Final Exam

Math 0220 (evening)   **Fall 2010**

100 points total   **Student's name:**

1. Find the limit, if it exists. If the limit does not exist explain why

(a) [7 points] $\displaystyle\lim_{x \to \left(\frac{\pi}{2}\right)^+} 2^{\sec x}$

(b) [8 points] $\displaystyle\lim_{x\to-1^-}\frac{3x+3}{5|x+1|}$

2. [10 points] A boy and a girl start from the same point. The boy walks east at 2 meters per second. The girl walks south at 1.5 m/sec. At what rate is the distance between the boy and girl changing after 40 seconds? Simplify your answer. Do not use radicals or irrational expressions in it.

3. [10 points] The graph of $f'(x)$ is given. Assuming that $f(0) = 2$ find the values $f(2)$, $f(6)$, $f(8)$, $f(10)$, and $f(12)$. (The part of the graph between 2 and 6 is a top half of a circumference.)
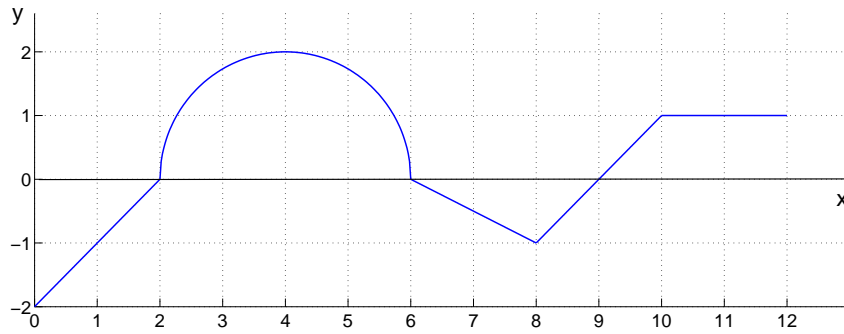
4. [10 points] Use the Midpoint Rule with $n = 5$ to approximate the integral $\int_{-1/2}^{9/2} x\sqrt{x+1}\,dx$.

Leave the answer in its exact form (with radicals).

5. [10 points] Find the average value of the function $f(x) = \sec^3 x \tan x$ on the interval $[0, \pi/3]$.

6. Evaluate the integrals

(a) [7 points] $\int 5\,t^4\,\ln t\,dt$

(b) [8 points] $\int 3\sin^5 x \, dx$

7. [10 points] Find the derivative of the function by any appropriate method $f(x) = x^{\tan^{-1} x}$

8. [10 points] Find equations of tangent and normal lines to the curve $y = x^{\frac{1}{2x}}$ at $x = 1$.

9. [10 points] Find the absolute maximum and absolute minimum of the function $f(x) = x - \sin 2x$ on the interval $[0, \pi/2]$. Use the fact that $\dfrac{\pi}{6} < \dfrac{\sqrt{3}}{2}$.

Bonus problem. [10 points extra] Evaluate the limit $\displaystyle\lim_{x\to 0^+} (\cos x)^{\frac{1}{\sin x}}$.