-3)^2 + 1$.

(b) The graph of $g(x) = x^2$ is given. Do step by step transformations of $g(x)$ to draw the graph of $f(x)$. On the second figure draw the graph of the function ax^2, on the third draw $a(x-h)^2$, and finally on the last plot draw the function $f(x) = a(x-h)^2 + k$.

Solution:

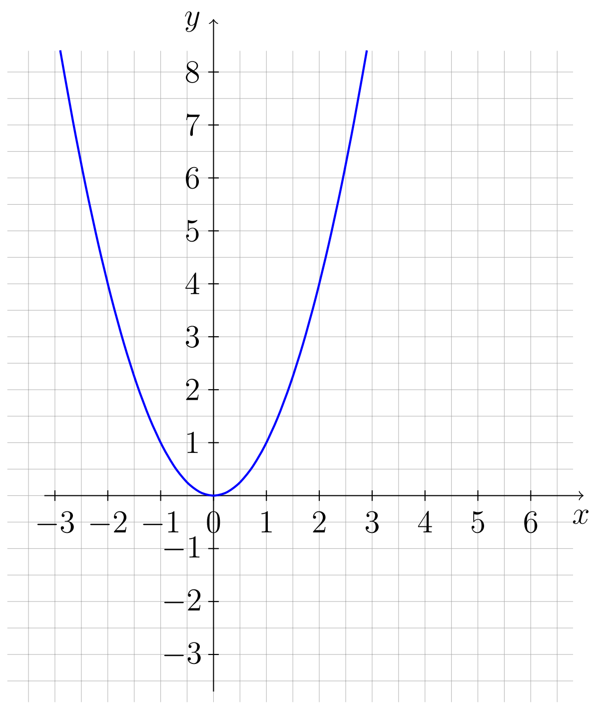

Figure 1: Graph of the function $g(x) = x^2$.

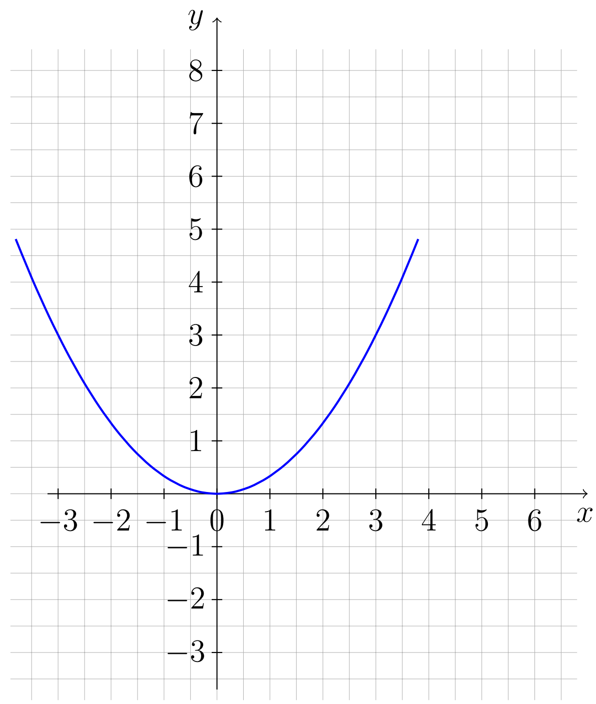

Figure 2: Graph of the function $ax^2 = \frac{1}{3}x^2$.

