e

Quiz 2 **Your name:** Sample

Your TA's name:

1. Find the limit, if it exists. If the limit does not exist explain why: $\lim_{x \to -1} \frac{3|x+1|}{4x+4}$.

$$\frac{3|x+1|}{4x+4} = \begin{cases} \frac{3(x+1)}{4(x+1)}, & x+1>0 \\ \frac{-3(x+1)}{4(x+1)}, & x+1<0 \end{cases} =$$

$$= \begin{cases} \frac{3}{4}, & x>-1 \\ -\frac{3}{4}, & x<-1 \end{cases}$$

$$\lim_{x \to -1^-} \frac{3|x+1|}{4x+4} = -\frac{3}{4} \neq$$

$$\neq \lim_{x \to -1^+} \frac{3|x+1|}{4x+4} = \frac{3}{4}$$

Hence $\lim_{x \to -1} \frac{3|x+1|}{4x+4}$ DNE.

e

2. Use the Intermediate Value Theorem to show that there is a root of the equation $\sqrt[3]{x} - 1 = -2x$ in the interval $(0, 1)$.

$\sqrt[3]{X} - 1 = -2X \iff \sqrt[3]{X} + 2X - 1 = 0$

Let $f(x) = \sqrt[3]{X} + 2X - 1$.

$f(0) = -1$, $f(1) = 1 + 2 - 1 = 2$

We can apply IVT since $f(x)$ is continuous on $[0,1]$. Take $N = 0$. Then $f(0) = -1 < 0 < 2 = f(1)$.

By IVT there is $c \in (0,1)$ such that $f(c) = N = 0$.

This c is a root of the given equation in $(0,1)$.

e

3. Find an equation of the tangent line to the curve $y = 2 + \sqrt{x}$ at the point $(1, 3)$. (Find a slope of the tangent line using its definition, that involves limit).

$$m = \lim_{h \to 0} \frac{2 + \sqrt{x+h} - 2 - \sqrt{x}}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{x+h-x}{h(\sqrt{x+h} + x)} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} =$$

$$= \boxed{\frac{1}{2\sqrt{x}}}$$

e

4. Find the derivative of the function $f(x) = \frac{x}{x-1}$ using the definition of a derivative.

State the domain of the function and the domain of its derivative.

$$f'(x) = \lim_{h \to 0} \frac{\frac{x+h}{x+h-1} - \frac{x}{x-1}}{h}$$

$$\frac{x+h}{x+h-1} - \frac{x}{x-1} = \frac{(x+h)(x-1) - x((x+h)-1)}{(x+h-1)(x-1)} =$$

$$= \frac{\cancel{(x+h)x} - (x+h) \cancel{- x(x+h)} + x}{(x+h-1)(x-1)} = \frac{-x+h+x}{(x+h-1)(x-1)}$$

Hence $f'(x) = \lim_{h \to 0} \frac{h}{h(x+h-1)(x-1)} =$

$$= \lim_{h \to 0} \frac{1}{(x+h-1)(x-1)} = \boxed{\frac{1}{(x-1)^2}}$$

Domain of $f(x)$ is $x \neq 1$ or $x \in (-\infty, 1) \cup (1, \infty)$

Domain of $f'(x)$ is $x \in (-\infty, 1) \cup (1, \infty)$.