Exam 2

Your name:

100 points total

Your TA's name:

1. Find the derivative of the function by any appropriate method

(a) [10 points] $f(x) = \ln \dfrac{e^x - 1}{e^x + 1}$

(b) [10 points] $y = x^{\ln x}$

2. [15 points] Find 2 numbers whose sum is 20 and whose product is as large as possible. For a proof use the optimization method.

3. [15 points] Find $f(x)$ if $f'(x) = x\left(3x - \dfrac{2}{x^2}\right)$ and $f(1) = 5$.

4. [15 points] What, approximately, is $\sqrt{9.01}$? Use linear approximation to find the answer.

5. [20 points] Suppose that the population of a colony of bacteria increases exponentially. At the start of an experiment, there are 400 bacteria, and one hour later, the population has increased to 600. How long will it take for the population to reach 900?

6. [15 points] Find the absolute maximum and the absolute minimum values of $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ on the interval $[-2; 1]$.

Bonus problem. [10 points extra] Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root.