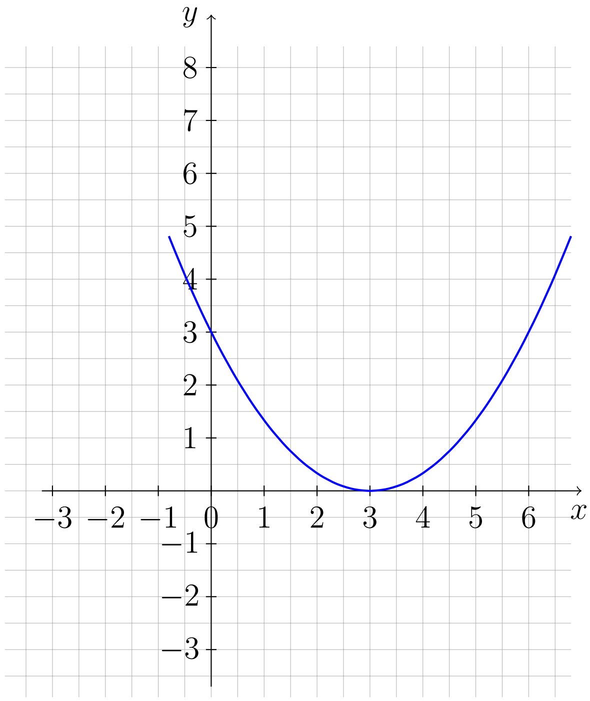

Figure 3: Graph of $a(x-h)^2 = \frac{1}{3}(x-3)^2$.

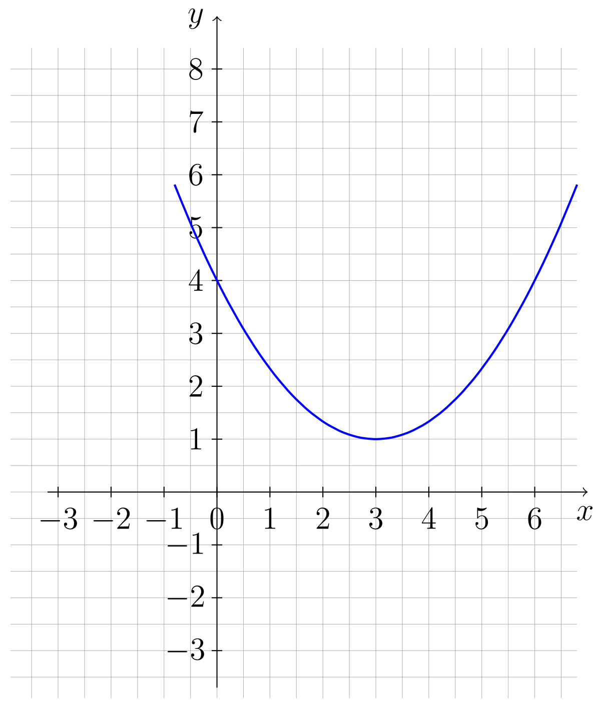

Figure 4: Graph of the function $f(x)$.

8. Solve the equation $\sqrt{x+3} - 1 = x$.

Solution: $\sqrt{x+3} - 1 = x \quad \Leftrightarrow \quad \sqrt{x+3} = x+1 \quad \Rightarrow \quad x+3 = x^2+2x+1$
$\Leftrightarrow \quad x^2 + x - 2 = 0$

The last equation has two solutions $x = -2$ and $x = 1$.

Check: $x = -2 \qquad \sqrt{-2+3} - 1 = 1 - 1 = 0 \neq -2 \quad \Rightarrow \quad x = -2$ is not a solution.

$x = 1 \qquad \sqrt{1+3} - 1 = 2 - 1 = 1 \quad \Rightarrow \quad x = 1$ is a solution.

Answer: $x = 1$.

9. Solve the equation $|x+3| - 2 = 6$.

Solution: $|x+3| - 2 = 6 \quad \Leftrightarrow \quad |x+3| = 8 \quad \Leftrightarrow \quad x+3 = -8$ or $x+3 = 8$
$\Leftrightarrow \quad x = -11$ or $x = 5$

Answer: -11 and 5.

10. (a) Solve the inequality $|2x-3|<5$ and write interval notation for solution set.

Solution: $|2x-3|<5 \;\Leftrightarrow\; 2x-3>-5$ and $2x-3<5$

$\Leftrightarrow\; 2x>-2$ and $2x<8 \;\Leftrightarrow\; x>-1$ and $x<4$.

The solution set is $(-1,4)$.

(b) Graph the solution set.

Solution: