Math 0031 **Midterm Exam 2**

Spring 2017 **S o l u t i o n s**

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1. Suppose that a polynomial function of degree 5 with rational coefficients has the numbers $-2,\ 3+2i$, and $1-\sqrt{6}$ as its zeros. Find the other zero(s).

   Solution: Since the coefficients are rational, the conjugates of $3+2i$ and $1-\sqrt{6}$ are also zeros. These conjugates are $3-2i$ and $1+\sqrt{6}$.

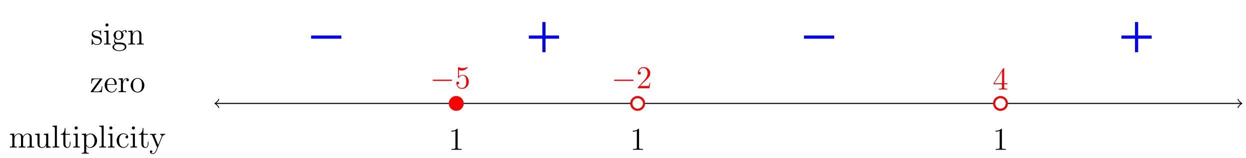
   There are only two other zeros because the polynomial function of degree 5 cannot have more than five zeros. (Note: If you did not explain why there are only two other zeros then your answer is not complete and your credit cannot be full.)

   Answer: $3-2i$ and $1+\sqrt{6}$.

2. Solve the inequality $\dfrac{3}{x-4} \leq \dfrac{1}{x+2}$.

   Solution: $\dfrac{3}{x-4} \leq \dfrac{1}{x+2} \iff \dfrac{3}{x-4} - \dfrac{1}{x+2} \leq 0 \iff \dfrac{3(x+2)-1(x-4)}{(x-4)(x+2)} \leq 0$

   $\iff \dfrac{3x+6-x+4}{(x-4)(x+2)} \leq 0 \iff \dfrac{2x+10}{(x-4)(x+2)} \leq 0 \iff \dfrac{2(x+5)}{(x-4)(x+2)} \leq 0$



   Answer: $x \in (-\infty, -5] \cup (-2, 4)$.

3. Consider the function $f(x) = \dfrac{x^2-x-2}{x^2-4}$.

   (a) Determine the domain of the function.

Solution: This is a rational function. Therefore it is defined everywhere except points where the denominator is zero: $x^2 - 4 = 0 \quad \Leftrightarrow \quad (x+2)(x-2) = 0 \quad \Leftrightarrow$ $x = -2,\ x = 2$

The domain of the function is $D = (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$.

(b) Determine the vertical asymptotes of the graph of the function, if any.

Solution: We factor numerator and denominator and try to simplify the function:

$$f(x) = \frac{(x+1)(x-2)}{(x+2)(x-2)} = \frac{x+1}{x+2}.$$

The graph of the function has vertical asymptotes when the denominator is zero, i.e. $x + 2 = 0$. Therefore $x = -2$ is the vertical asymptote.

(c) Determine the horizontal asymptotes of the graph of the function, if any.

Solution: After simplification $f(x) = \dfrac{x+1}{x+2}$.

The degree of the numerator is the same as the degree of the denominator. The ratio of leading coefficients is $\frac{1}{1} = 1$. Therefore $y = 1$ is the horizontal asymptote.

(d) Determine the oblique asymptotes of the graph of the function, if any.

Solution: The degree of the numerator is the same as the degree of the denominator. Therefore the graph of $f(x)$ has no oblique asymptotes.

(e) Sketch the graph of the function $f(x)$. Draw all asymptotes. Mark the asymptotes and points outside the domain, if any.

Solution: Note that the point $(2, 0.75)$ has to be removed from the graph of $f(x)$ since 2 is not in the domain.

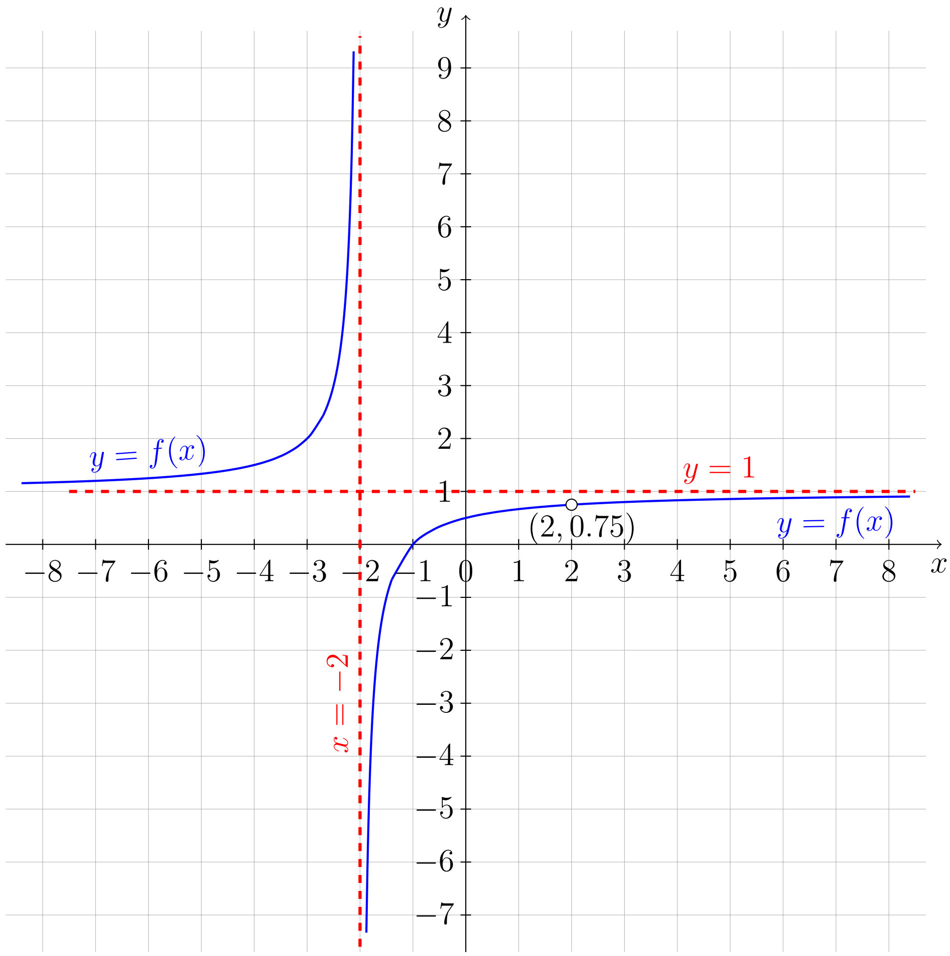
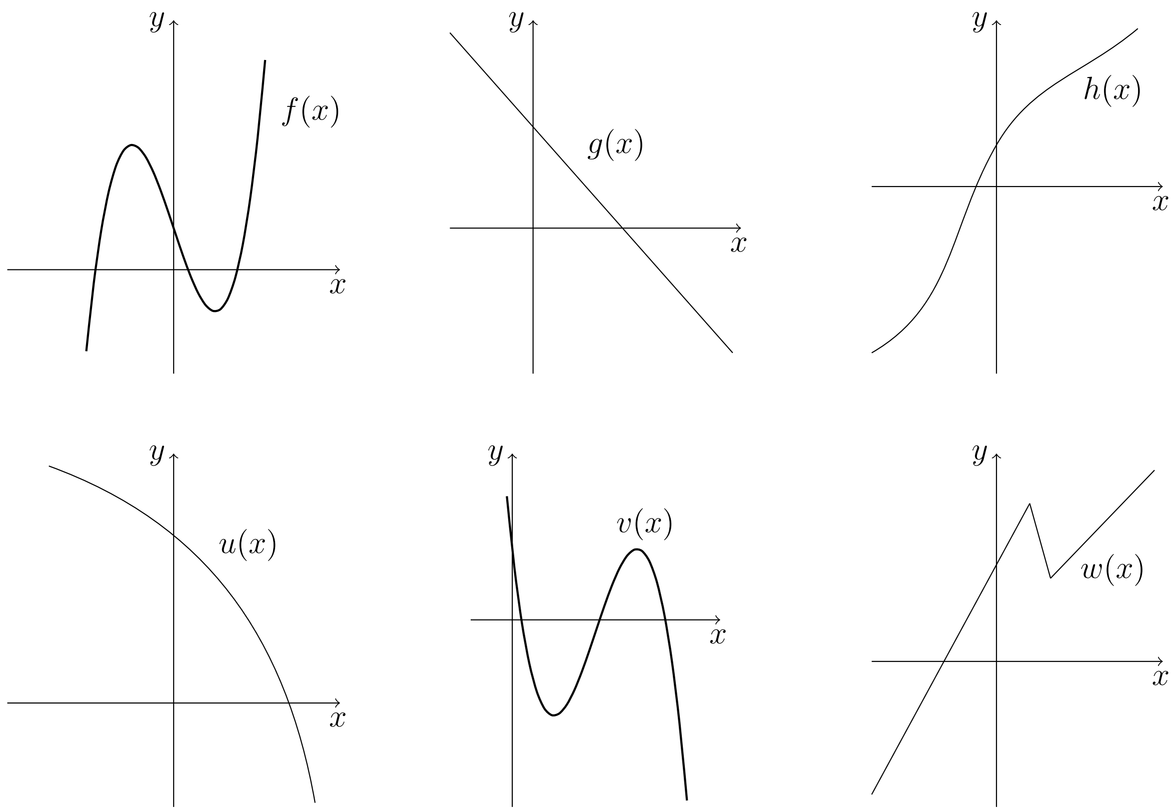


Figure 1: Graph of the function $f(x)$.

4. Graph of the functions $f(x)$, $g(x)$, $h(x)$, $u(x)$, $v(x)$ and $w(x)$ are given



Determine which functions are one-to-one and mark them below:

○ $f(x)$ ○ $g(x)$ ○ $h(x)$

○ $u(x)$ ○ $v(x)$ ○ $w(x)$

Solution: One-to-one functions pass the horizontal line test.
They are $g(x)$, $h(x)$ and $u(x)$.

5. The function $f(x) = \left(\frac{x-1}{2}\right)^3$ is one-to-one. Find its inverse.

Solution: 1. $y = \left(\frac{x-1}{2}\right)^3$ 2. $x = \left(\frac{y-1}{2}\right)^3$

3. $x = \left(\frac{y-1}{2}\right)^3$, $\sqrt[3]{x} = \frac{y-1}{2}$, $2\sqrt[3]{x} = y - 1$, $y = 2\sqrt[3]{x} + 1$.

4. $f^{-1}(x) = 2\sqrt[3]{x} + 1$.

6. Find

(a) $\log 0.001$ *Solution:* $\log 0.001 = \log 10^{-3} = -3.$

(b) $\log_8 2$ *Solution:* $\log_8 2 = \log_8 8^{1/3} = \frac{1}{3}.$

(c) $\log_5 \sqrt[4]{5^3}$ *Solution:* $\log_5 \sqrt[4]{5^3} = \log_5 5^{3/4} = \frac{3}{4}.$

(d) $\dfrac{\log_3 8}{\log_3 2}$ *Solution:* Using the Change-of-Base formula:

$$\frac{\log_3 8}{\log_3 2} = \log_2 8 = \log_2 2^3 = 3.$$

7. Simplify

(a) $\log_2(16 \cdot 32)$ *Solution:* $\log_2(16 \cdot 32) = \log_2 16 + \log_2 32 = 4 + 5 = 9.$

(b) $\ln \frac{a^3}{b} - \ln \frac{a^2}{b^2}$ *Solution:* $\ln \frac{a^3}{b} - \ln \frac{a^2}{b^2} = \ln \frac{a^3}{b} \div \frac{a^2}{b^2} = \ln \frac{a^3}{b} \cdot \frac{b^2}{a^2}$

$= \ln ab = \ln a + \ln b.$

(c) $6 \log x^{3/2} \sqrt[3]{y^5}$ *Solution:* $6 \log x^{3/2} \sqrt[3]{y^5} = 6 \log x^{3/2} + 6 \log y^{5/3}$

$= 6 \cdot \frac{3}{2} \log x + 6 \cdot \frac{5}{3} \log y = 9 \log x + 10 \log y.$

8. Solve equations

(a) $4^{3x-7} = 16$ *Solution:* $4^{3x-7} = 16 \quad \Leftrightarrow \quad 4^{3x-7} = 4^2 \quad \Leftrightarrow \quad 3x - 7 = 2$

$\Leftrightarrow \quad 3x = 9 \quad \Leftrightarrow \quad x = 3.$

(b) $\log_3(x + 16) - \log_3 x = 2$

Solution: $\log_3(x + 16) - \log_3 x = 2 \quad \Rightarrow \quad \log_3 \dfrac{x + 16}{x} = \log_3 9 \quad \Leftrightarrow \quad \dfrac{x + 16}{x} = 9$

$x + 16 = 9x \quad \Leftrightarrow \quad 8x = 16 \quad \Leftrightarrow \quad x = 2$ is a possible solution.

Domain: $x + 16 > 0$ and $x > 0 \quad \Leftrightarrow \quad x \in (0, \infty).$

Answer: $x = 2.$

(c) $\ln(x+10) - \ln x = \ln 6$

Solution: $\ln(x+10) - \ln x = \ln 6 \quad \Rightarrow \quad \ln \dfrac{x+10}{x} = \ln 6 \quad \Leftrightarrow \quad \dfrac{x+10}{x} = 6$

$\Leftrightarrow \quad x + 10 = 6x \quad \Leftrightarrow \quad 5x = 10 \quad \Leftrightarrow \quad x = 2$ is a possible solution.

Domain: $x + 10 > 0$ and $x > 0 \quad \Leftrightarrow \quad x \in (0, \infty)$.

Answer: $x = 2$.