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Quiz 2 **Your name:** Sample

Your TA's name:

1. Find the limit, if it exists. If the limit does not exist explain why: $\lim_{x\to-2^-}\left(\frac{5x+10}{2|x+2|}+x\right)$.

$$\frac{5x+10}{2|x+2|} = \frac{5(x+2)}{2|x+2|} = \begin{cases} \frac{5(x+2)}{2(x+2)}, & x+2>0 \\ \frac{5(x+2)}{-2(x+2)}, & x+2<0 \end{cases} =$$

$$= \begin{cases} \frac{5}{2}, & x>-2 \\ -\frac{5}{2}, & x<-2 \end{cases}$$

$$x\to -2^- \iff \begin{matrix} x\to 2 \\ x<-2 \end{matrix}$$

Hence $\lim_{x\to-2^-}\left(\frac{5x+10}{2|x+2|}+x\right) =$

$$= \lim_{x\to-2^-}\left(-\frac{5}{2}+x\right) = -\frac{5}{2}-2 = -4\frac{1}{2}$$

$$= -4.5$$

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2. Use the Intermediate Value Theorem to show that there is a root of the equation $x \sin x = 1$ in the interval $\left(0, \frac{\pi}{2}\right)$.

$x \sin x = 1$

Let $f(x) = x \sin x$, continuous

Then $f(0) = 0$, $f(\frac{\pi}{2}) = \frac{\pi}{2} \cdot 1 = \frac{\pi}{2}$

$f(x)$ is continuous on $[0, \frac{\pi}{2}]$, and

$f(0) < 1 < f(\frac{\pi}{2})$.

Take $N = 1$ and apply the IVT.

Then there must be c such that

$0 < c < \frac{\pi}{2}$ and $f(c) = 1$.

The last means that c is a root

of the equation $x \sin x = 1$ in $(0, \frac{\pi}{2})$.

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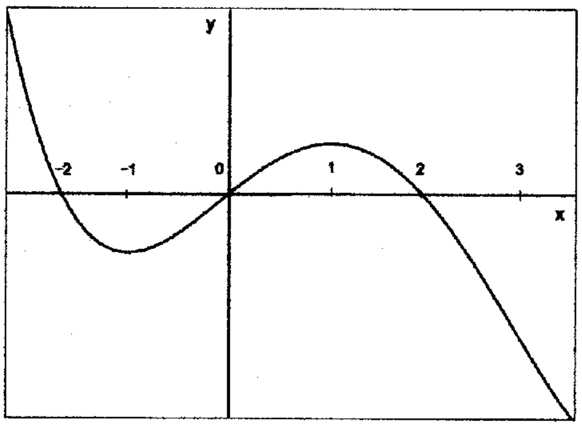
3. For the function $f(x)$ whose graph is given, arrange the following numbers in the increasing order and explain your reasoning

5, $f'(-2)$, $f'(0)$, $f'(1)$, $f'(1.5)$.



All the given derivatives are slopes of the curve in the corresponding points.

$f'(-2) \approx -1.5$, $f'(0) \approx 0.8$, $f'(1) = 0$

$f'(1.5) \approx -0.8$

Increasing sequence of numbers:

$f'(-2)$, $f'(1.5)$, $f'(1)$, $f'(0)$, 5

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4. Differentiate the function $g(t) = \dfrac{t^3 + 2t - 5}{\sqrt{t}}$.

State the domain of the function and the domain of its derivative.

(You were right in the class you do not have to use the definition. This is a problem from 2.3)

$$g(t) = t^{5/2} + 2t^{\frac{1}{2}} - 5t^{-\frac{1}{2}}$$

The power rule gives

$$g'(t) = \frac{5}{2} t^{3/2} + 2 \cdot \frac{1}{2} t^{-\frac{1}{2}} - 5\left(-\frac{1}{2}\right) t^{-\frac{3}{2}}$$

or

$$g'(t) = \frac{5}{2} t^{3/2} + \frac{1}{\sqrt{t}} + \frac{5}{2t\sqrt{t}}$$